HAND-BOOK

OF

PROFESSIONAL INSTRUCTIONS

FOR THE

TRIGONOMETRICAL BRANCH

Survey of India Department.

PREPARED BY

COLONEL G. STRAHAN, R.E.,

DEPUTY SURVEYOR GENERAL, TRIGONOMETRICAL BRANCH.

UNDER THE DIRECTION OF

COLONEL H. R. THUILLIER, R.E.,

SURVEYOR GENERAL OF INDIA.

PRINTED AT THE TRIGONOMETRICAL BRANCH OFFICE, DEHRA DUN.

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PREFACE.

The Handbook of Professional Instructions of the Trigonometrical Branch is divided into nine parts; the first eight of these are devoted to the details of the principal operations of a geodetic survey as carried out in India and the ninth deals with various miscellaneous matters. The nine parts are as follows:—

Part	I.	The Measurement of Base-lines.
,,	II.	Triangulation—Principal and Secondary.
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,,	V.	Astronomical Latitudes.
,,	VI.	Tidal Observations.
,,	VII.	Levelling Operations.
,,	VIII.	Pendulums.
,,	IX.	Miscellaneous.

Of these the first and eighth subjects being too extensive for treatment in a handbook, are very briefly referred to; little more being given than references to such standard works as are regarded as the best authorities on these operations.

Part II is a compilation of the various instructions on triangulation issued from time to time for the guidance of the officers of the Trigonometrical Branch brought up to date; and it embodies in a form easy for reference the results of their experience since the commencement of the Trigonometrical Survey. In arranging this part of the Handbook, and especially in the chapter that treats of the computations, I am much indebted to Mr. Cole (in charge of the Computing Office) for his assistance.

Part III describing the observations and computations connected with astronomical azimuths is almost entirely due to the same officer.

For Part IV treating of Electro-Telegraphic Longitudes, I am myself solely responsible.

Part V has been contributed by Captain Burrard, R.E., and Parts VI and VII have been compiled by Mr. Eccles, the former from "A Manual for Tidal Observations" by Major Baird, R.E., and Professor G. H. Darwin's Report to the British Association of 1883; and the latter from Colonel Walker's "Memoranda on Levelling Operations".

These last three Parts necessarily describe the operations as carried on with the particular equipments now in use, and therefore are not of universal application; but it is hoped that the general principles involved have been so far explained, that any reasonably intelligent officer could, with instruments of a different design, adapt his procedure accordingly.

Part IX is in a great measure a repetition of Chapter III of the Handbook of the Topographical Branch; but as that work may not necessarily be in the possession of officers of the Trigonometrical Branch, it has been considered advisable to reprint some portion of it *mutatis mutandis* for insertion in this Handbook.

There are, and probably always will be, many differences of opinion about the relative advantages of various modes of conducting such operations as are here described, but the rules laid down have been found by experience to be the best adapted to the requirements of Indian Geodesy, and should therefore not be lightly departed from by officers of the Survey Department.

DEHRA DUN: *July, 1891.*

G. STRAHAN, COLONEL, R.E.,
Deputy Surveyor General,
In charge Trigonometrical Branch,
Survey of India.

While this Handbook was in the press, a new Greenwich Ten-year Catalogue was published, giving the places of 4059 stars in Right Ascension and Declination on 1st January 1880. For this Catalogue the quantities e, f, g, h, l, and e′, f′, g′, h′, and l′, have not been computed, and moreover Airy's Day Numbers are not given in the Nautical Almanac for 1891 and 1892; and presumably will not be given for subsequent dates. The method of computing star places as explained on pages 74, 106, and 178, and exemplified in page 2 of Form P. 32 for the Azimuth Computations, and in Form No. 4 of the Electro-Telegraphic computations, is therefore no longer applicable. The reduction must be made in future by means of the "Quantities for Correcting the Places of Stars" given for every day of the year in the Nautical Almanac under the heading "Apparent Places of Stars", the use of which is rendered sufficiently clear by an example worked out in the "Explanation" at the end of the Nautical Almanac. Professional Forms 32, 36 and 37 have been revised so as to suit the above mentioned alteration.

SURVEY OF INDIA DEPARTMENT.

HANDBOOK OF THE TRIGONOMETRICAL BRANCH.

PART I.

The Measurement of Base-Lines.

1. There are few scientific operations on which care and ingenuity have been more lavishly expended than on the measurement of Base-lines, though it may at first sight appear to be but a simple matter to measure the distance between two fixed points. The necessity for extreme accuracy arises from the fact that in extending a chain of triangles from a measured base, any small error in the latter is increased in the former in the exact proportion by which the length of the chain differs from that of the base-line. Many methods have been tried at various times, but one of the most trustworthy and the only one likely to be adopted in the immediate future in the Trigonometrical Branch, is the measurement by the Colby compensation bars and microscopes. A brief history of the measurement of the old Indian Base-lines will be found in Chapter V of Volume I of the "Account of the Operations of the Great Trigonometrical Survey of India", to which the reader is referred.

2. A detailed description of the Colby apparatus would be too long for insertion in this Handbook, nor is it necessary, as it may be found in many standard works, *viz.*, in Captain Yolland's "Account of the Measurement of the Lough Foyle Base, 1847", in Colonel Everest's "Measurement of the Meridional Arc of India, 1847", in Captain Clarke's "Account of the Principal Triangulation of the Ordnance Survey, 1858", in Sir Thomas Maclear's "Verification and Extension of LaCaille's Arc of Meridian, 1866", and in the 3rd Volume of the "Course of Mathematics" formerly in use at the Royal Military Academy, Woolwich. Chapter VI of Volume I of the "Account of the Operations of the Great Trigonometrical Survey of India" also contains a brief description of the apparatus and the precautions to be observed in using it, including the all-important process of comparing the bars and microscopes with the 10-foot and 6-inch standards.

3. The measurement of a base is so important and so *special* an operation that it would always be entrusted to one or more of the most experienced officers of the Department, who would necessarily have at

hand for reference some of the extensive literature on the subject; it falls therefore somewhat beyond the scope of this Handbook which is intended for the guidance of young and inexperienced officers when called upon for the first time to superintend the ordinary operations of geodesy.

PART II.

Triangulation, Principal and Secondary.

CHAPTER I. PRINCIPAL TRIANGULATION.

Definition of Principal Triangulation.

1. The principal triangulation consists of a system of chains of triangles, conforming generally in direction with meridians or parallels of latitude, and tied together by other chains at their extremities. The values of side length are derived from base-lines measured in different parts of India, and the geodetic elements of latitude, longitude and azimuth are brought up through the triangulation from a station of origin, which has been chosen at Kaliánpur.* The latitude and azimuth were obtained there by astronomical observations, and the longitude was deduced through the triangulation from the Madras Observatory. The heights are referred to *mean sea level* as origin; for a definition of which term, *vide* Parts VI and VII of this Handbook.

Definition of Secondary Triangulation.

2. In the course of the execution of these chains, the positions of neighbouring places of note, and of prominent points, are established by triangulation of less precise character which is called "Secondary." The points so fixed are intended for the use of detail surveyors.

Direction of chains.

3. The direction in which a series of triangles should proceed depends on the object for which it is undertaken; but it is customary in India to avoid an oblique direction, and to conform strictly to a meridional or longitudinal line.

---

* *Vide* Chapter XI, Volume II of the "Account of the Operations &c."

4. In geodetic operations, the determination of the elements of the earth's figure being the object in view, both directions are of equal value; for it is only by a comparison of the positions of the stations as obtained from the triangulation with those resulting from suitable astronomical observations that these elements can be arrived at. The meridional chains suffice in conjunction with astronomical determinations of latitude to determine the figure of the meridian, and the longitudinal chains combined with observations of differences of longitude (effected in the present day with extreme accuracy by means of the Electric Telegraph, *vide* Part IV of this Handbook) lead to a knowledge of the magnitude and form of the Equator, and of the circles of latitude parallel to it.

Employment of chains of triangles in determining the figure of the earth.

5. The initial line from which the series starts (whose length will have been determined either by direct measurement, or deduced by triangulation from a measured base) being selected, the next operation will be to choose the stations in a judicious manner in the required direction. As the azimuth of the initial line is always known approximately from the previous triangulation, there is no difficulty in determining this direction. In starting, however, from a measured base it will be necessary, unless an azimuth has been observed in connection with it, to determine the direction of the meridian by any of the methods given at pages 78 to 81 of the 3rd edition of the "Auxiliary Tables to facilitate the Calculations of the Survey of India."

Determination of the direction.

6. A Plane-table reconnaissance, where possible, will be found of great assistance in laying out the stations. The initial base should be plotted on the board on any convenient scale (4 miles = 1 inch will probably be found suitable); and from either end of it, when the table has been properly set up and orientated either by compass, or by preference by the back ray to a previously fixed station, rays should be drawn to conspicuous points likely to prove suitable sites for stations in advance; from these again when visited the same process should be carried on, advantage being taken at the same time of all opportunities of fixing prominent hill peaks, trees, buildings &c., likely to be utilized in the secondary work. This reconnaissance or Plane-table triangulation is of great use in identifying points observed, as well as in selecting stations, and should on no

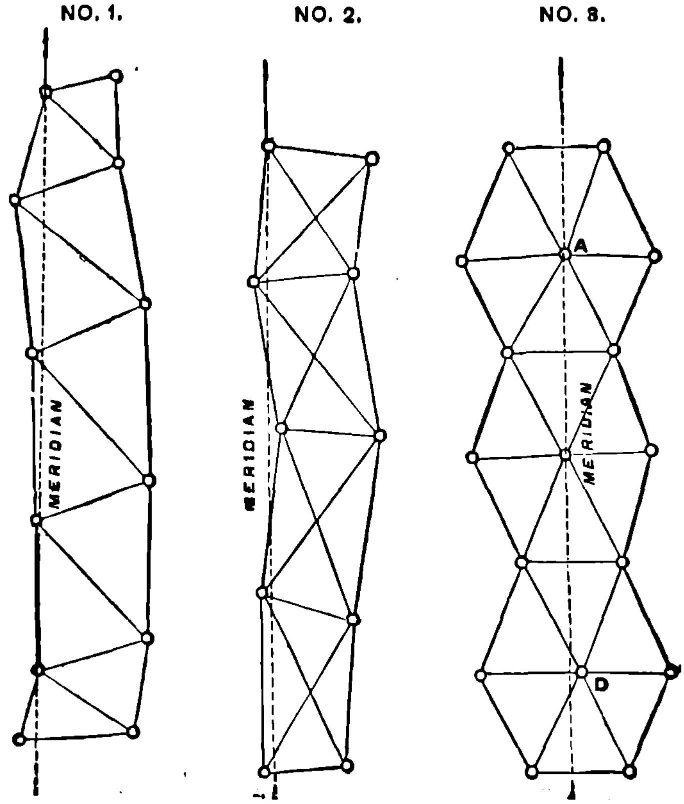
Use of the Plane-table in fixing stations.

account be neglected. It may even in skilful hands be employed in executing a more or less good preliminary map of the districts visited, should none exist. It is assumed that the reader is familiar with the use of the Plane-table: much useful information on this point will be found in the Handbook of the Topographical Branch. The operations of selecting and constructing stations are described at some length in Chapter II of Volume II of the "Account of the Operations of the Great Trigonometrical Survey of India" to which the reader is referred.

Varieties of figures composing the Series.

7. A series of principal triangles may be either single or double as represented below, wherein No. 1 is a single series. A double triangulation may be arranged in several ways; for example, No. 2 is a series of successive quadrilaterals, No. 3 is a series of successive polygons. In practice it frequently happens that these varieties of figure are combined together in various ways, according to the nature of the country, and the local facilities it affords; but as each method has its peculiar advantages and disadvantages, it is proper that their relative merits should be duly considered, in order that the most judicious selection may be made which circumstances admit of.

NO. 1. NO. 2. NO. 3.



8. The single series possesses advantages of economy of time, labor, and money, as well in the field operations as in the office computations; and it is, consequently, the most proper kind of series to be adopted when circumstances impose a strict attention to these restrictions. For instance, in a level country, like the plains of Hindustan, in which, from the absence of natural elevations, trigonometrical operations are both costly and tedious, every consideration of economy combines to recommend a single series as the most eligible arrangement. On the other hand, the advantage of accuracy will always be found greatly in favor of a double series, which, moreover, supplies a check of the most efficient kind at every stage of the work.

Relative advantages of different figures.

9. The relative advantages of quadrilaterals, polygons, and single figures, are fully entered into at pages 38, 39 and 200 of Volume II of the "Account of the Operations of the Great Trigonometrical Survey of India", a work which should be in the hands of all surveyors employed on triangulation. The conclusion there arrived at is that as a general rule a symmetrical hexagon is the best figure for triangulation in a flat country, and a symmetrical quadrilateral the best in a hilly country.

Continued.

10. In all principal series azimuths must be observed at intervals of about 1°. In order that these observed azimuths may be as free as possible from those errors which are caused by local attraction, it is desirable that no great mass of high mountains should stand either east or west of the station of observation. Local attraction acting in any direction except in the meridian will, by dislevelment of the theodolite, cause uncompensated errors in the observations for azimuth. As astronomical latitude is advantageously measured at the same stations as azimuth, it is always advisable to avoid local attraction in the direction of the meridian also, for its full effect comes in as error in the resulting latitude.

Local attraction.

11. In choosing stations it is extremely important to avoid intermediate obstacles situated on, or close to, the ray, because if the ray grazes the side of a mountain it will certainly be deflected by the vapours which arise from the sloping ground, whereby the observations will be more or less injuriously affected by lateral refraction; and if the ray grazes close over

Grazing rays.

the top of a mountain or over intervening ground or even over a village or huts, then the angles horizontal and vertical will be greatly disturbed. If circumstances, however, concur in admitting of no other choice, it will be desirable always to provide a check by introducing a polygon or quadrilateral at that part of the work, and a secondary station should also be established on the intervening hill, which may be treated as an auxiliary point, for taking vertical angles, whereby the comparative heights may be determined independently of direct observation.

Size of triangles.

12. A Principal Series should consist of triangles as large as the features of the country admit of without grazing rays, by which arrangement the number of stations, and probability of accumulation of error will be least. Considerations of economy likewise impose the same restriction, inasmuch as the additional points required for topographical purposes can be obtained at less cost by means of secondary triangles, in which accumulation of error is an evil that need not be dreaded, the limits of inaccuracy being sufficiently controlled by the principal triangulation. No general rule can be applied with respect to the magnitude of triangles, which must necessarily vary with the configuration of the surface of the country. In a mountainous region with gigantic features, (composed of lofty peaks and deep valleys) it would be as difficult as it would be injudicious to select small triangles; and in a flat country it would be absurd to struggle against natural difficulties in the vain endeavour to establish a series of great magnitude. In hilly countries, twenty to thirty miles is a convenient distance for principal stations, and such distances can usually be observed with facility, in all ordinary conditions of atmosphere, by means of heliotropes and Argand lamps. When hills are table-topped, as is generally the case where the stratification of the rocks is horizontal, it may be necessary to shorten the sides to less than twenty miles. An open country with detached peaks may frequently afford distances of thirty to forty miles with the fairest prospect of good observations: in a level country, such as Bengal, ten miles is the most favorable length of side, and the limits should be considered eight to fifteen miles.

Symmetry of triangles.

13. The triangles selected should be symmetrical, that is, as nearly equilateral as possible, such being the best condition for ensuring accuracy when all the angles are measured; but as it is impracticable to obtain in the field

triangles exactly equilateral, the rule in practice is, to admit of no angle under 30°, or above 90°. This rule applies to all the triangles which compose a series, whether principal or secondary; but, in the case of secondary triangles which do not form part of a long series, wider limits are admissible. These limits must also of necessity be extended in the case of a quadrilateral, because the four triangles of which it is composed obviously cannot be all equilateral. The best condition for that figure is when it approximates to the square, in which form no angle will be much above 90°, and none much less than 45°.

Proper sites for stations.

14. The stations selected should be on the highest peaks, but if these are inaccessible, it becomes necessary to adopt a lower point. Every effort should, of course, be made to reach the summit when practicable. In the case of a lower point being used, care must be taken that the view is clear in the direction of the stations in advance and that it is suitably situated for the use of topographical surveyors. This precaution has occasionally been neglected greatly to the inconvenience of detail surveyors wishing subsequently to utilize the stations for secondary work.

Construction of stations.

15. There has been at different times considerable variety in the structures built to mark the sites of the principal stations: experience has shown that one of the three classes here described will be suitable in almost all cases that can occur in Indian practice, but of course under exceptional circumstances modifications may be made where really required, but not merely at the caprice of executive officers.

First. Hill stations:—

Where the soil is composed of rock, a dot surrounded by a concentric circle is cut on the solid rock *in sitû*, otherwise a large stone similarly marked is buried in the ground. Over the mark a small *paka* platform or pillar is built to a convenient height, and of sufficient diameter to accommodate the theodolite stand: on the summit of this pillar another mark-stone is inserted, and fixed truly vertically over the lower one. Besides the upper mark-stone it is usual to imbed in the pillar three picked, flat, heavy stones, for the tripod of the instrument to stand upon. These are called "foot stones," and they should be duly levelled, which will save trouble afterwards in adjusting the instrument.

Surrounding this pillar and separated from it by an annular space, a platform is built, on which the observer and his assistants stand. The dimensions of this platform should be at least as large as the observatory tent, and it will generally be found convenient to make it somewhat larger—12 feet × 15 feet will prove a suitable size. The annular space round the central pillar is for the purpose of isolating the instrument, and preventing any shake caused by the observer's movements; it should be filled in with loose sand, otherwise screws or other parts of the apparatus may be lost by falling into it. The distance between the two mark-stones should be recorded, but all measurements and observations should be referred to the upper mark-stone, and are so recorded in the angle books. In alluvial country where stone is scarce, it will be better to mark the circle and dot on bricks, as flat dressed stone is likely to be stolen by the inhabitants of such districts for domestic purposes.

Secondly. Tower stations:—

These are generally resorted to in flat countries to enable the observer to surmount trees and other obstacles to the view, and to overcome the curvature of the earth. The most approved form is a hollow rectangular tower of *paka* masonry. This is first built up to within 3 feet of its ultimate height, and upon two of the opposite walls (say north and south) are laid two stout beams, and crosswise on these near their centre four smaller ones. This platform of four small beams or battens serves as a foundation on which a perforated circular pillar of masonry to support the theodolite stand is built up to the required height. The walls of the tower are then carried up to within about 6 inches of the level of the top of the pillar when two similar beams to support the flooring for the tent and for the observer to move about on are placed on the east and west walls, and a firm railing added for the sake of the observer's safety. The dimensions of the various parts of these towers will vary according to circumstances, and it is impossible to lay down any hard and fast rule. The following formula will probably be found suitable in most cases:—

Thickness of wall at bottom in feet $= 0\cdot 2 \times h^{\frac{4}{5}}$, h being the height in feet.

Inside the walls should be vertical, and outside they should taper to give a thickness of about 1½ feet where the beams for the observer's platform rest. The interior dimensions should be 7 feet × 11 feet, giving a space on the platform of 10 feet × 14 feet which is amply large enough for the observer and his assistants: indeed with the modern theodolites, which are considerably smaller than the old 36-inch and 24-inch instruments, less space than this might easily suffice; but the size of the observatory tent must be the final guide to the dimensions required. The perforated circular pillar on which the theodolite stands must be 2½ feet diameter for the new 12-inch instruments, and somewhat larger (about 3 feet) for the older pattern of 24-inch theodolites. The perforation is to enable the plumb-line to pass down through it to the mark-stone on the ground level. A second mark-stone is buried in the ground vertically below this surface one, as in the case of ordinary hill stations. It is objectionable to build solid pillars for a station exceeding 3 or 4 feet in height, as in course of time deflection is sure to occur to a greater or less extent, and so to vitiate the true centering of the instrument. Plate I shows, by sketches not drawn to scale, the general arrangement of a tower station as above described.

In Fig. 1 is shown the tower built up to the level of the cross beams supporting the isolated pillar; and also the details of the pillar itself. Fig. 2 represents the completed tower. The aperture at the bottom is to facilitate the centering of the theodolite over the mark-stone by a plumb-line. Fig. 3 shows a method of raising the theodolite boxes, and other stores, to the summit of the tower; care must be taken that the feet of the shears are bedded in holes sufficiently deep to prevent them slipping. If the tower is a low one the shears will be unnecessary, and the arrangement in Fig. 4 will probably be found more handy. It represents the observing platform extended sufficiently beyond the walls of the tower to enable the boxes to be drawn up without scraping against the brickwork. Access to the summit of the tower is gained by ladders.

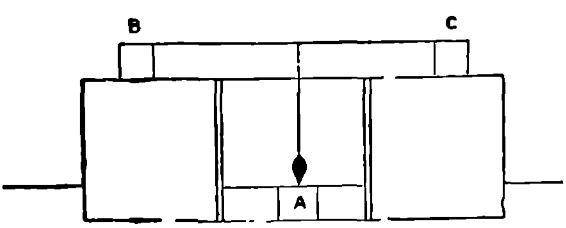
Thirdly. Trestle stations:—

These have been found a cheap and convenient substitute for tower stations, where timber and workmen are easily obtained, and the height required is not excessive. Plate II shows the construction of a trestle station, as designed and built by Major Branfill, on the South-East Coast Series in 1874-75. It consists essentially of a braced tripod stand 17 feet high resting partly on the masonry platform, and partly on the summit of the isolated pillar. This stand is surrounded by a braced scaffolding, to which at the proper height is attached a platform for the observatory tent, and which is carried up to a considerable height above this, for the purpose of showing the lamp or heliotrope to other stations. In Major Branfill's case palmyra trees which had been cut down during the process of ray clearing were used, and found to answer well. It appears from his report that lashings only were used for joining the various parts of the structure, the cost of the whole being only about Rs. 30. No difficulty would be experienced in raising the theodolite boxes on to a platform of this description.

16. The simplest practical method of determining the height to which a tower must be raised is to erect on the site of the station a temporary scaffolding of bamboo or other suitable material, sufficiently high to bring clearly into view at the time of minimum refraction the lowest of all the neighbouring signals that have to be observed. It may be useful to remember that if h is the height of a tower, the ray from it will graze the earth's surface at a distance d approximately, such that $d = \sqrt{\frac{3h}{2}}$, d being measured in miles and h in feet.

Adjustment of mark-stones.

17. The method of adjusting mark-stones is as follows:—Let A be the mark at the foundation, either engraved on the rock, or on a heavy embedded stone. Let the external part of the platform be built up to the intended height of



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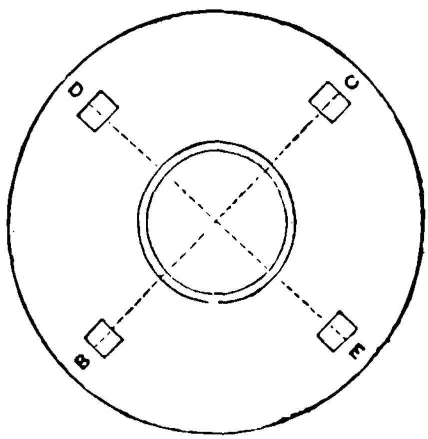
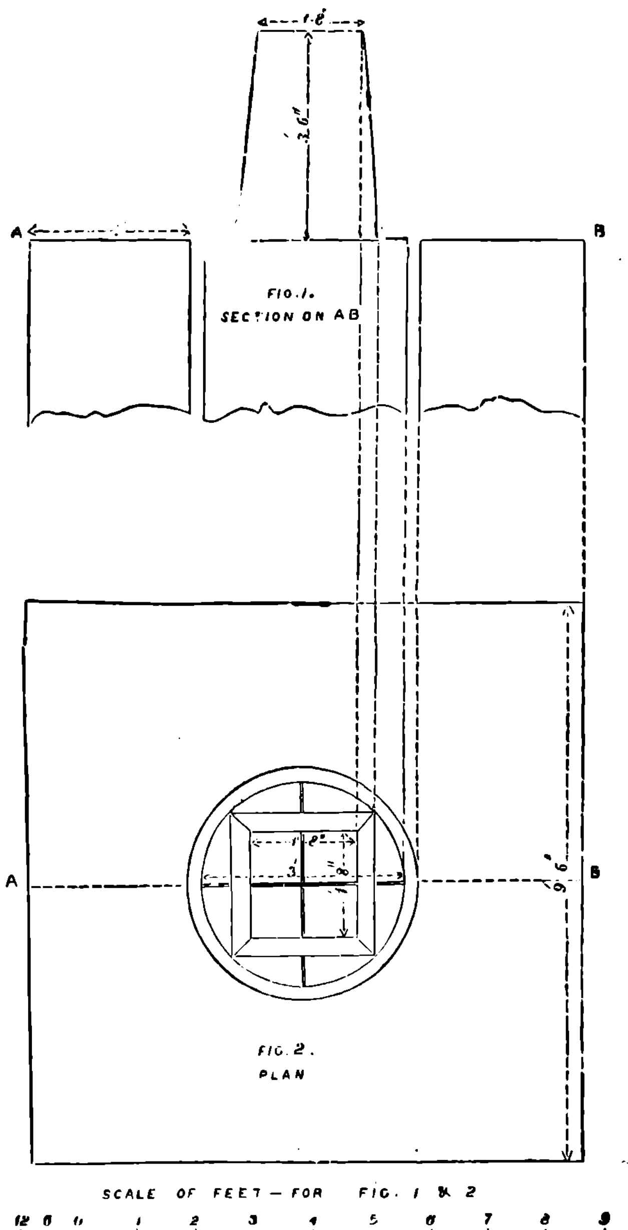


PLAN

the next mark, and place upon it four heavy stones, arranged in a quadrilateral figure, in such wise that threads $B\ C$ and $D\ E$ stretched diagonally across may intersect near the centre. Adjust these threads to correspond with a plumb-line suspended over the mark A, and when the coincidence is complete, mark the four exterior stones, by pencil lines, or lines scratched with a knife. Arrangements must now be made for protecting these stones, either by covering them over, or appointing a man to watch each, while the pillar in the centre is being built up nearly to the level of *B C*, at which height the next mark-stone should be fixed and adjusted, to correspond with the cross threads before adverted to.

Construction of protecting pillar.

18. At principal stations as soon as the observations are completed the mark-stone must be protected and concealed from view by a masonry pillar, which should be built in such a manner that future surveyors may use it instead of a stand to rest their theodolites on, and centre the instruments with almost as great accuracy as if they were to set them up visibly over the mark. Grooves must first be cut on the surface of the existing isolated pillar in straight lines from the circumference to the edge of the mark-stone, and in such directions that if they were carried over the stone, they would intersect each other at right angles on the mark. Then a rectangular masonry pillar, of which each side should equal seven-tenths of the diameter of the circular pillar, must be raised on the latter pillar to a height of three feet six inches, tapering upwards, so as to make the summit twenty inches square. The sides of the rectangular pillar will be parallel or perpendicular to

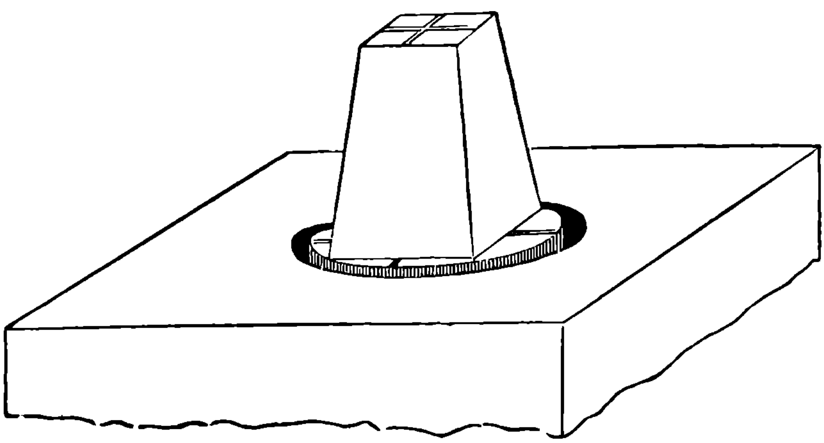


the grooves on the circular pillar, the extremities of which latter will be left visible for reference. The accompanying figures 1, 2 and 3 exhibit all these details very clearly. Lines to fix the exact position of the mark on the upper surface of the pillar may be drawn by means of a simple square bar of iron or wood about four feet long, fitted with a plummet at either end, both suspended from the same face of the bar. The plummets must, by moving the bar, be brought first over one pair of grooves and then over the other, lines being drawn in both cases along the edge of the bar used as a ruler. In the case of tower stations it will be sufficient to pile some loose stones or rubbish over the mark-stone and then to brick up the aperture at the foot of the tower.

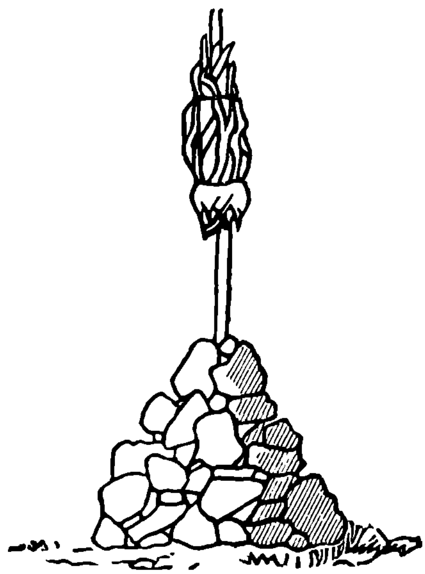
Construction of protecting pillar.

19. Grooves should be cut into the surface of the rectangular pillar as in that of the circular. The masonry should be of the best burnt brick and mortar procurable; the exterior surfaces of the bricks to be neatly cut and finished, and unplastered. Figure 3 represents a completed pillar. A pyramidal pile, seven feet square at base and six feet high, either of earthwork

FIG. 3.



and stones or *kacha* bricks cemented and plastered with mud, should be built over the pillar for its protection, and to prevent water from getting into the annulus between it and the platform. If the station is likely to be subsequently required for secondary work it is usual to set up over the protecting pillar a pile of stones supporting a pole with brushwood tied on to it as in the annexed diagram. The pile of stones should be about 5 or 6 feet high and the pole about as much more. The station must then be made over to the care of the senior Native official of the village in whose lands it is situated. For this purpose, a document must be drawn up in duplicate in accordance with the form provided for the purpose, *viz.*, O.1.Δ, the original being sent to this office for record, while the duplicate is left in the hands of the person to whom the charge of the station is committed. The authorized form is as follows :—



(1) The station named of the Great Trigonometrical Survey of India, situated in latitude longitude on the lands of the village in Pargana District is hereby placed in the charge of*
and will remain under his charge and that of his successors until further orders.

(2)† He is authorised to prevent all persons from injuring the station, but he will permit any person who may have occasion to visit it for Survey purposes, to remove the covering pile of stones or earthwork, and refer to the station mark, which is indicated by the intersection of the cross lines on the surface of the masonry pillar. When such a person has completed his observations, he will be required to erect a fresh pile of earth and stones, 7 feet square at base, and 6 feet high over the pillar.

(3) Any person wilfully injuring the station, or neglecting to comply with the terms of this document, will be liable to be prosecuted by the

* Here give name and official rank of individual in question.

† In the case of tower stations para. (2) should be scored out.

District Officer, to whom a report must be made without delay by the official in charge of the station.

Given under the orders of the Government of
dated 30th October, 1865.

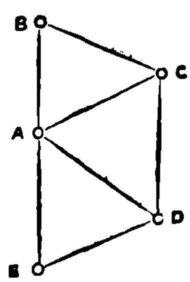
Date of Transaction
Signature of Survey Officer
Signature of Native Official

Final observations.

20. The selection of stations is merely preparatory to the final observations, and is called carrying on an Approximate Series. It is usual to choose a few triangles in advance, and then detach a party to continue the Approximate Series, and prepare the stations ahead, while the final observations are progressing in rear. This division of labour is of great advantage in accelerating progress, and, if judiciously arranged, with regard to local circumstances, and the organization of the party, is productive of the best results.

Measurement of horizontal angles.

21. The angles at a station are taken thus: supposing the observer to be at *A*, and the signals at *B, C, D, E* all visible, the instrument is carefully levelled and adjusted, and so fixed that some station, *B* for instance, reads 0°, or zero in microscope *A*: *B* is then called the zero station. Suppose the telescope to be brought up from the left hand of *B*, and turned gently, so that *B* may enter the field of view, and come near the centre wire, but not pass over it; the instrument must then be clamped, and the bisection of *B* completed, by using the slow motion screw. All the micrometers, or verniers, are now read, and the assistant records the readings in a fair legible hand in the Angle Book. The observer should then look again into the telescope to see that *B* remains bisected. If found correct the telescope is to be carefully unclamped and moved towards *C*, care being taken not to overshoot it. The clamping, bisecting, and reading is done as before, and similarly also for *D* and *E*. A complete set of observations is thus obtained at zero 0° by a continuous motion from left to right. Now, after overshooting the station *E* the



telescope is brought back by a continuous motion from right to left, to each station in succession, and the readings recorded. This will give a second set at zero 0°. A third set, or, as it is commonly called, round, is then taken in a similar way, and the three rounds, if accordant will suffice for that zero; if the discrepancies in the values of any angle exceed 2″ another measure of that angle must be taken. It is by no means a matter of indifference (except at the centres of polygons) which station is selected as the zero station. The left and right hands should be taken alternately at the successive stations for reasons given at pages XII to XVII of the Introduction to the Great Indus Series, Volume III.

22. When the signals to be observed lie all round the station observed at, as at the centre of a polygon, the observer must be careful to complete each swing by intersecting the same signal as he commenced with. Thus if he begins with *B* in going round from left to right he will end with *B*, and again beginning with *B*, having first overshot it, he will swing round from right to left and end on *B*.

23. Now turn the telescope through 180° in a vertical plane, and round 180° in azimuth, so that if the face of the vertical circle were previously to the left hand, it will now be to the right hand; *B* will then read 180°, and this is called zero 180° *FR*, the former position being zero 0° *FL*, *i.e.*, face left. Proceed as before, and take three sets of observations, the motion of the instrument being in one set continuous from left to right, in the next from right to left, and in the third from left to right.

Zero changes.

24. Having thus obtained six sets of observations for one position of the instrument a "change of zero" must be made, *i.e.*, the horizontal limb must be turned round through some definite arc, thus altering the readings of all the stations by that amount. In this position also six sets are to be secured when a further "change of zero" and another six sets are to be taken, as far as is desirable. The angle through which the limb is shifted at each change of zero and also the number of changes vary with the different classes of instrument. The following table gives the details for the three kinds of Theodolite commonly employed. Full information on this subject is given in Volume II, Chapter IV, to which the reader is referred.

Five Microscope Theodolite:—

$$\frac{0^\circ\ 0'}{180^\circ\ 0'},\ \frac{79^\circ\ 12'}{259^\circ\ 12'},\ \frac{158^\circ\ 24'}{338^\circ\ 24'},\ \frac{237^\circ\ 36'}{57^\circ\ 36'}\ \text{and}\ \frac{316^\circ\ 48'}{136^\circ\ 48'}.$$

Three Microscope Theodolite:—

$$\frac{0^\circ\ 0'}{180^\circ\ 0'},\ \frac{70^\circ\ 1'}{250^\circ\ 1'},\ \frac{140^\circ\ 2'}{320^\circ\ 2'},\ \frac{210^\circ\ 3'}{30^\circ\ 3'},\ \frac{280^\circ\ 4'}{100^\circ\ 4'}\ \text{and}\ \frac{350^\circ\ 5'}{170^\circ\ 5'}.$$

New 12-inch Theodolites with two microscopes:—

$$\frac{0^\circ\ 0'}{180^\circ\ 0'},\ \frac{210^\circ\ 1'}{30^\circ\ 1'},\ \frac{60^\circ\ 2'}{240^\circ\ 2'},\ \frac{270^\circ\ 3'}{90^\circ\ 3'},\ \frac{120^\circ\ 4'}{300^\circ\ 4'}\ \text{and}\ \frac{330^\circ\ 5'}{150^\circ\ 5'}.$$

Precautions.

25. Having described in a general manner the method of observing the angles in a Geodetic Survey, it is necessary now to remark upon the chief precautions necessary to be taken, in order to ensure the greatest degree of accuracy, which the means at our disposal admit of.

Continued.

26. The observer ought to be skilful, scrupulous, and patient and should cultivate, by incessant practice, the power of vision, and delicacy in handling instruments. The instrument should be kept in good order, and handled with the greatest care. Separate instructions on this head are given below.

Continued.

27. The stability of the foundation of the pillar and due isolation of the instrument are essential elements of accuracy, as already remarked in para. 15. The instrument should also be carefully centred over the station mark, and made truly level, for effecting both of which purposes appropriate apparatus is attached to every instrument.

Levelling.

28. The operation of levelling is thus performed. Let the level on the body of the instrument be placed parallel to a line joining two foot-screws, and, by means of these, bring the bubble to float in the centre of its tube. Then turn the instrument round 180° in azimuth, and if the bubble continues to float in the centre, that diameter of the limb which is parallel to the line joining the two foot-screws must be truly level; otherwise half the difference of the reading at each end of the bubble is the error,

which ought to be corrected by the foot-screws, and the other half difference of the reading is the error of the level itself. The latter error need not be corrected at all, if it amounts only to a few divisions, because it can always be allowed for in performing the adjustment. For instance, if the right hand end of the bubble reads a divisions, and the left hand end reads b divisions, in one position of the instrument, and after a semi-revolution in azimuth the same ends of the bubble read a' and b', then $\frac{a + a'}{2}$ and $\frac{b + b'}{2}$ are the readings which the bubble ought to have when the instrument is truly level. Now turn the instrument round 90° in azimuth, and, by means of the third foot-screw, bring the bubble to read $\frac{a + a'}{2}, \frac{b + b'}{2}$. The diameter at right angles to the former one will now be approximately levelled, and if this operation be repeated two or three times, the adjustment will be perfected. The process should always conclude with the third foot-screw. If the error of the level be very large, that is to say, if the reading of one end of the bubble differs considerably from the reading of the other, it can be rectified by the small capstan-headed screws attached to the level for that purpose. The proper time for effecting the practical correction is when the instrument is nearly level; but if the error be small, it is better, as before stated, to leave it alone, and allow for it in levelling, because the materials of the instrument have a tendency to settle into a position of equilibrium, in which they will remain steady, but, if frequently disturbed, the screws will in time work loose, and no confidence can then be placed in the permanence of the adjustments.

Adjustment of level.

29. The apparatus attached to the level for the purpose of adjusting it in due relation to the axis of the instrument, is very different in various instruments; for, in some, there may be a single screw at one end for raising or depressing that extremity of the level; in other instruments there are two antagonizing screws at one end of the level, and one must be released before the other is tightened.

Continued.

30. In most modern instruments, however, there are three screws at each end, of which the central of each set is a drawing screw, and the two external ones are pushing screws. In this case, if one end of the level requires

lowering, release the central screw, and tighten the external ones. On the contrary, for raising that end of the level, release the external, and tighten the drawing screws. It is a general rule, in working antagonizing screws, that is to say, screws that produce opposite results, that one should be released before the other is tightened, otherwise the screws are liable to be bent, or the threads broken, and the efficiency of these delicate instruments to be completely destroyed.

31. *Continued.* The criterion of the instrument being level, is, that the reading of each end of the level remains constant during a complete rotation in azimuth. The accuracy of the adjustment should therefore always be tested before final angles are taken, by reading the level in a position parallel to two foot-screws, and then successively at 90°, 180° and 270° in azimuth. A difference of about one division may be considered of no account in taking terrestrial angles, but the most scrupulous care must be taken in levelling the body of the instrument when stars, or other elevated objects, are observed.

32. Centering. The process of levelling is liable to disturb the centering of the instrument over the station mark, which should therefore be duly looked to. Instruments are centered either by means of a plummet, or by a look-down telescope.

33. *Continued.* The point of suspension of the plummet may, perhaps, not coincide precisely with the centre of the axis; therefore, if the plummet revolves with the instrument when the latter is turned round in azimuth, the centre will not agree after a semi-revolution, and must be corrected for half the difference. If the point of suspension of the plummet is not attached to the axis of the instrument so as to revolve with it there is no method of correcting the eccentricity.

34. *Continued.* When a look-down telescope is employed for centering, the instrument must first be duly levelled, the eye-piece of the look-down must be adjusted to distinct vision of the wires, and the object-glass adjusted to distinct vision of the station mark, so as to be free of parallax. Now, by means of the centering screws, move the instrument till the station mark is duly bisected by the cross wires. Then turn the instrument

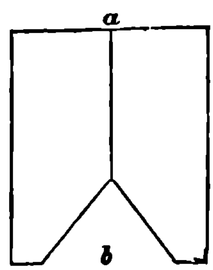
round 180° in azimuth, and if the station mark is seen on one side of the wires, half the difference is the error of centering, the remainder being the error of collimation of the wires. Adjust the wires accordingly, and also the centering, after which relevel, and repeat the process, which will generally suffice to perfect the adjustment. The advantage of a look-down telescope over the plummet is two-fold;—first, it is not disturbed by currents of wind; secondly, the adjustment is more minutely performed, by reason of the magnifying powers of the telescope. This latter advantage is, however, more apparent than real, as it is of no use to centre the theodolite more accurately than the signals observed.

Adjustment of transit axis.

35. The body of the instrument being duly centred and levelled, the next care of an accurate observer is to render the transit axis truly horizontal. The telescope of every theodolite, with its attached vertical circle, is supported on a horizontal axis, the ends of which, called "pivots," rest in angular supports, termed **V**s. The imaginary line, joining the centre of the pivots, is the axis on which the telescope rotates, and, in order that the latter may describe true verticals, the axis must be truly horizontal. This adjustment is effected by raising or lowering one of the **V**s by the appropriate screw, placed beneath the **V** for that purpose,—the amount of adjustment being regulated by the indications of a riding or striding level, which must be placed upon the pivots during the process of levelling them.

Continued.

36. The riding level is furnished with two feet, which are cut into the shape of notches, or inverted **V**s, and there is either a single screw or two antagonizing screws at one end of the tube, whereby to adjust the bubble to parallelism with the line of bearing of the feet. There is also generally attached to one end of the riding level a little cross level, by means of which the inverted feet **V**s are made to rest always with the same points on the pivots, and any slight discrepancy in the verticality of the feet **V**s is thereby prevented from having any effect. If the figure in the margin represent an inverted foot **V**, and *a b* be the line bisecting the angle of the **V**, then it is clear that if *a b* be truly vertical, each side of the **V** will rest



equally on the pivot; but if the similar line $a' b'$ of the other foot be not truly parallel to $a b$, then, on a slight declination of the line $a b$, the level will rest on the pivots by three points only, instead of four, and the bubble will, by reason of the obliquity of the **Y**, run up towards one end. The remedy for this evil is to render the two feet of the level truly parallel with respect to the imaginary lines bisecting the inverted **Y**s. For this purpose release the screws which attach the feet to the tube of the level, and gently turn the feet until, by successive trials, it is found that the bubbles remain stationary, even when the level is inclined a little on one side or the other of true verticality, as respects the bisecting line of the **Y**s. The proper bearing on the pivots will thus be maintained, and the riding level will give true indications, even without the aid of a cross level.

37. To level the transit axis—Uncover the pivots by removing the clips which retain the pivots in their **Y**s. Introduce the riding level gently between the radii of the vertical circle, and place it upon the pivots. Adjust it to the cross level, if it have one, and read off the ends of the bubble. In order to distinguish one end of the bubble from the other, it is usual to call the end next the little cross level L, and the opposite end O; but if there be no cross level, it is as well to mark the letters L and O on the tube, as characteristic marks. It is also usual to call the divided face of the vertical circle F, and the other face P. Now, suppose, in the first instance, that the L end of the level is at F, and the readings of the bubble in that position are l and o, gently take off the riding level, and reversing it, end for end, replace it on the pivots, so that L, which before was at F, will now be at P; read off the bubble, and let the readings be l' and o', then half the difference between the reading of the same ends in the two positions will be the inclination of the transit axis, to be adjusted by the proper screw under one of the **Y**s, whereby that **Y** may be raised or lowered. If $\frac{l - l'}{2}$ be greater or less than $\frac{o' - o}{2}$ the difference is occasioned by unequal expansion of the level, and the mean must be adopted as the true error. The other half difference of the readings is the error in the level itself, which, if it amount to any considerable quantity, may be practically corrected by the appropriate screws attached to the level.

Continued.

38. It is inadvisable to alter the inclination of the transit axis for an error amounting only to four or five seconds, because frequent tampering with the screws will in time loosen them, and destroy the permanency of the adjustments, the stability of which is a most important circumstance in making observations, because it is of no avail to render an instrument free from error at the commencement of a set of observations if the adjustments do not continue permanent throughout.

Continued.

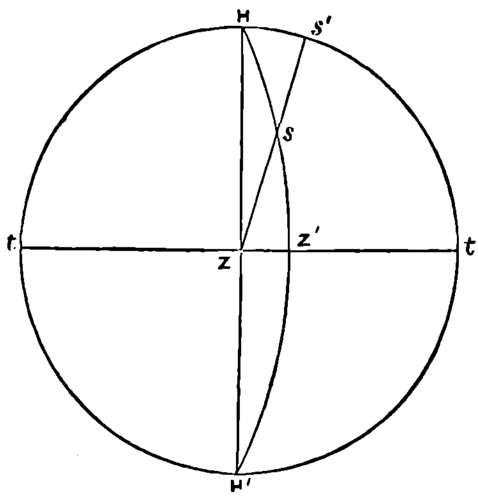
39. The value of the divisions of the scale of all the levels employed should be determined either at the Mathematical Instrument Office, Calcutta, or at the Trigonometrical Branch Office, Dehra Dún, by an instrument called a bubble tester, before the party takes the field; but if for any reason this is impracticable, it may be ascertained by affixing the level to the frame of the vertical circle, or making it ride parallel to the telescope, and then taking the readings of the microscopes in two positions of the bubble, whence comparing the number of divisions of the level scale run over by the bubble with the corresponding angular motion of the vertical circle, as measured by the microscopes, the value of one division of the level scale will be obtained by simple proportion, as shown in the following example:—

Values of level scales.

Temperature.	Vertical Microscopes.			Angular Difference.	Level.					Computed value of one division of the Level Scale.
					Readings.		Differences.			
	D.	E.	Mean.		L.	O.	L.	O.	Mean.	
	° ′ ″	″	° ′ ″	″	d	d	d	d	d	″
Fahr. 70·7	7 25 47·0	41·0	7 25 44·00	...	85·6	48·4	...	...	...	...
	7 26 20·5	14·1	7 26 17·30	33·30	50·9	83·0	34·7	34·6	34·65	0·9610
	7 26 12·1	6·0	7 26 9·05	8·25	59·9	73·9	9·0	9·1	9·05	0·9116
	7 25 62·1	56·2	7 25 59·15	9·90	70·1	63·8	10·2	10·1	10·15	0·9755
	7 25 52·2	45·0	7 25 48·60	10·55	81·0	53·1	10·9	10·7	10·80	0·9769
	7 25 44·2	37·3	7 25 40·75	7·85	89·9	44·3	8·9	8·8	8·85	0·8870
	7 26 11·1	4·9	7 26 8·00	27·25	61·2	73·0	28·7	28·7	28·70	0·9495
										5·6615
	Mean value of one division of the level scale, ...						...		...	0·9436

40. If in the figure in the margin, which represents an orthographical projection on the plane of the horizon, Z be the true zenith, and $t\ t'$ be the direction of the transit axis, then, when that axis is level, the telescope will rotate in the vertical plane $H\ Z\ H'$ at right angles to $t\ t'$, and when the telescope is elevated 90° it will point to Z, the true zenith of the place. But if the transit axis be inclined to the plane of the horizon, the telescope will decline from the point Z by an angle $Z\ Z'$ equal to the amount of the dislevelment of the axis. The telescope will now rotate in an oblique plane $H\ Z'H'$, and if S be an elevated object, the telescope will refer its position to H on the horizon, instead of to S', which is the true horizontal point appertaining to S, as determined by the true vertical $Z\ S\ S'$ passing through Z and S. The angle $H\ Z\ S'$, or corresponding arc $H\ S'$ is the error in azimuth occasioned by the dislevelment of the transit axis, and in the spherical triangle $S\ Z\ Z'$ right angled at Z', we have $\cos S\ Z\ Z' = \tan Z\ Z' \cot Z\ S$, or as the arcs $Z\ Z'$ and $H\ S'$ are both small, amounting in fact to a few seconds, we have

Effects of dislevelment.



$$H\ S' = Z\ Z' \times \tan \text{altitude}.$$

41. This formula clearly shows that when the altitude is 0°, an inclination of the transit axis produces no error in the azimuth readings; and also the error in azimuth must be very small when an object is little elevated, as is the case with terrestrial objects. It is also evident that the error lies in the direction in which the pivot is higher, so that if the left hand pivot be higher, the error will be *minus*, and corresponding correction *plus*; but, on reversing the face of the instrument, the pivots change positions with respect to the observer, and the errors will have an opposite sign; consequently, if the mean be taken between an observation face left, and another face right, the influence of an inclination in the axis will be entirely eliminated, and this will also happen with regard to depressed objects, in which the signs of the errors will be opposite

Continued.

to those in the case of elevated objects. Hence, if angles be taken between an elevated and a depressed object, those on a single face will be burdened with the sum of the errors due to the depression and elevation, but the mean of the angles between face left and face right will be free from error.

Further adjustment.

42. The next adjustments to be undertaken are those of the telescope itself, which are four in number, and all have reference to the wires, or ocular lines, situated near the eye end. These adjustments are,—

1st—Distinct vision of the wires,
2nd—Freedom from parallax,
3rd—Collimation in azimuth,
4th—Verticality of the vertical wires, and horizontality of the horizontal wire.

Distinct vision of wires.

43. The only adjustment of a personal nature is that for distinct vision of the wires, which varies with the focus of the observer's eye, and the eye-piece must therefore be pushed in or drawn out of its cell, until distinct vision is obtained. It is advisable to direct the telescope to the sky, or hold a piece of paper obliquely in front of, and a short distance from, the telescope, so that the wires being projected on a blank field, may be viewed by the eye, undistracted by other objects. The wires will be truly in the focus of the eye-piece when they appear sharply defined, and all the little specks of dust on them are seen clear and distinct.

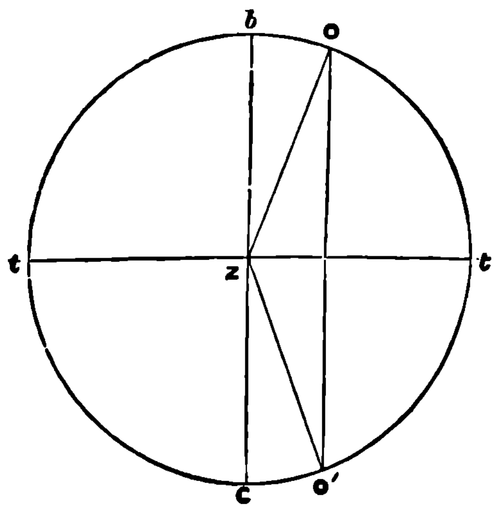
Elimination of parallax.

44. Now direct the telescope upon a distant object, a heliotrope for instance, which bisect with the vertical wire. Move the eye gently to one side, and if the object still appears bisected there is no parallax, and the wires are truly in the focus of the object-glass. If, however, on moving the eye to one side, the image of the object appears to move with the eye, then the focus of the telescope lies beyond the wires, and the object-glass and wires must be approximated to each other: this is called far parallax. If, on the other hand, the image appears to move in a contrary direction to the eye, the imperfection is called near parallax, and may be rectified by augmenting the distance between the object-glass and the wires: this adjustment is effected in some instruments by moving

the object-glass, in others by moving the tube which contains the wires. Small instruments usually have a pinion and rack to regulate the adjustment of the focus, but this is not the case with the larger class of instruments.

Collimation.

45. The line of collimation in a telescope may be defined to be the line joining the centre of the wires and the centre of the object-glass. In a theodolite or alt-azimuth instrument, the telescope rotates on the transit axis, and the line of collimation in a telescope thus mounted should be at right angles to the transit axis, otherwise, it will not describe a great circle, when the transit axis rotates. For, let *t t* and *o z* be the directions of the transit axis and line of collimation respectively; and suppose the line *o z* to rotate round the axis *t t* previously made truly level, it is clear that it will not pass through the zenith at all, but after a semi-revolution the point *o* will arrive at *o′*. The line of collimation will in fact describe a cone, the base of which will be a small circle parallel to the vertical circle, which intersects the transit axis at right angles.



Continued.

46. Having defined what the line of collimation is, the next thing to be considered is, the practical method of adjusting it. There are several ways in which this adjustment may be effected, and of these the following is the one usually adopted in the field with moderate-sized instruments.

Continued.

47. Direct the telescope to a distant fixed object which should be nearly on the observer's level, and bisect it with the wire. Then lift the telescope out of the Vs, and replace it with reversed pivots, so that the right hand pivot may now be at the left hand. This must be done with delicacy, taking care not to shake the instrument, as the success of the observation depends

on the Ys remaining unmoved. If the line of collimation be true, the object will appear bisected in the second position of the telescope, otherwise it will be seen to one side of the wires, and half the apparent is the real error to be corrected by the appropriate screws at the eye-end of the telescope. Two or three repetitions will suffice to perfect this adjustment, and when it has once been effected, it should not again be tampered with unless the error of collimation amounts to more than 3″ or 4″.

Measurement of amount of collimation.

48. It is very clear that, by pursuing the process described in the foregoing paragraph, the amount and direction of collimation can easily be measured on the limb of the theodolite, and may be used as an element of reduction, instead of practical adjustment. All that is necessary is, after intersecting the distant fixed object, to read off the microscopes, then lifting out the telescope, and replacing it with reversed pivots, again bisect the object, and read off the microscopes. One half the difference of the readings will be the error of collimation, and its sign may be known from the following consideration:—Suppose the instrument, in the first instance, to have stood face left, with the pivots in their usual Ys, and suppose the readings in that position to have been in excess of those obtained after reversal, then the error of collimation is + as respects all observations on face left, and − as respects those on face right. Consequently, in applying the reductions dependent on this error, the corrections will have opposite signs, *viz.*, − for face left, and + for face right.

Correction for altitude.

49. If the distant fixed object, by means of which the collimation has been determined, is situated above or below the horizon, the collimation error thus obtained must be reduced by multiplying it by the cosine of the elevation or depression of the object observed. On the other hand, the amount of the collimation error at the horizon being known, the azimuthal error at any altitude may be obtained by multiplying by the secant of the altitude.

Elimination of collimation.

50. This clearly shews that an error of collimation produces its least effect on azimuthal readings, when the altitude of the observed object is nothing. It is moreover evident that the system of observing with the face of the instrument alternately left and right must necessarily eliminate the effect of the error, because it lies in contrary directions on the two faces.

Effects of erroneous collimation.

51. The permanence of the direction of the line of collimation is a most important consideration, as any change during the course of the observations will prevent the errors cancelling each other. It is found in practice that the line of collimation is liable to a slight fluctuation of a few seconds, on account of the unequal expansion of the metal composing the telescope, but the errors incidental to this fluctuation have a tendency to compensate during a long series of observations.* In observing stars for azimuth, the greater their elevation, the greater is the error produced in the resulting azimuth by unadjusted collimation.

Inconvenience in applying the above method.

52. The previously described method of adjusting for collimation is that which is usually practised with moderate-sized instruments; but it was found inapplicable to the large theodolites. The difference of the readings on opposite faces, as obtained by observing an object alternately face left and face right, gives double the error of collimation, and such is the accuracy with which these instruments are graduated, that its amount and direction can always be inferred with great confidence from the regular observations, and may easily be corrected, when required, by the appropriate screws at the eye-end of the telescope. This method, however, does not appear to be altogether satisfactory for instruments with three microscopes or verniers, because the microscopes, on reversing the face, will stand 60° apart from their previous positions, thus embracing the greatest change in zero which the instrument is capable of, and introducing the full amount of uncertainty due to graduation. Another objection to this method in instruments which are not provided with a full vertical circle, is the extreme difficulty of reversing the pivots of a large heavy instrument without some jar or shake, the occurrence of which there is no means of determining, and which, if it exists, vitiates the whole observation.

Gauss's method of collimating.

53. Gauss's method of collimating is free from the foregoing objections, being equally independent of errors of graduation, and of the practical difficulty of reversing pivots. The principle of it is as follows:—Suppose two telescopes each fitted with a cross of spider lines at the solar focus in

* As a precaution against unequal expansion, it is proper to clothe the telescope and axes with woollen cloth, and to protect the instrument from cold currents of air, &c., and from the prejudicial effects of solar rays.

the ordinary way, be placed approximately in the same straight line, object-glass to object-glass. An observer looking through one telescope will then see, by a well known optical principle, the cross in the other as well as in his own, and by a slight motion of either telescope the two crosses may be made to coincide. The lines of collimation of the two telescopes are then exactly parallel (if not coincident) and if produced indefinitely will cut the celestial sphere at the two opposite poles of a great circle. Now suppose the telescope whose collimation is to be tested to be placed somewhere between these two, sufficient room being left for this purpose, with its line of collimation parallel to theirs, it is evident that if its cross coincides with that of one of the collimators in one position it will, if properly collimated, coincide with that of the other after it has been rotated 180° round the transit axis.

Continued.

54. The practical execution of this method of collimating consists in placing two small theodolites, one on each side of the instrument to be collimated, so that all three instruments shall be nearly in the same straight line.* To effect this, set the telescope of the centre instrument level, and place a theodolite so that its wires may be seen on looking through the centre instrument. Now turn the centre telescope 180° in a vertical plane, and place the other theodolite so that its wires may also be seen. Then remove the centre telescope, by lifting it out of the Vs, whereby the two external telescopes may be adjusted upon each other, so that their wires shall mutually intersect; after this they are not to be disturbed. Replace the centre telescope, and directing it upon one of the small theodolites, intersect the image of its wires and then turn over 180° in a vertical plane, so that the telescope may now point to the other theodolite; and, if the wires of the latter are found to be also intersected, the line of collimation is truly at right angles to the axis of rotation, otherwise one half the apparent is the true deviation to be adjusted, or measured by the microscopes, as the case requires. It is obvious that this method, with a little modification, may be made applicable to collimation in altitude†.

* A few tenths of an inch are not important because it is the parallelism of the rays, and not the exact centering of the object-glasses, upon which the method depends.

† For example: By means of the vertical tangent screw intersect the wires of one of the auxiliary theodolites, turn round 180° in azimuth, and the telescope will point to the other auxiliary theodolite. One half the apparent deviation in altitude will be the error of collimation to be adjusted by the screws acting on the wires. After which the micrometers should be adjusted to zero. This method is seldom employed, as with an altitude and azimuth circle, the index error can easily be corrected by means of observations taken on both faces as will be explained below.

55. The distance between the telescopes is necessarily limited by the fineness of the wires and the magnifying powers in the auxiliary theodolites, the wires of which are not likely to be visible at a greater distance than ten or twelve feet; consequently, they must be placed about five feet from the telescope to be collimated, or at any distance greater or smaller, which on trial may be found suitable. To render the wires of the latter distinct, an assistant should reflect light into its eye-piece, and for this purpose a slip of paper held obliquely will answer very well. The wires are seen more plainly when plain glass is substituted for the eye-piece of the telescope.

Continued.

56. The fourth adjustment in the telescope referred to in para. 42 consists in rendering the wires in the telescope perfectly vertical and horizontal. The systems of wires used in the focus of telescopes are very various, a common form consists of a single vertical crossed by a horizontal wire, which is the best for the principal triangulation, as the objects observed are generally luminous points. For observing flag-staves, spires, and other perpendicular objects, a St. Andrew's cross is employed in connection with one horizontal wire. As the intersection of three lines, however, occupies a large undefined space, it is preferable that the horizontal wire should not pass through the intersection of the oblique lines, but form a small triangle therewith. In this case horizontal angles should be observed to the centre of the cross and vertical angles on the horizontal wire. A system of wires consisting of five, seven, or more vertical wires and one horizontal wire (or occasionally two very close ones) is usually applied to astronomical circles. These wires are generally fixed at a proper angle on a perforated plate, which admits of a small play for the purpose of adjustment. To verify the horizontal wire all that is necessary is to set the telescope after levelling it on a distant steady defined object near the horizon, and move the instrument in azimuth, when the object should appear to move along the wire from one extremity of the field to the other. To adjust the vertical wire all that is necessary is to move the telescope in altitude, so that a distant object may appear to traverse or run along the vertical wire. If the object does not remain intersected at all parts of the field, the error ought to be rectified by moving the wire-plate in the appropriate direction. The relative position of the wires, with respect to each, other having been fixed by the maker, it is only possible to

Adjustment of diaphragm.

adjust one wire by means of the wire-plate, but if there be a micrometer at the eye-end, its wire can generally be adjusted separately.

Method of wiring diaphragms.

57. The wires should be stretched tightly, for if they be slack their intersection will not remain permanent during the revolution of the telescope on its axis. As wires are liable to be broken, to become slack by damp, or uneven by accumulated dust, it is necessary that every surveyor should be able to replace them when required. The best substances to apply to large instruments are spider lines, and to small ones, the fibres of raw silk. To procure spider lines, prepare some card frames, as shewn in the margin. Next look out in the garden, or among trees and bushes, for some healthy spiders, and take one up on the edge of the card frame. Then gently shake the frame, to detach the spider, which will hang therefrom, as shewn in fig. 1. Wind up the fibre, so that the turns may be rather widely apart, *vide* fig. 2. When the end of the card is reached, make a notch (*a*), in which insert the end of the thread, and then cut off the rest with a pair of scissors. If more cards are wanted, do not let the spider fall to the ground, where it would be covered with dust, but lay hold of the line at *b*, and place it in the notch of another card, which wind up as before. The cards should afterwards be placed between leaves of clean paper, to preserve them from dust. Having obtained the fibres, they may be fixed as follows :—Take out the wire-plate, remove the old wires, and clean the varnish from the engraved cuts. This must not be done by scraping with a knife, but by using spirits of wine and warm water, wiping the old varnish off, until the cuts appear quite clean. Now take a card, examine the fibre with a magnifying glass, and select a clean uniform piece. Take two little balls of wax, which attach to the extremities of the selected fibre, and cut off the remainder. If one of

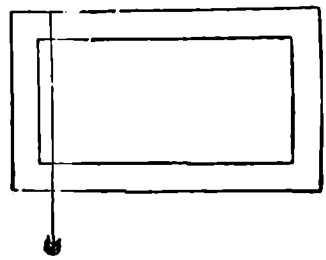


FIG. 1.

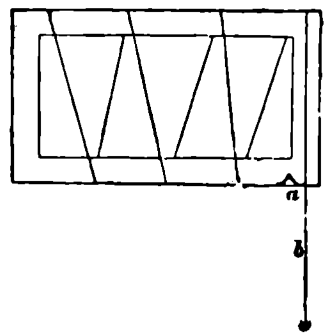


FIG. 2.

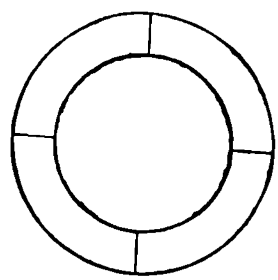


FIG. 3.

the balls of wax be held in the hand, and the other be allowed to hang freely, the fibre will become straight and untwined. Now take a camel's hair brush, dipped in clean water, and rub the fibre gently, for the purpose of cleaning and damping it. Put the wire-plate upon a little block of wood, *vide* fig. 5, and place the fibre upon it, taking care to examine with a magnifying glass that the wire falls into the proper cuts. The two little wax balls hanging on either side serve to stretch the thread, and keep it in its place. The cross wire is prepared in a precisely similar manner, and as many more as may be required. Then, taking care that they are all truly adjusted in their respective lines or cuts, let a drop of varnish fall upon each cut, and put a tumbler over the apparatus, to protect it from dust. In twenty-four hours the varnish will have set, the ends of the fibres may then be cut off, and the wire-plate carefully replaced in the telescope. This last is the most difficult part of the undertaking, because it requires very delicate handling to replace a wire-plate without breaking a wire, the method of proceeding may however be acquired by patience and practice. The best varnish to use is copal, but sealing wax dissolved in spirits of wine, friar's balsam, or laudanum, will answer. Fibres of raw silk are most suitable to small instruments, and these can easily be applied by following precisely the same rules. The previous damping of the fibres ensures their becoming tight and well stretched when dry. The best time to apply fresh wires is during the rains, when there is no dust flying about, and the atmosphere is rather damp. All the instruments should be carefully examined at that time, and those which require it should be fitted with fresh fibres. Freedom from dust also renders this season the most favorable for cleaning the axis and other working parts of the instrument which require to move glibly.

FIG. 4.

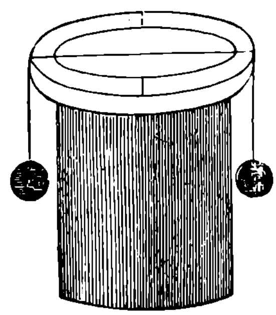


FIG. 5.

58. The next subject for consideration is the azimuthal microscopes, upon the proper adjustment of which the value of the azimuthal observations greatly depends.

Adjustment of microscopes.

59. When a circle is graduated into spaces of 5′ each, the rough reading of the degrees and fractional parts of the degree, as far as to the last 5′ space, is performed by an index having a single stroke upon a piece of metal, which is adjustable, and generally fixed on the clamp-plate. The remaining minutes and seconds are shewn by the micrometer microscopes, of which there are generally three or five attached to the body of the instrument in old theodolites and two in the modern ones. These are firmly affixed to radiating arms, and are placed at equal intervals round the circle. The reading microscope, as now constructed, is a species of compound microscope consisting of three lenses, one of which is the object lens, and the other two form a positive eye-piece; the amplifying lens being omitted, as the field is not required to be extensive, and the measure is made near its centre. When firmly fixed and truly adjusted it is capable of sub-dividing the minute into seconds and fractions of a second with great accuracy and facility. It admits of the divisions on the limb being well illuminated, and does not injure them by friction, as happens with the vernier, over which it possesses all the advantages of convenience, combined with high magnifying power and micrometrical nicety of measurement. Its value, however, is entirely dependent on its being kept in proper working order and true adjustment, for which reason a knowledge of the principles of its construction is essential to an observer who aspires to great precision.

Description of microscopes.

60. The arc usually measured by a microscope is a divided space of the value of 5′, to be sub-divided into 300″ by five revolutions of the micrometer-screw, which therefore has its circular head graduated and numbered from zero by 10″, 20″, 30″, &c., up to 60″, every fifth stroke being longer than those of the other graduations, and terminating with a lozenge. Now, in order that the screw may be competent to measure a 5′ space without excess or defect, it is clear that the image of the latter must be magnified so as to occupy precisely the length of five revolutions of the screw. Suppose that an object (the divided limb of the circle for instance) is placed exactly at the solar focus of the object lens, the rays passing through the lens become parallel, and the image will be formed at an infinite distance; but if it be placed beyond solar focus, an image is formed within the tube, and the two points where the divided limb and its image are situated, are called conjugate foci, the latter of which recedes upward as the other approaches the object lens. If we call the distance of the limb from the lens f, and

Continued.

the distance of its image from the same lens F, the length of the image will exceed that of the object in the ratio of F to f, or $\frac{F}{f}$ will represent the magnified state of the image. Hence it is manifest that the expression $\frac{F}{f}$ will have an increased value if we either augment F or diminish f. This is the fundamental principle to be attended to in adjusting the *runs, viz.*:—If the object-glass be protruded, it will approach nearer to the limb, whereby the size of the image will be increased and will require more traverse of the micrometer-screw of the microscope to measure it, and this will also be the case if the whole microscope be made to descend towards the limb.

Conditions of adjustment.

61. Before describing the method of performing the adjustments, it may be necessary farther to premise that the microscope is in correct adjustment when the following conditions are strictly fulfilled, *viz.*, when the image of the divided limb and the micrometer wires are so distinctly visible together that no parallax can be detected by varying the position of the eye, in which state of good vision, five revolutions of the screw must exactly measure one of the 5′ spaces of the limb, or ten revolutions measure two 5′ spaces.

Distinct vision of wires.

62. To obtain distinct vision of the wires, place a slip of white paper on the limb, and illuminate it by means of the reflector, the wires will thus be projected on a blank field, in which state the eye is better able to judge of their distinctness. Now slide the eye-piece in its cell, until the wires appear sharply defined. This is a personal adjustment, and varies with the focal length of the observer's eye.

Elimination of parallax.

63. If, on removing the paper, the limb is not visible, the microscope must be moved bodily up or down,* till the limb is seen distinctly. The means of effecting this differ in the various classes of instruments in use, and cannot therefore be described in detail here; but no difficulty in manipulation is likely to arise. Having rendered the lines on

* It can easily be found out whether the microscope requires to be lowered or raised to produce distinct vision, because, if, on thrusting in the eye-piece, distinct vision is produced, there is far parallax, and microscope must be lowered, and *vice versá*. After this experiment the eye-piece must be carefully restored to the position adapted to distinct vision of the wires.

the limb visible simultaneously with the wires, bring one of the divisions to the centre of the field, and intersect it with the micrometer wire. Now move the eye to the left or right, and if the intersection remain steady, there is no parallax; but if the object appears to shift its place with respect to the wires, then notice in which direction it moves and correct the parallax according to the instructions given in para. 44.

Adjustment continued.

64. The microscope having thus been freed from parallax, it will be necessary next to place a division of the limb, 0° for example, exactly under the centre of the microscope, and then to measure with the micrometer the number of revolutions and divisions between $359^\circ\ 55'$ and $0^\circ\ 5'$. If due care be taken to note the way in which the graduation on the micrometer head runs from 0 to $10''$, $20''$, $30''$, it will be found that one cut or stroke on the limb will be arrived at by revolving the micrometer in the same direction with the numbering of the seconds on the head, while the other stroke will be reached by a motion of the micrometer contrary to the order of graduation.* Subtract the reading of the latter from the reading of the former, and the difference will be the number of minutes and seconds measured by the micrometer. If this fall short of $10'$, the magnifying power is too small, and the object-glass must be protruded, in order to augment the image; but if the measured space exceed $10'$, the object-glass must be screwed upwards to diminish the image. Now, in the former case, protruding the object-glass will throw the image higher up the microscope, and produce near parallax, to correct which the whole microscope must again be bodily moved farther from the limb, and this again will diminish the run, so that the first correction should be a little over done to counteract the subsequent adjustment for parallax, and the whole process must be repeated two or three times, until the two conditions, *viz.*, correctness of run and freedom from parallax, are very closely satisfied.

Further precautions.

65. It must, however, be remembered that the divisions of the limb are themselves affected by error of graduation, to eliminate the effect of which it is desirable to take a mean of several $5'$ spaces. Moreover, the plane of

* Note that the final intersection of a division on the limb with the micrometer should always be made by screwing up against the spring, whether the motion of the microscope as a whole is with or against the order of graduation.

the circle may not be exactly at right angles to the axis of rotation, in which case all parts of the circle will not be at the same distance from the object lens, and there will be a difference of run occasioned by the circle approximating nearer to, or receding farther from, the microscope. It is also to be recollected that the excellence of the observations depends, not on a single microscope, but on the joint operation of all, so that although each may be near the truth, still the sum of their errors may be considerable; whereas, if the errors neutralised each other, the magnitude of the individual errors would not be of so much importance. To produce, therefore, the greatest degree of accuracy, the adjustment must be perfected with due regard to the foregoing considerations.

Final correction of one micrometer.

66. In general, with an instrument that has been previously in use,* the adjustments will be found already near the truth, and all that is required is occasionally an alteration of the run of a single microscope, so as to compensate the combined errors of the others. Now as the value of one micrometer has only the weight of one-third in producing the mean, so the correction to be applied to an individual microscope must amount to three times the sum of the errors of all the microscopes. For instance, if the mean value of three micrometers amount to $10' + 2''$ for a $10'$ space, then $3 \times 2'' = 6''$ is the correction to be applied to a single microscope to reduce the sum of the whole to $10'$, and this correction is much more easily applied than a smaller one to each.

Rule for obtaining the mean value of all the runs.

67. The rule to be adopted is therefore as follows:—Set the instrument to 0° by the index, and a whole degree stroke will then fall nearly under the zero of each microscope. Now, as the micrometer in observing is limited to less than a $5'$ range on each side of zero, it is unnecessary to go beyond that extent on either side. It is also immaterial to read the whole degree stroke in the centre, because the sum of the $5'$ spaces will be given at once by the reading of the extremes; consequently, it will suffice to measure the $10'$ spaces under micrometers *A*, *B*, *C*, respectively, repeating each measure at least once, for the sake of precision, and to avoid mistakes. Now turn the instrument round 180° in azimuth, and measure

* Unless the microscopes have been dismantled, or taken off for the purpose of being cleaned, or of allowing the axis to be cleaned, or for any other similar reason.

the 10′ spaces as before. The mean value of each microscope will thus be obtained from the mean of 10′ spaces at opposite diameters of the limb, and therefore in the case of three-microscope instruments the mean value of the runs of all the microscopes will be got from six 10′ spaces at every 60° of the limb apart, or from twelve independent 5′ spaces, which should suffice. Now, if the mean error of the microscopes amounts to more than 1″ in 10′, it will be advisable to correct that microscope, which is most erroneous, by a quantity equal to three times the combined error.* In performing this adjustment the rules for avoiding parallax (as given in para. 63) ought to be strictly adhered to.

Example.

68. The following example will render this subject quite clear, and it is only necessary to add that the runs should be taken at every principal station, before beginning the measurement of the angles, and duly recorded in the angle book, according to the form subjoined.

EXAMPLE.

Observations to determine the runs of the Micrometers belonging to Harris' 15-inch Theodolite in use with the Karára Meridional Series, 19th April, 1845.

Reading of the Index.	HORIZONTAL MICROSCOPES.									REMARKS.
	A.			*B.*			*C.*			
	Lowest Reading.	Highest Reading.	Difference.	Lowest Reading.	Highest Reading.	Difference.	Lowest Reading.	Highest Reading.	Difference.	
°	″	″	′ ″	″	″	′ ″	″	″	′ ″	The circle is divided into 5′ spaces, and the runs were taken from 5′ below to 5′ above zero, the reading of the centre division being omitted as of no effect upon the means.
0	32·6	24·6	9 52·0	30·6	31·1	9 60·5	42·5	45·8	9 63·3	
"	32·7	25·0	9 52·3	30·9	30·8	9 59·9	42·3	46·0	9 63·7	
180	30·2	27·9	9 57·7	34·0	36·1	9 62·1	25·2	22·5	9 57·3	
"	30·0	28·2	9 58·2	34·3	35·9	9 61·6	25·4	22·2	9 56·8	
Mean of each,			9 55·05	...	...	9 61·03	...	...	9 60·28	
Mean of three Microscopes,				...	...	...	...	...	9 58·79	
Error on 10′ run,			...	...	...	...	...	...	0 1·21	

* This correction, if the amount be only a few seconds, will not require the object-glass to be disturbed, because it will generally suffice to raise or lower the whole microscope, until the required value of the runs is correctly obtained. The rationale of this proceeding is that the microscope is more sensitive, as regards the value of the run, than it is with respect to parallax, or distinct vision. On account of the spherical aberration of the lenses, the exact position of the point of distinct vision is undefined to a small extent, and within this small limit the run may be sensibly varied without producing any appreciable parallax. This peculiarity will perhaps be rendered clearer by applying numerical values to the expression $\frac{F}{f}$ which represents the magnified state of the image. Let $\frac{F}{f} = \frac{4}{1}$, and let the microscope be raised 0·01 of an inch, which will make $f = 1·01$. This will reduce the image in the focus in the proportion of 4 to 3·96, in which ratio the run will also be reduced, consequently as 4 to 300″ so is 3·96 to 297″·0. Therefore the descent or ascent of such a microscope will produce a variation of 3″ for 0·01 of an inch perpendicular rise, while the latter quantity would hardly be perceptible in the shape of parallax.

EXAMPLE—(*Continued*).

Adjusted *A* Microscope to read about 3″·6 more, after which the runs were as follows :—

Reading of the Index.	HORIZONTAL MICROSCOPES.									REMARKS.
	A.			*B.*			*C.*			
	Lowest Reading.	Highest Reading.	Difference.	Lowest Reading.	Highest Reading.	Difference.	Lowest Reading.	Highest Reading.	Difference.	
°	″	″	′ ″	″	″	′ ″	″	″	′ ″	
0	20·5	16·4	9 55·9	40·5	40·9	9 60·4	52·4	55·9	9 63·5	
,,	20·6	16·3	9 55·7	40·8	40·9	9 60·1	52·5	55·8	9 63·3	
180	18·1	19·6	9 61·7	44·2	46·0	9 61·8	35·3	32·4	9 56·9	
,,	18·0	19·8	9 61·8	44·8	46·5	9 61·7	35·3	32·3	9 57·0	
Mean of each,			9 58·73	...	...	9 61·00	...	...	9 60·18	
Mean of three Microscopes,				...	...	...	...	...	9 59·97	
Error on 10′ run,			...	...	...	...	...	...	0·03	
Error on 5′ run,			...	...	...	...	...	...	0·015	

69. The vertical microscopes are similarly adjusted, and their runs should be taken at every station, and duly inserted in the angle book according to the following form :—

Runs of vertical microscopes.

Reading of the Index.	VERTICAL MICROSCOPES.						REMARKS.
	A.			*B.*			
	Lowest Reading.	Highest Reading.	Difference.	Lowest Reading.	Highest Reading.	Difference.	
°	″	″	′ ″	″	″	′ ″	
F. R. 0	27·5	23·0	19 55·5	32·0	22·1	19 50·1	The vertical circle of this instrument is divided to every 10′.
,,	28·0	23·9	19 55·9	31·1	23·0	19 51·9	
F. L. 0	24·2	18·2	19 54·0	34·0	35·5	19 61·5	
,,	24·7	17·7	19 53·0	34·1	35·5	19 61·4	
Mean of each,	...	...	19 54·6	...	...	19 56·2	
Mean of two Microscopes,...			...	...	...	19 55·4	

Adjusted *A* Microscope to read about 9″ more after which the runs were as follows :—

°	″	″	′ ″	″	″	′ ″	
F. R. 0	15·2	19·7	19 64·5	20·2	11·7	19 51·5	
,,	15·6	19·8	19 64·2	19·8	11·5	19 51·7	
F. L. 0	12·8	15·6	19 62·8	21·0	22·3	19 61·3	
,,	12·5	15·1	19 62·6	21·4	22·5	19 61·1	
Mean of each,	...	...	19 63·5	...	...	19 56·4	
Mean of two Microscopes,...			...	...	...	19 60·0	
Error on 20′ run,		...	...	...	...	0·0	

70. The foregoing operations are very liable to disturb the lateral adjustment of the microscope, as this is regulated by the three screws, which serve to move the microscope bodily a short space, so as to bring it over its proper division on the limb. After having, by means of these lateral screws, brought the microscope nearly over its proper division, the remaining minute portion of lateral adjustment can be readily effected by moving the comb, for which purpose there is a screw at the reverse end of the micrometer box. If the comb be moved or adjusted in the slightest degree, care must be taken to set the zero of the micrometer head to correspond with the zero of the comb. For this purpose take the milled nut of the micrometer head in one hand, and the graduated head in the other hand, then turn the latter round (taking care not to turn the micrometer screw also), and when zero on the head corresponds with the index on the micrometer, the adjustment is complete. The graduated head only retains its place by friction, therefore it is necessary to be careful in working the micrometer not to lay hold of the graduated head, but to use the milled nut which is given for that special purpose.

Final adjustment of position.

71. The microscopes should be fixed 120° apart in a three microscope theodolite, and should coincide with a whole degree stroke when the index is at 0°; but on account of the errors of graduation, it is advisable, in order that the minutes appertaining to microscope *A* may also answer for *B* and *C*, that the latter should be set to read 30″ or 40″ in excess, whereby the minute given by *A* will be common to *B* and *C*, provided care be taken to add 60″ to the readings of the latter when they fall into the next minute above. Suppose *B* and *C* are set to read 30″ and 40″ more than *A*, then if the reading of *A* is 0° 3′ 50″, that of *B* will be recorded as 80″, and that of *C* as 90″, which number of seconds are referable to 0° 3′. If this precaution be not taken the minutes, and sometimes the degrees, may require to be separately registered for each microscope, because, on account of errors of graduation, readings would frequently be obtained, as shewn in the following example, which would prove inconvenient for registry :—

Distance between microscopes.

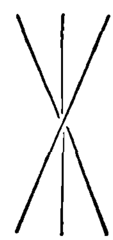
A	*B*	*C*
0° 0′ 10″	359° 59′ 50″	359° 59′ 55″
62 11 5	10 45	10 55

Adjustment of vertical microscopes.

72. The vertical microscopes must, besides being set diametrically opposite to each other, be freed from index error; that is to say, when the telescope is pointed to an object in the horizon the microscopes must be at zero, or if the telescope be pointed to an object of known elevation the microscopes must be set to give that elevation. The true elevation of an object may be obtained by observations on both faces, the mean of which when corrected for level gives the required elevation.

Final tests.

73. There remain yet to remark a few other points which should be attended to. For instance, it is clear that the micrometers should act in a tangential direction to the circle; therefore, if during the foregoing adjustments the direction of the microscope should by chance be changed, all that is necessary is to turn the microscope gently round in the collar until the micrometer is brought to act in the proper direction, when the collar-screws should be tightened, to render its position permanent. If the wires form a cross, and the divisions on the limb are lines, then the tangential direction ought to be correctly attained, when the angles of the cross wires are truly bisected by the divisions, as shewn in the margin. Other instruments may be differently constructed, but the peculiarities of such cases will suggest the method of placing the micrometer in a tangential direction.



Continued.

74. It is usual also at this time to take care that each microscope is set over the same part of the division lines, that is to say, the microscopes should be equidistant from the centre of the circle.

Avoidance of lost motion.

75. It has been mentioned in a footnote to para. 64, that the intersection of a division should always be made by screwing against the spring. This is done in order that the shoulder of the micrometer head may be made to press steadily on its proper bearing. When the micrometer head is released it would leave its bearing, and the screw would not be retracted but for the action of the spring, upon the efficiency of which the uniformity of the motion of the micrometer during the releasing process entirely depends. It has, however, been found in practice that, owing

to irregularities of friction, or other causes, the spring does not act upon the screw uniformly and instantaneously. The head may be released by a small quantity before motion takes place. This is called lost motion, and occurs in all screws when the action is suddenly reversed. No reliance can therefore be placed on the action of a micrometer, unless the spring and screw are acting in opposition, whereby the bearing of the female screw is preserved, and uniformity and steadiness of action is maintained in immediate obedience to the rotation of the micrometer head.

Selection of division of limb for intersection.

76. It is not to be supposed, from what has just been remarked, that the micrometer is never to be turned to the left, that is to say, released; because it is clear that, if constantly turned one way, the wire would soon arrive at the extremity of the field. The screwing up motion is indispensable only for the purpose of finishing an intersection. The rule is, to select that line or division which appears to fall nearest to zero in the micrometer, whereby the excursions of the wire are limited within a range of 5′. If the screw requires to be released, in order that the wire may reach the line which it is intended to intersect, then all that is necessary is to overshoot that line, by carrying the wire about a minute beyond it, and then to reverse the motion. Complete the intersection by the screwing up rotation.

Precautions as to illumination.

77. The graduation of an instrument consists of very fine engraved lines; and it is obvious that if the light falls obliquely on the graduation, one side of the lines will be illumined, and the other will cast a shadow. The apparent centre of the line, will in this case not coincide with the true centre, and if the graduation be coarse, the discrepancy produced may amount to two or three seconds. The obvious precaution to be taken is to throw light perpendicularly on the graduation, taking care that the field is equally illumined, and that both sides of the division lines are seen equally distinct, and sharply defined. Care should be taken in illuminating the field of the telescope at night to avoid any side light upon the wires, and not to alter the illumination between observations of two adjacent signals.

Necessity for shelter.

78. No good observations can be expected if currents of air are allowed to impinge on the instrument, or if it is subjected to the influence of solar rays, or

any other causes calculated to disturb the equilibrium of the expansible materials of which an instrument is composed. Independent of this evil, it is also to be remarked that, if the wind is allowed to blow on the telescope, it frequently produces a lateral deflection to the amount of a few seconds. To guard the instrument from disturbing causes of this nature, it is sheltered by a tent, called an observatory tent, the observations being taken through small windows protected by movable *purdahs*.

Care in recording.

79. It is not, however, sufficient to be skilful and scrupulous in the manual and optical processes of observing; equal care and attention must be bestowed on the registry of the data in the angle books, otherwise the care and skill bestowed on the observations will be wasted.

Duties of recorder.

80. The observer should give out the degrees and minutes in an audible voice, and the recorder should repeat them while he is writing them down: unless this precaution be taken, errors may arise from peculiar pronunciation or defective hearing. Similarly, the readings of the micrometer heads should be given out distinctly to the nearest tenth of a second, and duly repeated by the writer. The figuring on the micrometer heads is seldom neatly executed, and a liability sometimes exists to read 30″ for 50″, and *vice versâ*, which mistake in one microscope of a three-microscope theodolite would affect an angle by nearly 7″. Errors of this and similar kinds can only be avoided by strict attention. The recorder should examine, when the observations are repeated, whether any reading is discrepant, and should give due notice to the observer of such an occurrence. The relative readings of the microscopes will generally serve to detect any error to the amount of a minute; in the reading of B and C, for instance, if these be set to read always in excess of A, it will be necessary occasionally to add 60″ to their indications. The recorder should always take out the angles while the observations are in progress, to enable the observer to note any errors at the time and to repeat a measure, if necessary, according to the precept laid down in para. 21. Some observers take no heed of the results while they are engaged in observing, but make out the angles afterwards, a practice which cannot be too strongly deprecated.

Original and duplicate angle books.

81. The book in which observations are recorded is called an angle book. Two of these books are kept on each series, one of which is used in the observatory and called the "Original Angle Book," the other is a transcript of the former and is termed the "Duplicate Angle Book." These should be paged, indexed and signed at the foot of each page. The original is retained in the Dehra Office and the duplicate ultimately sent to the Calcutta Office.

Angle books continued.

82. The angle book should on no account be suffered to fall into arrears. The original should be examined by two computers, and attested by their signatures, as well as by that of the observer. The duplicate should be compared with the original by two persons, and attested by their signatures. It is a standing rule, that, in order to avoid mistakes, all computations and comparisons should be performed independently by two persons, who should each sign both copies; and, unless such precautions have been observed, the results are considered untrustworthy as final work. Observations should never be recorded on loose scraps of paper, or in pencil in rough books. The original record is always the most valuable document, and the labour of copying, besides being expensive, causes delay. It is usual to add up all the angles observed on any one zero together, and divide them by the number, by which means as many sets of means are obtained as there are zeros, each mean being considered as an integral observation. It is a fixed rule never to reject an observation, unless there be some obvious error in it, an explanation of which should be inserted in the column of remarks: the circumstance of its differing, however widely, from the mean is not a sufficient cause for its rejection.

83. Having now remarked upon all the minutiæ to be attended to in the observatory*, it still remains to inculcate the necessity of the signals being carefully adjusted over the station marks.† To insure this condition being fulfilled, the men who work the lamps and heliotropes should be trained to perform their parts with skill and fidelity.

Residual errors.

84. Notwithstanding all the precautions which may be taken in making the observations, and recording them with fidelity and precision, it will be found that

* It may here be remarked that in observing azimuthal angles, the vertical clamp and tangent-screw should never be used, as it is liable to deflect the telescope laterally. The telescope should be raised or lowered to the proper altitude of the object by the action of the hand only, the vertical clamp being left quite free. Similarly, in observing vertical angles it is unnecessary to clamp the azimuth circle; the vertical clamp and tangent-screw should alone be employed.

† A condition as indispensable to accuracy as the centering of the instrument itself.

every mean angle is nevertheless burdened with small residual errors, which it is impossible entirely to eliminate, and which must therefore be dispersed in the subsequent computation, agreeably to the rules which will hereafter be given. These errors arise partly from imperfections in the instrument, and in the observer, and some part of them may also be attributed to unavoidable sources; such, for instance, as the displacement by refraction to which an observed object is liable on account of the rays of light passing through a medium of very variable density and temperature.

Uncertainty of refraction.

85. There is reason to believe that refraction takes place chiefly in the vertical plane; but owing to the smoke and various vapours perpetually rising from the earth probably not having an uniform density at a given height above the surface, the ray in its passage through these must unquestionably be to some extent, liable to lateral as well as vertical refraction. We see, in fact, both by night and day that this cause is perpetually in operation; for the small disc of an argand lamp, which is only twelve inches in diameter, and in a clear, settled atmosphere, is reduced to a luminous point, swells out sometimes for several nights in succession into a broad ill-defined blaze, subtending occasionally two minutes of the horizon, and vibrating more like a sheet of fire than an object intended for accurate intersections, whilst by day the visible disc of the heliotrope, though only two inches in diameter, is even wilder and more straggling.

Time for observing horizontal angles.

86. The only method of overcoming these sources of irregularity is to await a favourable state of the atmosphere, and be prepared to profit by it when it offers itself. Such occasions occur for day observation almost every sunny day for a shorter or longer period, between half-past three P.M. and sunset. The lamps are also frequently well adapted for observation from sunset to past midnight, especially in the early part of the season, but at other times they are steadiest from after midnight till about sunrise.

Precautions against lateral refraction.

87. In hilly countries, where the stations are placed on lofty peaks, with deep intermediate valleys, objects are less disturbed by atmospheric causes than under other circumstances. Very good observations may then be obtained for

an hour or two after sunrise; but, in general, however promising the apparent state of the atmosphere may be at sunrise, it can rarely be depended on. It is at this period of the day that *mirage* is most conspicuous. Two or three heliotropes are sometimes seen piled one above the other, or placed side by side, sometimes uniting together to form columns or pillars of fire subtending several minutes, although the rays of light pass through an aperture limited to two inches in diameter. This kind of lateral refraction is, however, too conspicuous to be overlooked; but there is another insidious sort which ought carefully to be guarded against, because at the time it is in operation the visible disc of the lamp or heliotrope appears at first sight small, well-defined, and steady; if, however, it be carefully bisected by the wire, and then watched, it will be found after the lapse of a few minutes gradually to diverge to one side, and after a shorter or longer time will again move off slowly in another direction. This treacherous state of the atmosphere is very frequently in operation, and the only remedy is to watch carefully, and adjust the position of the telescope so that the object may appear to diverge equally on each side of the wire. With this precaution good observations can be made, but each intersection occupies several minutes of time, and the observations proceed very slowly.

Test of good work.

88. The accordance, or otherwise, of angles taken on the same zero, enables an observer to judge of the accuracy of his intersections, and the average errors of the triangles will show how far care and precaution have succeeded in eliminating the imperfections of the instrument, and other sources of error. Even with the best theodolites an observer may be satisfied with his work, if his triangular error does not average over one second.

Deduction of heights of stations.

89. The formulæ from which the relative heights of the stations of a Trigonometrical Survey are deduced are explained in the third Edition of the Auxiliary Tables, page 24 *et seq.*, and more fully in Chapter XIII, Volume II of the "Account of the Operations of the Great Trigonometrical Survey of India," and it will be unnecessary to enter here upon this part of the subject. The procedure in the field is limited to the careful measurement, 1st, of the vertical angles, with the corresponding level readings, and 2nd, of the heights of the telescope of the theodolite and of the signal above the platforms whose relative heights are required.

Reciprocal vertical angles.

90. It is generally assumed that the refraction at both stations is the same; but, in order to justify this assumption as far as possible, the back or reciprocal vertical angles ought to be taken at one and the same instant.* But as this is not always possible, the next best method is to be careful to observe them about the same hour from noon, for then the errors caused by terrestrial refraction are more likely to compensate each other. The best period of the day for observing verticals is the time of minimum refraction, which usually occurs from about 2-45 to 3-45 P.M. Long experience has proved that reciprocal vertical observations, taken at that period of the day, notwithstanding the unsteadiness of the heliotropes, give the most accordant results, and may therefore be confidently relied on. If the stations are not visible at the time of minimum refraction, the precaution should be taken of observing them simultaneously, by means of two instruments and two observers.

Number of observations necessary.

91. Four sets of observations, two with the face each way, are sufficient at each station, except in extensive plains, where a second and third day's measures are desirable. All stations that are visible should be observed, as verificatory results may thus be secured. Vertical observations taken with the face each way, that is to say, with the face of the vertical circle in one instance to the left hand, and in the next to the right, are called collimated observations, because the mean of the two faces gives a vertical angle free from error of collimation, which is not the case with observations on a single face.

Method of observation.

92. The method of observing is as follows:—The instrument should be duly levelled and adjusted previous to the time of minimum refraction; and as soon as signals are steady, or nearly so,† direct the telescope to one of the principal stations, and having clamped the vertical circle, bisect the signal with the horizontal wire, and note the hour and the minute at which the observation was made. Now note the readings of the two ends of the level that is attached to the vertical microscopes,‡ calling

* This was done on the Great Indian Arc, in crossing the plains of Hindostan, because the vertical angles were there subject to the influence of extraordinary refraction and *mirage*, on account of the rays grazing along the surface of the ground.

† Heliotropes are seldom perfectly steady till 4 or 4½ P.M., but at that period they are beginning to rise at the rate of a second per minute. Although they may appear a little agitated at 3 P.M., still the refraction is then least, and the observations give the best results. In fact, any error arising from the agitated aspect of the heliotrope will be much less injurious than refraction, and indeed may be counteracted in a great measure by multiplying the observations, because the agitation of the disc does not produce a constant error in one direction, as happens in the case of uncertain refraction.

‡ Theodolites differ in the manner of attaching this level but its indications must always be read whatever its construction.

them "eye end" and "object end" respectively, and read off the microscopes of the vertical circle, according as the vertical angle may be an elevation or a depression, and let the readings be duly recorded in the angle book; release the clamp; turn over the telescope 180° in a vertical plane, and the instrument round 180° in azimuth; bisect the same station again, and record the readings of the level and microscopes as before, together with the hour and minute of time at which the bisection was made. The mean of the two observations will give one collimated vertical angle. Repeat the last observation as before, and then reverse the instrument, and take another observation, which will complete the second collimated vertical angle. If these two be accordant they will suffice, otherwise a third, and even a fourth, ought to be taken. Now direct the instrument to another station, and take the vertical angles in the same manner, and in that way proceed until all the surrounding stations have been observed; but if the signals begin to rise, which will be ascertained from the depressions diminishing, or altitudes increasing, it will be necessary to desist for that day, and reject any observation which may have been made after the time of minimum.

Reciprocal observations.

93. On arrival at a station which has been observed from one in rear, the back or reciprocal observations must be taken within a minute or two of the corresponding times. If any atmospheric change should appear to take place intermediately to the observations of the reciprocal vertical angle, the equality between the refractions, which the nature of the problem demands, is likely to be destroyed; and therefore too long a space of time ought not, when it can be avoided, to be allowed to elapse before the reciprocal observations are made.

Different methods of recording elevation and depression.

94. It must be borne in mind that if the instrument gives altitudes on one face, it will give zenith distances on the other, and the graduation on the micrometer head will, in this case, be according to the order of the figuring; consequently, the system of reading altitudes on one face, and zenith distances on the other, is, perhaps, the safest course to pursue. Occasionally, however, altitudes are read on both faces, by taking the seconds from the micrometer head in an inverse order, thus 50″ is read 10″, 40″ is read 20″, and so on, which gives altitude instead of zenith distances. This was the method followed on the Great Arc, and several

other Series, and with care and experience it may be practised with facility, and without liability to mistakes. With regard to depressions it may also be remarked that the face of the instrument which gives altitudes according to the direct order of graduation, will give nadir distances in the same direct order for all objects below the horizon; and on the opposite face which gives zenith distances according to direct order of graduation, depressions may be read off in direct order. Instead of nadir distances, however, depressions may be read off at once by supposing the divisions on the micrometer head reversed, *viz.*, 50″ = 10″, 40″ = 20″, &c., as in the case of altitudes.

Continued.

95. Those instruments which give altitudes and nadir distances in direct order on one face, and zenith distances and depressions on the other, are numbered in four quadrants from 0° to 90°, following each other round the circle; that is to say, 0°, 10°, 20°, &c., to 90°, 10°, 20°, &c., to 90°, 10°, 20°, &c., to 90°, 10°, 20°, &c., to 90°; the last 90 being the zero of the first quadrant. Other instruments however are divided from 0° at the eye and object end to 90° both ways. These figures, as far as the circle is concerned, give altitudes and depressions on both faces; but if the ordering of the graduation on the micrometers on one face is direct for elevations it will be reverse for depressions, and *vice versâ* on the opposite face.

Varieties of graduation.

96. On account of the variety of graduation which obtains in different instruments, no fixed rule can be laid down, and the peculiarities of each must therefore be studied carefully by the observer.

Rules for recording elevations and depressions.

97. The following rules obtain in all instruments:—For altitudes the micrometer should be moved to the division apparently above the zero of the micrometer at the eye end, and to the division below zero of the microscope at the object end. It is, of course, exactly *vice versâ* with respect to depressions; that is to say, the divisions which appear below zero at the eye end, and above zero at the object end, should be chosen. Now, if on revolving the micrometer head, so that the wires shall proceed from zero in the microscope to the divisions above indicated, if it be remarked that the graduation on the micrometer head follows in the natural order of numbering, then the reading must be taken direct, otherwise it must

be made reverse. After studying each instrument a rule can generally be framed like the following which applies to Troughton and Simms' 36-inch theodolite, and probably to others by the same makers, *viz.* :—

F. L. { Elevations read direct.
Depressions read reverse.

F. R. { Elevations read reverse.
Depressions read direct.

A rule such as this, having once been carefully deduced for the particular instrument in use, will prevent subsequent mistakes.

Precautions in adjusting vertical microscopes.

98. The same precautions which are necessary in adjusting and reading the azimuthal microscopes are also indispensable in using vertical microscopes. They should be freed from parallax, and adjusted for runs; they should also act in the direction of a tangent to a circle, and all measurements should be made with that motion of the screw which draws against the spring; moreover, they ought to be set truly horizontal, with respect to the axis of azimuth motion and the division of the vertical circle, as they will then be free from index error, or nearly so.

Elimination of collimation in vertical microscopes.

99. Although the mean of two observations with the face each way gives a result independent of collimation and level error, still, when the former is large much inconvenience is felt. All that is necessary to free the instrument from this error, or rather to render that error very small, is to take the altitude or depression of any fixed terrestrial object on both faces, and deduce the mean, which will be the true vertical angle. Then, while the telescope remains firmly clamped, with the object truly bisected on its horizontal wire, move the microscopes by their appropriate screws, till they give the true reading of the vertical angle, as deduced from both faces. In all instruments some method of moving the microscopes through a small space laterally is provided, varying according to the ideas of different makers; by means of this adjustment the whole microscope must be moved, until the cross wires coincide with the proper division on the limb. This adjustment, however, will not generally allow of the microscope being set much nearer than a minute, and the remaining part must be effected by altering the graduated head of the micrometer, and then setting the comb to correspond therewith, as described in para. 70, when describing the adjustments of the azimuthal microscopes.

100. In para. 89 it has been remarked that the height of the instrument, and of the signal above their respective platforms must be recorded. The former should be carefully measured by the observer, and the latter by the men in charge of the heliotropes and lamps, so that on arrival at each station, the heights of the signals observed may be duly verified, and noted in the form subjoined.

Heights of eye and object.

EYE STATIONS.			OBJECT STATIONS.			
NAME.	Height of		NAME.	Height of		By whom Measured.
	Marks above Ground.	Instruments.		Heliotrope.	Lamp.	
	Feet.	Inches.		Inches.	Inches.	
Imlia ...	0·0	69·75	Asrofpur ...	26·	...	Ramdeen.
	8		Thana ...	30·	...	Jeri.
	24		Ragaopur ...	82	...	Bikari.
			Samnadeo ...	30	...	Manbod.

101. Hitherto we have only considered the case of stations at which the back, or reciprocal, vertical angle can readily be observed; but it is clear that if the amount of refraction were known, the relative heights could be deduced from verticals taken at one extremity of a ray: this method is however not considered sufficiently rigorous for determining the heights of principal stations, though it is largely used in secondary work and will be more fully explained in the next Chapter which describes that class of operations.

Computation of refraction.

102. It has been mentioned in para. 56, that luminous signals are always employed in principal triangulation. These are of two kinds, *viz.*, heliotropes used for reflecting the sun's rays by day, and lamps with argand burners, to consume either oil or kerosene, for night use. A heliotrope (called also heliostat and heliograph) consists of a plane circular mirror, so mounted on a small tripod as to be movable on two axes, one vertical and the other horizontal, whose directions pass accurately through the centre of the mirror, at which point a small hole about $\frac{1}{10}$ of an inch diameter, is drilled through the glass. Each heliotrope is accompanied by a small

Description of signals.

sight-vane, consisting either of an aperture in a board with two cross wires spanning it; or of a leaden weight (or even a lump of clay) of 2 or 3 lbs., into which is inserted vertically a straw or very thin slip of wood, on which a piece of thread is tied loosely, so as to be movable up and down at pleasure. The latter very simple contrivance has been used for many years in the Topographical Branch, and found to be perfectly serviceable, and of a construction so simple and easily improvised anywhere, as to leave nothing to be desired on that score. In principal triangulation the sight-vane for the heliotrope is attached to the top of the lamp box as shown at *A* in Plate III. Heliotropes are made of various sizes,—8 inches diameter will perhaps be found the most convenient, being very portable, and yet quite capable of being used as a signal at 60 or 70 miles. For distances less than 20 miles it will be found advisable to reduce the light by means of diaphragms. A two-inch or three-inch aperture suffices for 15 miles, if the heliotrope be carefully aligned. It is convenient for the observer to carry with him a heliotrope of larger size, say 12 inches diameter, for the purpose of flashing to the signallers, when he wishes them to show their own heliotropes.

Method of aligning heliotropes.

103. The process of aligning a heliotrope, or so placing it that the sun's rays are reflected in any desired direction, is very simple, and soon learned by ordinary *khalásís*. It is carried out as follows:—The sight-vane is carefully centered over the station mark, and the heliotrope then placed at a convenient distance from it, (about two feet is suitable) and facing the point on which it is to be aligned, in such a manner that the *khalásí* can see both the sight-vane and the distant mark, through the hole at the centre of the heliotrope. By moving the heliotrope slightly, the distant mark, the cross wires of the sight-vane, and the hole, are then placed in the same straight line. The mirror should now be turned towards the sun by means of its motion on the two axes, (the tripod remaining fixed) in such a direction that the image of the hole, which is a small black speck in the middle of the bright reflection of the mirror, falls on the cross wires of the sight-vane. An observer at the distant mark will then see the sun reflected in the mirror.

Continued.

104. If the heliotrope is situated nearly in a line between the observer and a low sun, it is found rather difficult to adjust it accurately; in such cases the difficulty may be got over by the use of a second heliotrope, which is so

placed as almost to face the sun, and reflect its rays on the first one, which can under these circumstances be made nearly to face the observer. A little ingenuity will enable the observer to arrange a code of signals for conveying a few simple orders to the heliotropers; it is unnecessary to specify such in detail, as each triangulator will probably prefer his own; but there is one that is almost universally used in the Department for calling in a heliotroper when no longer required, and that is, the display of two heliotropes simultaneously about 50 yards apart.

Adjustment of lamp.

105. In Plate III are given drawings of a lamp as now used for a night signal. Formerly country oil used to be burned, but in the present day kerosene is found more convenient. The part of the lamp that has to be centered over the mark-stone is the middle of the glass disc in the front of the lamp box. Two nicks *G G*, one at the top and one at the bottom of the box, are so made that the line joining them bisects this glass disc. When properly adjusted the flame of the lamp should be very nearly centrical with regard to this glass, and the reflector so placed that its axis may coincide with the ray to the observer's station. This adjustment can only be done by estimation, but the construction of the lamp is such that, when the box itself is properly aligned by means of the sights on its upper surface, the positions of the flame and reflector are very closely correct.

Continued.

106. The approximate adjustment of the alignment of the lamp is performed by means of the eye-hole *B* and a pin *C*. The apparatus stands on three legs *T*, two only of which are seen in the drawing. The plummet *P* hangs from the top of the heliotrope sight-vane, and when correctly placed its line should pass accurately through the cross wires of the sight-vane of the heliotrope, through the two nicks on the lamp box, and through the station mark. The heliotrope *H* stands on a board *D* which slides in a groove, and is removable for convenience of packing. The boards *E E*, which are screwed on to the top of the box, are intended to receive a second heliotrope for double reflection as described in para. 104. The four small battens *F F F F* are grooved to receive a sliding panel to protect the glass, when the lamp is not in use.

Description of stations.

107. The last point upon which it is necessary to dwell in the instructions for principal triangulation, is the urgent necessity for a full, clear, and intelli-

gible description of each of the principal stations of observation. There is no reason why the trigonometrical and astronomical stations of the Survey of India should not, with ordinary care, last for centuries; but, in order to be of real service at any future time, the sites should be named and described in such a manner, that they can be readily identified by any reasonably intelligent person. It is not pretended that this can always easily be done, especially in the thinly-inhabited, forest-clad districts met with in many parts of India and Burma, but the observer must ever bear in mind that it is a point of the highest importance, and strenuous endeavours must be made to secure it.

Continued.

108. *Firstly* as to the name. When the station is situated on a hill with a well-known local name, it will be best to retain that name. In other cases the name of the nearest village, or of the village on whose lands it is situated, must be given to it. *Secondly* as to the description. This must embrace an account of the details of its construction, the height and size of the platform and pillar, the number and positions of the mark-stones embedded in them, also the materials of which they are built, and whence they were brought. If on a hill, the best means of ascending it should be specified; and the village, if any, from which the ascent should be made, or supplies obtained, should also be entered in the description. The approximate distances and bearings of several neighbouring villages, temples, hills, rivers or other permanent objects should be stated, and any other information given which would simplify the identification of the site subsequently, not omitting the names of the province, district, thána, tahsíl, pargana or other local sub-division in which the station is situated. The description, which should be written at the time of visiting the station, must be in a clear and legible hand in both the original and duplicate angle books; names must be hand-printed, or at the least written in a large round hand, due care being taken to ensure orthography in the transliteration of native words. An authorized list of the correct spelling of proper names can be obtained for most districts, and should be in the possession of the party for ready reference.

Stations revisited.

109. When any stations of previous triangulation, which have been already described and closed according to the directions given in paras. 18 and 19, are visited, the condition in which they are found must be stated, in order to obviate any uncertainty whether the original site has been recovered or not; the

measures taken for subsequent protection should also be described, or any alterations that may have been made in the construction of the station. Occasions have arisen in which some doubt has been experienced as to which of two closely contiguous stations has been occupied. The observer should be alive to such contingencies and invariably notice in his description of stations any such possible cause of confusion.

CHAPTER II. SECONDARY TRIANGULATION.

Intersected points.

1. The object of secondary triangulation is purely topographical. The principal series provides too few points for the detail surveyors, and the secondary work supplements the principal by breaking down its large triangles into smaller ones, and fixing in addition a great number of minor points by rays taken to them from two or more stations. These minor points are generally known in the Department as *intersected points*, and are not as a rule visited by the triangulator, who contents himself with fixing them by the intersection of two or more rays, whence the name of *intersected points*.

Chains or networks.

2. The secondary triangulation may be laid out either in chains or a network, or a combination of the two, as may seem most suitable for the particular piece of country under survey. Where any large river with a broad valley intersects the district, a series may with advantage be carried along it, with stations on either bank, the only disadvantage being the necessity for frequent crossing and recrossing. This can, however, be partly obviated by the judicious co-operation of two observers, one on either side of the river. Most commonly, however, a network is the more convenient plan of proceeding.

Vernier theodolites.

3. The rigorous principles laid down in Chapter I may be considerably relaxed in the case of secondary work. In the first place vernier theodolites of the better class have been generally considered sufficiently good for the measurement of the angles. They are considerably lighter than the microscope theodolites, and simpler in construction and better adapted for the use of Assistant Surveyors, to whom the secondary triangulation is generally entrusted.

General adjustments.

4. The adjustments of vernier theodolites are, for the most part, the same as those of microscope instruments, differing only in the case of the verniers themselves: these require no adjustment, being immovably fixed at the proper distances apart by the maker. The descriptions given in paras. 27–56 of the previous Chapter apply to both classes of instruments; and any observer, who has mastered them will have no difficulty in applying his knowledge to the simpler form of theodolite.

Adjustment of vertical verniers.

5. The adjustment of the vertical verniers needs a few remarks. The true vertical angle to any distant well-defined point must be determined by observations on both faces, corrected for level error, as described in para. 99, (the correction for level may be avoided by bringing the bubble to the middle of its run for both observations by means of the clips, before reading the verniers). The verniers must now be set to read as nearly as possible the value of this vertical angle, and the telescope made to intersect the distant object *by the clips only*. It will now be found, probably, that the bubble of the level attached to the verniers is displaced from the centre of its run; it has only to be brought back by its own adjusting screws, when the process will be complete. It will never, except by pure chance, happen that both verniers can be made to read the true vertical angle; in practice there is always some discrepancy between them. The best plan is to make the mean of their readings equal to the true vertical angle, by which their combined index error will be reduced to the lowest quantity possible; but it will be always safer to observe vertical angles on both faces, when great accuracy is aimed at.

Angles to be observed.

6. It is very desirable, though not indispensable, that all the three angles of every triangle should be observed. One of the most important reasons for observing all the angles, especially in secondary work, is to avoid gross errors, such as 10°, resulting from an erroneous reading of the figures on the limb: in the case of a simple chain such an error might escape detection, and lead to endless trouble and uncertainty if it occurred in a triangle of which two angles only were measured. In the case of a network such a fault could be localized almost with certainty, and therefore the measurement of the three angles is not of such vital importance; still, however, it renders the work more complete and

satisfactory, and should not be neglected except on good grounds. In fixing intersected points it is advantageous, when practicable, to observe them from more than two stations. If observed from one only, it is obvious that they cannot be localized at all; if from two, they are fixed by one triangle only, and there is no certainty that the same point has been intersected from both stations; whereas if observed from three or more stations, the data are sufficient to give two or more values of one of the rays, and therefore to furnish a test of the correctness of the identification, by the agreement or otherwise of the several values.

Number of zeros.

7. The number of zeros may be considerably reduced on comparison with those required for principal work. Two zeros and two changes of face should suffice even for fairly large triangles: a twelve-inch or fourteen-inch theodolite is generally preferred for this class of work. Luminous signals should be employed for any rays exceeding 7 or 8 miles in length; for shorter ones the pole and brush, such as is shown in the wood-cut in para. 19, Chapter I, is suitable in hilly country, if it stands up against the sky-line. In flat country, or where the back-ground, against which the signal is likely to be projected, is nearly the same colour as itself, a white-washed *gharra* may be placed on the top of the pole, or the pole and brush may be altogether superseded by a small bell tent.

Centering the theodolite and signals.

8. The care in centering the theodolite and the heliotropes should not be relaxed, as the menials employed will in all probability become careless about this process, if they observe that the officer in charge is not always equally scrupulous about its accuracy; and it should be remembered that, as the signallers are under no sort of supervision when on detached duty, it is doubly necessary that they should acquire a *fixed habit* of always centering with the greatest attainable nicety, whether really requisite or not for the particular operation in hand. Heliotropes when used for distances of 4 or 5 miles should be reduced by stops to about 2 inches diameter; if the light thus diminished is still too dazzling, an effectual way of subduing it is by holding a single fold of linen or cambric a few inches in front of the object-glass of the telescope.

Selection of stations.

9. Stations for secondary triangulation should be so selected as to obtain as far as possible an uninterrupted view on all sides. If the station is situated on a

forest-clad hill top, it will be found a very convenient plan to leave one large tree standing as close as possible to it, after clearing the jungle away; such a mark is frequently found of great use to plane-table surveyors. The same remark applies to hill tops used for intersected points. The platforms for the more important stations must be built in much the same fashion as the principal stations, with an isolated central pillar and a mark-stone, unless the mark can be cut on the rock *in sitû*. Masonry tower stations are never likely to be required, and should not be built without special sanction.

Description of intersected points.

10. In the column of remarks in the angle book a brief description of each intersected point, when necessary for identification, should be given; such as the particular corner or minaret of a building observed, a single tree, or pile of stones on a hill top &c., &c., and if such points are lettered or numbered, great care must be taken that each point is always distinguished by the same number, when observed from two or more stations. The horizontal angles between the stations and intersected points may either be taken on one zero without change of face, in which case the readings should be checked by the recorder calling out the setting of the limb, and the observer noting, after the limb has been put to that setting, whether the point is duly intersected: or they may be taken on one zero with one change of face, in which case the above check is unnecessary, as the recorder can see at a glance whether the two readings differ by 180° as they ought. The latter method is certainly the better, the only objection being that it takes up slightly more room in the angle book, and adds a few more figures to the computations. The means of horizontal angles between secondary stations should be retained to one place of decimals of seconds, and between stations and intersected points to the nearest second only. In observing objects such as large trees, rounded hill tops &c., which are not capable of accurate intersection, it is unnecessary to record the reading of more than one vernier on each face.

Vertical angles.

11. Vertical angles should in all cases be taken on both faces, otherwise the error of collimation, which may be considerable, comes in purely as error in the vertical angle observed. Vertical angles to *stations*, as distinguished from *intersected points*, should be taken twice on each face, *viz.*, once with a motion of the telescope from below upwards, and once with reversed

motion, *i.e.*, from above downwards. The observation of vertical angles should be limited to about one hour on either side of minimum refraction, at least where any accurate determination of height differences is aimed at. In observing heights of buildings or trees or hill tops &c., it is a matter of considerable importance to specify clearly in the angle book the point selected, and also to give the height of the instrument and signal above the station platform.

The reciprocal vertical angles.

12. In secondary triangulation, as in principal, it is desirable to observe reciprocal vertical angles, at very nearly the same hour of the day at both stations; but in the former class of work the rule must necessarily be relaxed in many cases, notably in that of intersected points. It was stated in para. 101, Chapter I, that if the amount of refraction were known the relative heights could be deduced from verticals taken at one extremity of a ray. It is usual to express the length of the ray in seconds of arc, and the constant of terrestrial refraction as a fraction of it, so that if r be the total amount of refraction, k the constant, and C the contained arc, $r = kC$, whence $k = \frac{r}{C}$: and, if we assume an equality between the refractions at the extremity of a ray, we can obtain the value of k from every pair of reciprocal observations at every station. The mean of all the values of k at any one station may, therefore be adopted as the constant of refraction at that point, and may, without risk, be applied to correct a single vertical angle to a secondary point, when its reciprocal observation is wanting. Let D be the observed depression of that secondary or intersected point, and C the contained arc due to its distance, then $D + kC = \Delta$ will be the true depression cleared of refraction. Let the unobserved reciprocal vertical be D^1 or $-E^1$, its corresponding value in *vacuo* being Δ^1, then as $C = \Delta + \Delta^1$ therefore $\Delta^1 = C - \Delta$, and the subtended angle $S = \frac{1}{2}(D - D^1) = \Delta - \frac{1}{2}C = D + kC - \frac{1}{2}C$ or for an elevation $S = E - kC + \frac{1}{2}C$. The use of Table XIV, 3rd Edition, Auxiliary Tables, enables the computer to obtain the subtended angle, without the intermediate step of computing the unobserved vertical angle.

Co-efficient of refraction.

13. The relative heights of intersected points thus obtained are quite good enough for the purpose required, if the precaution is taken of making the observa-

tions of the vertical angles at about the same interval from noon, as the principal verticals from which the value of *k* is derived. The value of *k* averages about ·066 in hilly country where the rays are quite free from all obstructions, and pass near no intervening objects: in flat sandy countries like Rajputana and parts of the Punjab it is very irregular, even being occasionally negative. For an interesting discussion of this question the reader is referred to Appendix No. 3, Volume II, and to the Preface to Synoptical Volume VII, where a table of the values of the co-efficient, found in actual practice, is given.

Protection of secondary stations.

14. It is seldom necessary to employ any very elaborate precautions for the protection of secondary stations. A rough pile of earth or stones is generally sufficient to protect them from all but wilful injury. This cannot easily be guarded against, except by placing them under the care of an official in the same way as principal stations, a course which the comparatively small importance of secondary work rarely warrants; all stations should, however, be carefully described in the angle book, and the direction and distance of neighbouring villages or other permanent marks noted for subsequent identification.

Ray tracing.

15. Somewhat akin to the secondary triangulation, as defined in paras. 1 and 2 of this Chapter, is the system known in the Trigonometrical Branch as *Ray tracing*. It was first instituted as a means of ascertaining the exact direction of a distant station, which happened to be concealed by intervening jungle, in order to regulate the clearing of the line. The method is thoroughly explained in Chapters II and VII, Part IV of Thuillier and Smyth's Manual of Surveying, a work which is assumed to be as a matter of course in the possession of every field party, and need not therefore be given in detail here.

Management.

16. The reader should consult the "Handbook of the Topographical Branch" for further hints on the management of secondary triangulation, as this class of work is more intimately connected with that Branch than with the Trigonometrical, which concerns itself chiefly with the more rigorous methods of principal triangulation for geodetic purposes.

CHAPTER III. COMPUTATION OF PRINCIPAL TRIANGULATION.

Weight.

1. The usual abstracts of angles having been made in the angle book, the primary value of each angle resulting therefrom, is the arithmetical mean of the zero means, and is generally symbolized by M. When the number of measures on each zero is not the same, a small correction has to be calculated to apply to M to obtain what is called the *corrected or concluded angle C*. If the number of measures on each zero are equal M and C are the same. Now no measured quantities are ever absolutely correct, though some may possess greater reliability, or, as we may say, greater *weight* than others; and it is usual in refined observations, when a result has been derived from several independent measures, to calculate its *weight* relatively to other results similarly obtained, by means of the accordance of the several measures.* In triangulation the weights thus obtained are made use of in the subsequent calculations, as will be explained hereafter. The weights of angles are calculated in a form designed for the purpose, *P. 8, Computation of Weights of Observed Angles.* (See the example at the end of this chapter). To employ this form enter in the first column the number of observations on the first zero, and in the second column the seconds of observation successively; then do the same for the second zero, and so on. In the third column enter the zero means, and in the fourth the differences of the several observations from the zero means to which they appertain, regardless of sign; each of these quantities is symbolized by $(e.o.)$. In the fifth column enter the squares of the quantities $(e.o.)$ In the sixth column enter the differences between the zero means and their general mean M, with the signs + or − as the case may be; these quantities are symbolized by $(e.g.)$ Square these quantities and enter them in the next column.

Now put

$$e^2 = \frac{\text{Sum of } (e.o.)^2}{\text{Number of measures} - 1};$$

$$c^2 = \frac{\text{Sum of } (e.g.)^2}{\text{Number of zeros} - 1}.$$

* Besides the term *weight* for denoting the reliability of any result deduced from observations there are other terms, *viz.*, *error of mean square* or briefly *e.m.s.*, and *probable error* or *p.e.* The relation between these is denoted as follows:—

$$\sqrt{weight} = \frac{1}{e.m.s.} = \frac{1}{\cdot 6745 \times p.e.}$$

The *error of mean square* may be defined as the square root of the mean of the squares of the apparent errors of observation. By Sir G. Airy, Chauvenet and German writers this is called the *mean error*.

Next divide e^2 successively by the number of observations on each zero and adding c^2 to each quotient take their reciprocals: these added together give the weight w, and hence the weight reciprocal $\frac{1}{w}$ or u.* Only two places of decimals are retained throughout the calculation.

Corrected or concluded angle.

2. The correction to the arithmetical mean M, to produce the concluded angle C, is next obtained as follows:—Look out the smallest of the reciprocals from which w is derived, and subtract it from each of the others, entering the differences in the column provided, and in the next column enter the corresponding values of ($e.g.$) with their proper signs. Sum the several products and divide by w; the result is the correction required.

Spherical excess.

3. The next step in the calculation of principal triangulation is that of the spherical excess of each triangle. This does not require that the angles and sides should be employed to any degree of nicety. A preliminary calculation of the sides of the triangles is made in form *P. 14, Computation of Secondary Triangles*. The angles are entered to the nearest second, and the amount by which their sum differs from 180° is divided equally among them, thus giving angles for computation. In filling in *P. 14,* the names in each triangle should be so arranged that the first two stations *include* the known side, and the third is *opposite* to it. In the column for *log sines* it is convenient to enter the *cosecant* of the third angle instead of the *log sine*. Vertically over the *log sines*, figure for figure, is entered the *log* of the known side. Now by adding together the *log side*, the first *log sine* and the *log cosecant* we obtain the *log value* of the side opposite the first station, and in like manner we find the other side. Proceeding in this manner preliminary values of all the sides may be obtained; now enter Form *P. 9, Computation of Spherical Excess,* and employing the mean latitude for the triangulation for entering Table III of the Auxiliary Tables, perform the calculation as indicated in the form; the result is the spherical excess. Five place logs are sufficient in Form *P. 9,* and the spherical excess is taken out to three places. For any further information and an example, see page 12 of the Auxiliary Tables, Third Edition.

* This is the $(e.m.s.)^2$.

Figural corrections to the angles.

4. The triangulation of India consists of chains formed of single triangles, and groups of triangles constituting quadrilaterals, polygons and compound figures; (*vide* Part II, Chapter I). Now if the angular measurements were absolutely correct it would follow that (1) every triangle would have the sum of its angles exactly equal to 180° + *the spherical excess*; (2) at every point where angles have been measured completely round the horizon, the sum would equal exactly 360°; and (3) the ratio of any two sides of the triangulation would be constant, whatever succession of triangles is employed to arrive at it. But the observed angles fulfil none of these conditions, except by mere chance, and the next stage in the calculations is to find the necessary corrections to the angles to satisfy them while maintaining another condition, *viz.*, that these corrections shall be mathematically the most probable. This is done by what is called the "Method of Least Squares." It will not be necessary here to enter into any explanation of the theory of the process, but only to explain the working of it which is departmentally known as "grinding the figure."

Firstly. As regards single triangles; it is obvious that they can only fulfil the first condition. Add the three angles together and subtract 180° + *the spherical excess*; the remainder, which will be either *plus* or *minus*, will be the triangular error or *e*. Now if we call the errors of the three angles respectively x_1, x_2 and x_3, and the corresponding reciprocal weights u_1, u_2, and u_3

$$x_1 = \frac{u_1 e}{u_1 + u_2 + u_3}, \quad x_2 = \frac{u_2 e}{u_1 + u_2 + u_3}, \quad x_3 = \frac{u_3 e}{u_1 + u_2 + u_3}.$$

These are the most probable *errors* of the angles, and to employ them their signs must be changed and the quantities applied to the respective angles.

Secondly. Quadrilaterals:—A quadrilateral consists of three triangles* connecting four points (*vide* page 4) and has eight observed angles, two at each point. For obtaining the errors of the angles a form is provided numbered *P. 10.* The conditions to be fulfilled are the first and third. For an example see the end of this chapter.

* There are apparently four triangles, but only three are independent.

At the top of the form is a typical figure for general use, with the angles numbered, and it is intended that the names of the stations of any figure under reduction should be written in against the station numbers, I, II, &c.; while in the right-hand upper corner are to be entered the observed angles and their reciprocal weights. Now carrying the eye to the left, provision will be found for calculating the three triangular errors e_1, e_2 and e_3, and in the same line the error e_4 or the error of the side ratio

$$\frac{\text{I}-\text{IV}}{\text{I}-\text{II}},\ \frac{\text{I}-\text{II}}{\text{I}-\text{III}},\ \frac{\text{I}-\text{III}}{\text{I}-\text{IV}}.$$

$$\text{or } \frac{\sin(X_2+X_3)}{\sin X_1},\ \frac{\sin X_6}{\sin X_3},\ \frac{\sin X_8}{\sin(X_6+X_7)}.$$

This ratio, if the angles were errorless, should obviously equal unity, *i.e.* its logarithm should equal 0. In filling in this part of the form, one-third of the spherical excess of the triangle must be deducted from each angle, and at the time of taking out the log sines, the changes for 1″ of arc should be entered in the columns headed *s*. 7-place Log Tables are used in the calculation, but an 8th place is found by interpolation and employed. The resulting error will be in the 8th place of decimals, and for convenience is multiplied by 10,000,000, the quantities *s* being treated in the same way. Next enter Table I and fill in the reciprocal weights u_1, u_2, &c., at the head of each column; also in the line D fill in, above the *s*'s, their values as given in the calculation of side error; the proper signs will be given by means of the signs in front of the symbols below.

5. The filling in of Tables II and III requires no explanation; it is only necessary to remark that **where + signs are printed in the form, the quantities are positive, but where − signs occur, the computer must judge for himself whether they should be turned into + or not.** This rule holds good for all the forms used in the Calculating Section of the Trigonometrical Branch Office. The quantities in Table III, when summed horizontally, form the co-efficients of f_1, f_2 &c., in Table IV. In this table the *e*'s are now for the first time brought into use and the sign = is to be understood as preceding each. Having filled in the first four lines of Table IV we have four equations which have to be solved by rule as below, the rule having been devised to save labour:—Multiply the first equation through by the

co-efficient of f_2 and divide by the co-efficient of f_1 and record it in the first line below, omitting the resulting co-efficient of f_1. The next line will be obtained by multiplying the first equation through by the co-efficient of f_3 and dividing by the co-efficient of f_1; but as the co-efficient of f_3 is 0, the resulting quantities will be 0, and they have been printed so in the form. The third line is obtained by multiplying the first equation through by the co-efficient of f_4 and dividing by that of f_1. The co-efficients need not be entered where no provision is made for them. Now add the co-efficients in the first line of the new group to those of equation 2 and record below; then add the co-efficients in the second line of the new group to those of equation 3 and record next and treat the fourth equation similarly. We have now three equations involving three unknown quantities, f_2, f_3 and f_4. Going through a similar process to that already described, we come to two equations involving two unknown quantities, f_3 and f_4, and lastly we reduce the equations to one involving one unknown quantity, f_4, and dividing out by its co-efficient we have the value of f_4. This brings us to the bottom of the form. Carrying the eye up to the right the remainder of Table IV is found. Taking the last line but one, fill it up from the first equation in the group involving two unknown quantities, f_3 and f_4, transferring the term involving f_4 over to the other side. Then substitute for f_4, add the two terms and divide by the co-efficient of f_3, and this gives f_3. To find f_2, employ the first of the group of equations involving the three unknown quantities f_2, f_3 and f_4 and enter the third line of the last portion of Table IV, and similarly for f_1. Now enter Table V and multiply the quantities in Table I by the values of f_1, f_2, &c., as indicated. The resulting values obtained from this table are the most probable *errors* of the angles. These are usually computed to four places of decimals of seconds and retained to three places in subsequent calculations. To ascertain that no mistakes have been made the errors are first entered in the test columns of the triangular and side error calculations. In the first case the sums should equal e_1, e_2 and e_3 respectively, and in the last putting

$$s_1\, x_1 + s_2\, x_3 + s_{6,7}\, (x_6 + x_7) = S'_l$$

and

$$s_{2,3}\, (x_2 + x_3) + s_6\, x_6 + s_8\, x_8 = S'_r.$$

$$S'_r - S'_l = e_4.$$

If these tests are not satisfied some mistake in the calculations has been made.

Thirdly. Polygons.—Polygons are formed by a number of triangles with their apexes all meeting at a point, which the triangles completely surround (*vide* page 4). The angles are subject to all three conditions specified in para. 4, page 60. The errors are computed in the form *P. 11.* which has been constructed for a polygon of six sides, but may be employed for figures of a greater or less number of sides with a little adaptation. The principle is the same as for the quadrilateral, and the form needs no explanation beyond what has been given for the quadrilateral.

Fourthly. Complex Figures.—If more complex figures have to be treated than quadrilaterals or polygons, reference to Chapter XVI of Volume II of the *Account of the Operations of the Great Trigonometrical Survey of India,* is recommended. In such cases special care must be taken to obtain all the equations and to avoid any redundant ones. This may always be done by building up the figure and forming the equations at the same time. The employment of the minimum number of rays required to fix all the points will, if the angles are all observed, give as many triangular equations as there are points less two. Every ray afterwards introduced gives an equation for each angle observed in fixing it, either triangular, side or central, and these should be settled on before introducing another ray.

Triangles.

6. After the angular errors have all been found, the form for the calculation of Principal Triangles, *P. 12,* is entered. The order in which the stations are recorded is the same as described under "Spherical Excess," para. 3, page 59. The observed angles are next entered to two places of decimals. The spherical excess of each triangle is divided equally among the three angles and entered with a — sign to three places of decimals. The angular errors with their signs changed are entered in the column for 'Correction' to three places, and the seconds of the angles for computation (which are plane angles) are the algebraical sum of the three columns. The sum of the three plane angles should exactly = 180°. The rest of the calculation is much the same as already described, except that although 7-place logs are used, an eighth place is always derived by interpolation and employed.

Latitude, longitude and azimuth.

7. The calculation of latitude, longitude and azimuth is fully explained in the Auxiliary Tables, in the explanation of Table V. The form employed is numbered *P. 16*.

Heights.

8. The form for computing heights is *P. 19*. In it are first recorded the astronomical date and mean time of observation, then the name of the stations, next the number of observations from which the vertical angle is derived and then the vertical angles at each station; the letter *I* in this column being intended to be changed into an *E* or *D* according as the angle is an elevation or depression. The filling up of the columns numbered 4, 5, 6 and 7 is clearly explained in the form itself. The calculation next proceeds in the lower part of the form, and is first carried as far as height of $B - A$, and the result is entered above in column 8: then by the addition of the known height of *A*, the height of *B* in column 9 is obtained. The mean value of *B* in column 10 is from two, three or more determinations from surrounding stations.

Refraction.

9. The remainder of the calculation deals with refraction. It is computed in the last four lines at the bottom of the form, and is equal to $\frac{1}{2}C - R + \beta$. The factor of refraction or $\frac{r}{C}$ is obtained from columns 11 and 12, and a mean value should be used for the determination of the heights of any intersected points which have been observed from the principal station to which the value appertains. In secondary calculation refraction is less needed, and the calculation is sometimes omitted, the co-efficient of refraction being assumed to be 0·07. For further particulars see the Auxiliary Tables and the explanations of Tables IX to XV, especially of Table XIII.

COMPUTATION OF SECONDARY TRIANGULATION.

Triangles.

10. Secondary triangles are of different characters and are treated accordingly. Some are executed in chains to supply the place of principal triangulation, the instruments used being the largest class of vernier theodolites, *viz.*, 10-, 12- and 14-inch. These triangles may be called 1st class. It is not customary to *grind* the figures formed by them; but in computing them spherical excess must first be found and divided equally among

the angles which are kept to one place of decimals of seconds, and then any triangular error that remains must be also equally distributed; after this the sides may be calculated in Form *P. 13*. It may be noted that the same angle for computation is obtained whether the spherical excess be computed or not; but the spherical angle, in contradistinction to the plane angle, is subsequently required in calculating latitude, longitude and azimuth.

Continued.

11. Secondary triangles obtained with smaller instruments do not have their spherical excess worked out; but any discrepancy between the sum of their angles and 180° is equally divided between the angles, which are retained to the nearest second only. Form *P. 14*. Such triangles are employed to form small net-works based on well established points, and the omission of spherical excess in the subsequent calculations of latitude, longitude and azimuth is of no practical importance.

Triangles fixing intersected points.

12. There is a third class of triangles fixing intersected points, that is, points which are not visited, but are observed from the extremities of a known base. In these triangles only two angles are measured, and the third angle is found by taking the difference of the sum of the two observed angles from 180°. Form *P. 15*.

Latitude, longitude and azimuth.

13. If the triangles are large and of such a character that the spherical excess has had to be computed, the form for principal latitudes and longitudes should be used, *viz.*, *P. 16*. For smaller triangles and for intersected points at a greater distance than 8 miles, *P. 17* may be employed, in which the third and fourth terms of the expressions for latitude, longitude and azimuth are omitted. For intersected points at distances not exceeding 8 miles, *P. 18* should be used, in which only the first terms of latitude, longitude and azimuth are computed.

Heights.

14. The calculation of heights for secondary work is in the same form as for principal heights, *viz.*, *P. 19*; but it is not usual to determine the value of refraction, a factor, 0·07, as mentioned on the preceding page, being generally adopted for heights where only one vertical has been observed. In the

calculation of height under these circumstances, sufficient instruction will be found in the Auxiliary Tables.

15. When all the calculations have been effected which have been previously described, an abstract of all the data obtained is made in Form *P. 24*.

Abstract.

MISCELLANEOUS COMPUTATIONS.

16. Besides the calculations already described there are one or two which have to be resorted to from time to time. These are:—Computation of a triangle by two sides and the included angle, in cases where a side is required as a base for further calculation. Form *P. 25* has been provided for the purpose.

A triangle from two sides and the included angle.

In this form a subsidiary angle x is first computed by the formula

$$\log \sin C + \log a + \operatorname{colog} b = \log \tan x,$$

and then A is computed from the formula

$$\log \sin C + \log \sin x + \log \operatorname{cosec} (C - x) = \log \tan A;^*$$

then $$B = 180^\circ - (A + C);$$

and the third side c is obtained from the formula

$$\log \operatorname{cosec} A + \log \sin C + \log a = \log c.$$

* The formula is derived as follows:—a, b and C are given.

$$\tan A = \tan\left(\frac{A+B}{2} + \frac{A-B}{2}\right)$$

$$= \frac{\tan\frac{A+B}{2} + \tan\frac{A-B}{2}}{1 - \tan\frac{A+B}{2}\tan\frac{A-B}{2}} = \frac{\cot\frac{C}{2} + \tan\frac{A-B}{2}}{1 - \cot\frac{C}{2}\tan\frac{A-B}{2}} = \frac{\cot\frac{C}{2} + \frac{a-b}{a+b}\cot\frac{C}{2}}{1 - \cot\frac{C}{2}\cdot\frac{a-b}{a+b}\cot\frac{C}{2}}$$

$$= \frac{2a}{\overline{a+b}\tan\frac{C}{2} - \overline{a-b}\cot\frac{C}{2}} = \frac{2a}{a\left(\tan\frac{C}{2} - \cot\frac{C}{2}\right) + b\left(\tan\frac{C}{2} + \cot\frac{C}{2}\right)} = \frac{a \sin C}{b - a\cos C}.$$

Put $$\frac{a}{b}\sin C = \tan x; \quad \text{then } \tan A = \frac{\tan x}{1 - \tan x \cot C} = \frac{\sin x \sin C}{\sin(C - x)}.$$

A station from observations to three known points.

17. Another useful calculation is the interpolation of a position from which three known points have been observed. Form *P. 26* is provided for this. If *A, B* and *C* are the three known points (see diagrams at bottom of form), and *S* the station to be interpolated, three cases occur, (1) when *S* is outside the triangle *A B C* and *C* is beyond the line *A B*; (2) when *S* is within the triangle *A B C*; (3) when *S* is without the triangle *A B C* and *C* is within the side *A B*. In the first case *S* may fall on, or very nearly on, the arc of the circle circumscribing the triangle *A B C*, which happens when the angles *A S B* and *A C B* are equal, or nearly equal, to 180°: in this case the problem is impossible.

Let the angles $A\ S\ C = \alpha$, and $B\ S\ C = \beta$,

and let the angles $C\ A\ S = x$, and $C\ B\ S = y$;

then

$$x + y = 360^\circ - (\alpha + \beta + C).$$

Now compute an angle ϕ from the formula

$$\log \sin \alpha + \log \operatorname{cosec} \beta + \log a + \operatorname{colog} b = \tan \phi,$$

and compute $\frac{x-y}{2}$ from the formula

$$\log \tan (\phi - 45^\circ) \log \tan \frac{x+y}{2} = \log \tan \frac{x-y}{2};$$

then x and y are the sum and difference of $\frac{x+y}{2}$ and $\frac{x-y}{2}$.

Next compute the angles *A C S* and *B C S*, the angle *A C S* being $= 180^\circ - x - \alpha$, and the angle $B\ C\ S = 180^\circ - y - \beta$.

Distances and azimuths of points of which the latitudes and longitudes are known.

18. The next calculations—those of azimuth and distance between points of which the latitudes and longitudes are known—are fully explained in the Auxiliary Tables, pages 22, 23 and 71, 72.

PLATE I.

Fig. 1.

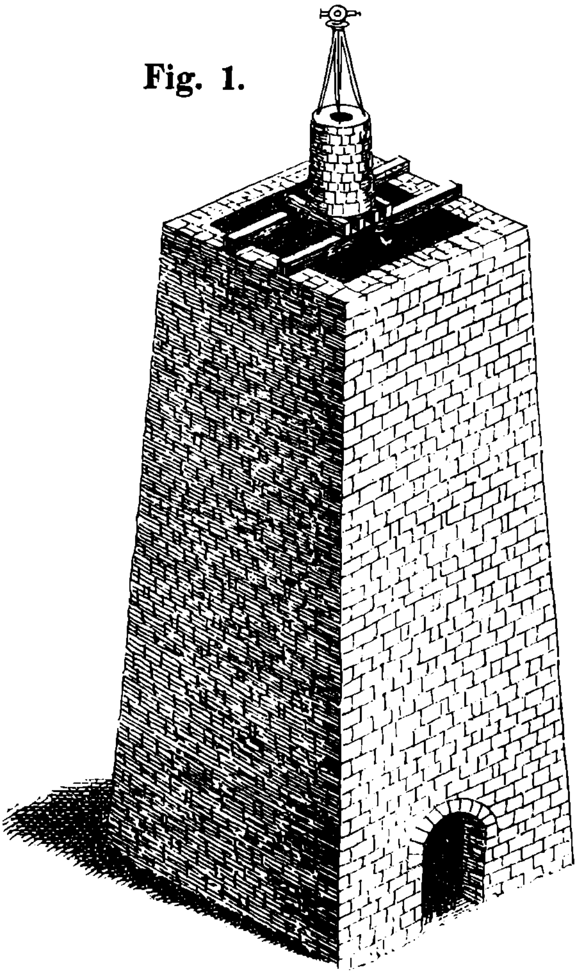


Fig. 2.

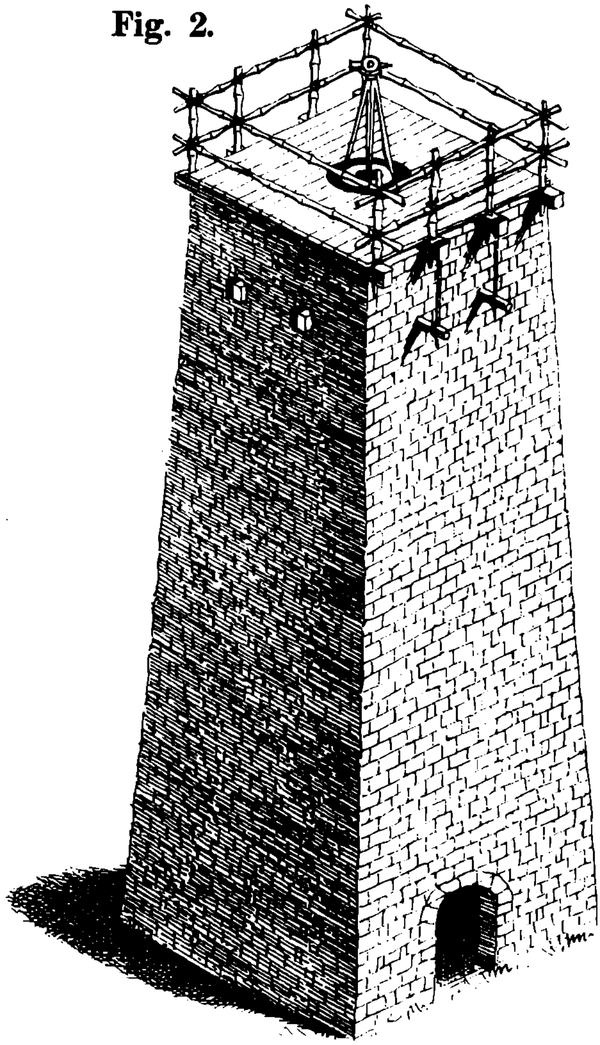


Fig. 3.

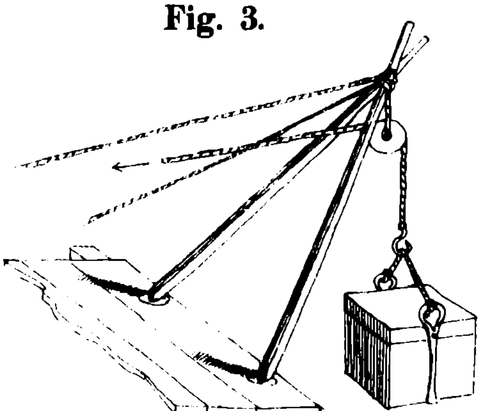


Fig. 4.

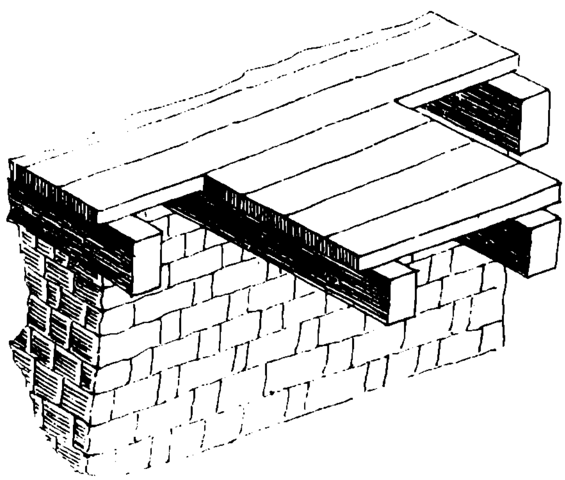


PLATE II.

REFERENCE.

A Masonry pillar and annulus, section and plan.

B Closing pillar in section.

C Wooden braced stand, elevation.

c'c' Masonry support for ditto, in plan.

D Observatory tent, } in elevation and plan.

E Theodolite box, } in elevation and plan.

FF Palmyra platform, } in elevation and plan.

G Lamp and Heliotrope

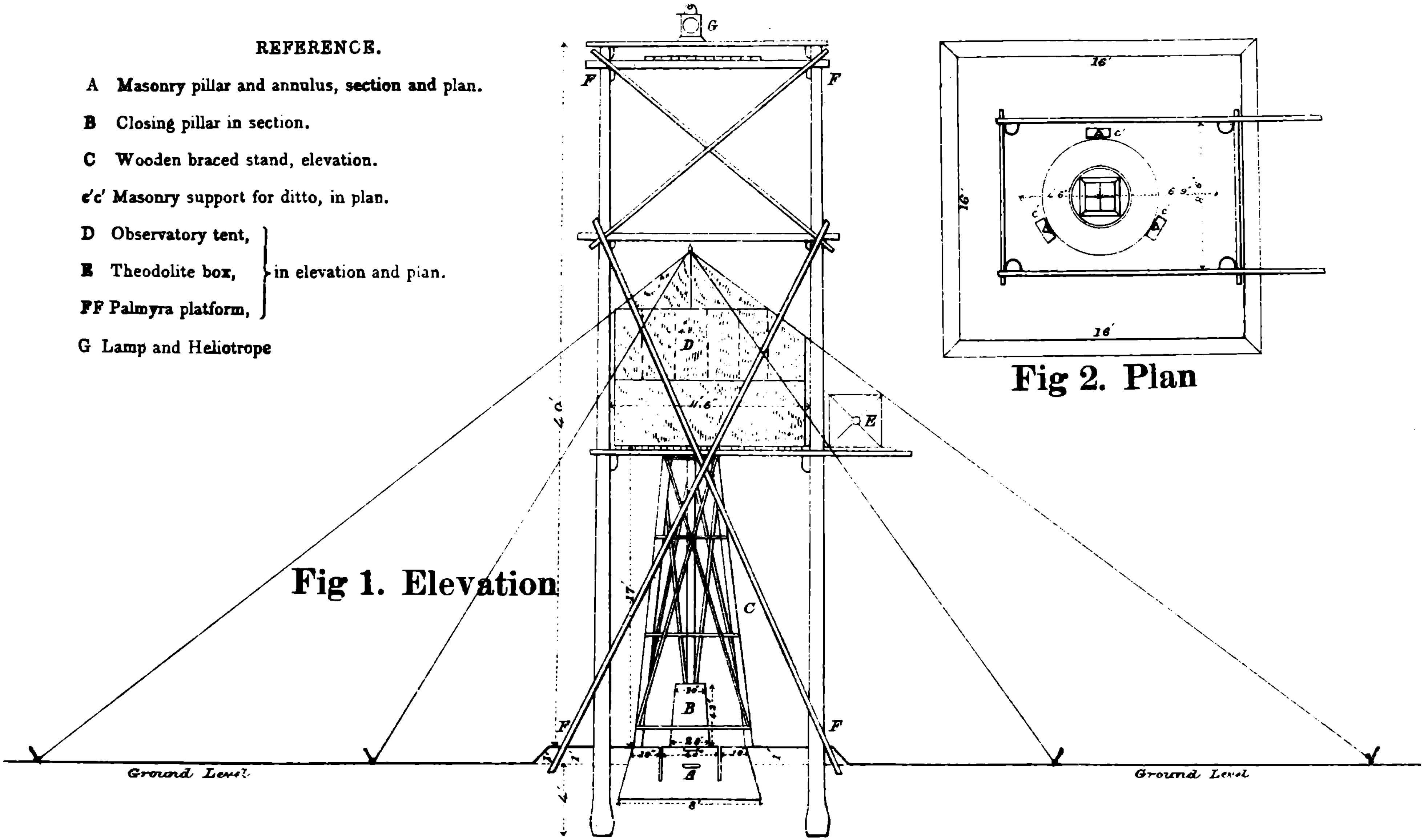
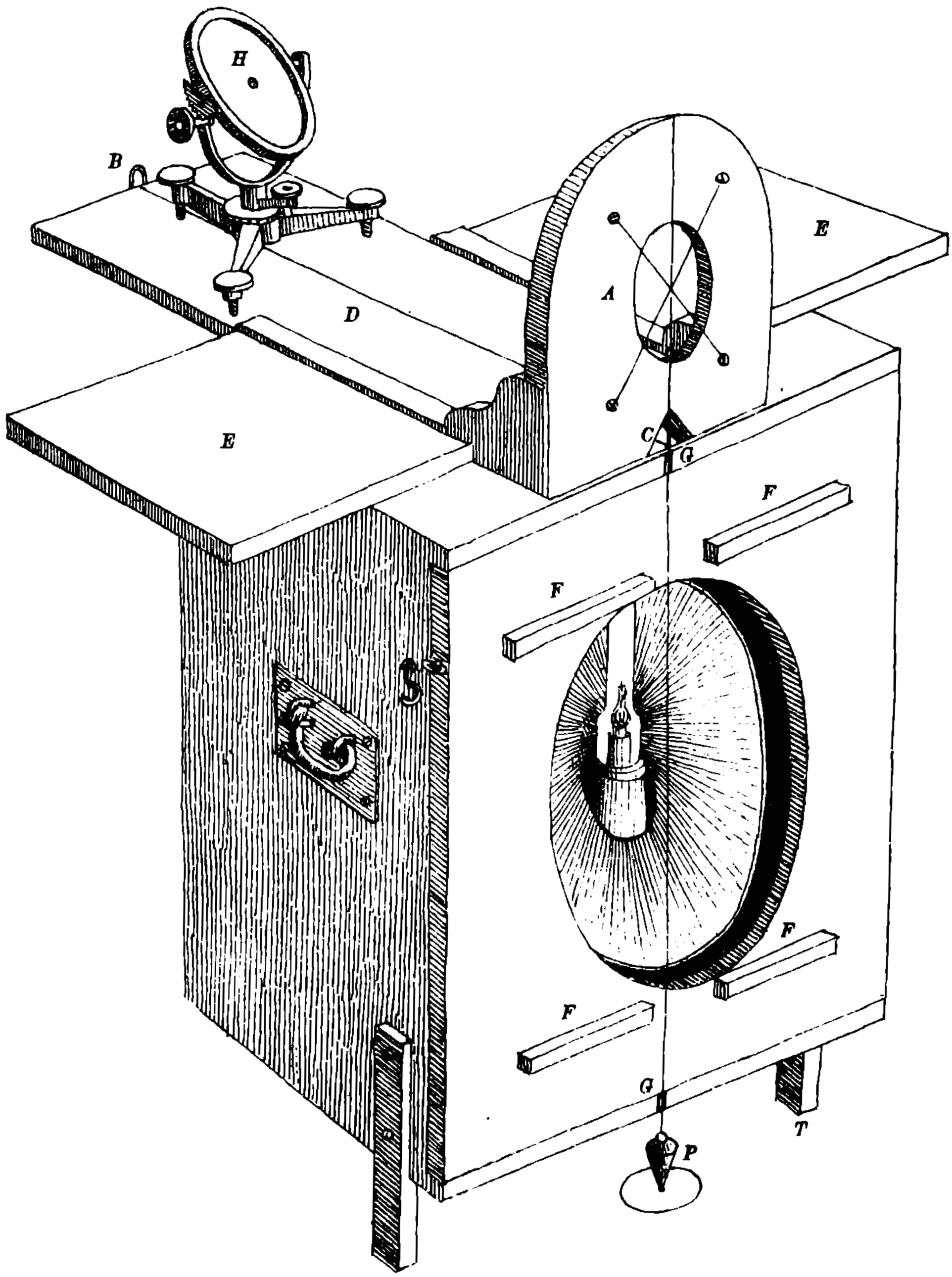


Fig 1. Elevation

Fig 2. Plan

Photozincographed at the Office of the Trigonometrical Branch, Survey of India, Dehra Dún, October 1889

PLATE III



Photozincographed at the Office of the Trigonometrical Branch, Survey of India, Dehra Dún, October 1889

Survey of India Department.

NO. —— PARTY ( ) SEASON 18

Computation of Weights of Observed Angles.

At Dhaula Hill *Station.*

Angle between

Kundal *H.S. and* Bhádrájan *H.S.*

General Mean 81° 14′ 28″.79

No. of Obsns. on each Zero = n	Seconds of Observation	Means of Zeros	Differences or e. o.*	(e. o.)²	Differences or e. g.*	(e. g.)²
	29·32		0·26	0·07		
	28·58		0·48	0·23		
3	29·28	29·06	0·22	0·05	+0·27	0·07
	31·16		0·87	0·76		
	30·28		0·01	0·00		
3	29·42	30·29	0·87	0·76	+1·50	2·25
	28·86		0·75	0·56		
	28·34		0·23	0·05		
3	27·14	28·11	0·97	0·94	−0·68	0·46
	29·10		0·52	0·27		
	28·66		0·08	0·01		
	27·48		1·10	1·21		
	28·92		0·34	0·12		
5	28·74	28·58	0·16	0·03	−0·21	0·04
	29·28		0·23	0·05		
	28·04		1·01	1·02		
3	29·84	29·05	0·79	0·62	+0·26	0·07
	29·32		0·23	0·05		
	29·36		0·27	0·07		
3	28·58	29·09	0·51	0·26	+0·30	0·09
	28·58		0·23	0·05		
	28·50		0·31	0·10		
3	29·34	28·81	0·53	0·28	+0·02	0·00
	28·58		0·29	0·08		
	30·48		1·61	2·59		
	28·26		0·61	0·37		
4	28·14	28·87	0·73	0·53	+0·08	0·01
	30·02		1·72	2·96		
	28·46		0·16	0·03		
	27·90		0·40	0·16		
4	26·82	28·30	1·48	2·19	−0·49	0·24
	27·78		0·00	0·00		
	26·80		0·98	0·96		
	28·44		0·66	0·44		
	27·54		0·24	0·06		
5	28·36	27·78	0·58	0·34	−1·01	1·02

Deduction of Weight

Divisors or n	†e^2 = 0·52; $\frac{e^2}{n}$ = Quotients	‡c^2 = 0·47; Quotients + c^2	Reciprocals of Quantities in preceding Column§
3	0·17	0·64	1·56
3	0·17	0·64	1·56
3	0·17	0·64	1·56
5	0·10	0·57	1·75
3	0·17	0·64	1·56
3	0·17	0·64	1·56
3	0·17	0·64	1·56
4	0·13	0·60	1·67
4	0·13	0·60	1·67
5	0·10	0·57	1·75

[Reciprocals] or Weight 16·20

Weight Reciprocal 0·06

Deduction of Correction to General Mean

Difference between *least* Reciprocal and that for "every other Zero"	e. g. corresponding to "every other Zero"	Product of Quantities in two preceding Columns
0·19	−0·21	−0·04
0·11	+0·08	+0·01
0·11	−0·49	−0·05
0·19	−1·01	−0·19

Sum −0·27

Sum ÷ Weight = Correction −0″·02

Corrected Angle 28″·77

[ ] signifies Summation

[(e. o.)²] = 18·27; [(e. g.)²] = 4·25;

[(e. o.)²] ÷ No. of Observations less one = e^2 = 0·52; †

[(e. g.)²] ÷ No. of Zeros less one = c^2 = 0·47; ‡

At .... *Station.*

Angle between

.... *S. and* .... *S.*

General Mean ° ′ ″

No. of Obsns. on each Zero = n	Seconds of Observation	Means of Zeros	Differences or e. o.*	(e. o.)²	Differences or e. g.*	(e. g.)²

Deduction of Weight

Divisors or n	†e^2 = ; $\frac{e^2}{n}$ = Quotients	‡c^2 = ; Quotients + c^2	Reciprocals of Quantities in preceding Column§

[Reciprocals] or Weight

Weight Reciprocal

Deduction of Correction to General Mean

Difference between *least* Reciprocal and that for "every other Zero"	e. g. corresponding to "every other Zero"	Product of Quantities in two preceding Columns

Sum

Sum ÷ Weight = Correction ″

Corrected Angle ″

[ ] signifies Summation

[(e. o.)²] = ; [(e. g.)²] = ;

[(e. o.)²] ÷ No. of Observations less one = e^2 = ; †

[(e. g.)²] ÷ No. of Zeros less one = c^2 = ; ‡

NOTES.—* *e. o.* denotes apparent error of *observation*: *e. g.* denotes apparent error of *graduation*. The former is the difference of an observation from the mean of its zero; the latter the difference of the zero mean from the general mean, *i.e.* zero mean *minus* general mean, and it should be entered *with its algebraical sign*. § The quantities entered in this column represent the *weights of the several zero means*. *Rule.*—Two places of decimals are nowhere to be exceeded, whether in means, differences, or squares.

Computed and Compared by 18

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Survey of India Department.

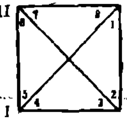
NO. ____ PARTY (________________) SEASON 18 __ - __

Reduction of Borta-Bonik *Quadrilateral by the Method of Minimum Squares.*

Explanation of Symbols.

- X_n denotes Concluded Angle.
- x_n ,, most Probable Error of X_n.
- u_n ,, Reciprocal Weight ,,
- s_n ,, change of Log Sin X_n for 1".
- $s_{m,n}$,, ,, ,, $(X_m + X_n)$ for 1".
- e denotes Spherical Excess.
- ε ,, right hand member, or Error, of equation.
- f ,, Factor (indeterminate).

Borta H.S. (or III) — Dhaula H.S. (or IV)



Sunda H.S. (or I) — Bonik H.S. (or II)

Statement of Data.

No.	X	u	No.	X	u
	° ′ ″			° ′ ″	
1	68 33 0.17	0.05	5	37 19 9.51	0.07
2	42 1 19.73	0.05	6	73 15 11.27	0.09
3	38 58 35.75	0.06	7	32 57 39.54	0.07
4	30 27 8.60	0.06	8	36 28 3.42	0.06

Determination of Right hand Members, or Errors, of Equations.

Triangular Errors = Sum of Angles − (180° + e).

Seconds of X Δ I, II, IV	Final Errors tested	Seconds of X Δ I, II, III	Final Errors tested	Seconds of X Δ I, III, IV	Final Errors tested
X_1 0.17	+0.087	X_3 35.75	+0.193	X_5 0.51	+0.240
X_2+X_3 55.53	+0.161	X_4+X_5 13.11	+0.190	X_6+X_7 50.81	+0.336
X_4 8.60	−0.050	X_6 11.27	+0.110	X_8 3.42	−0.059
Sum 4.30		Sum 5.13		Sum 3.74	
e 4.102		e 4.637		e 3.223	
ϵ_1 = + 0.198	+0.198	ϵ_2 = + 0.493	+0.493	ϵ_3 = + 0.517	+0.517

Side Error = 10,000,000 × (Sum of Log Sines on right − Sum of Log Sines on left).

	X − e/3	Log Sine	δ	Test: Final Error	Test: Change in Log Sine
	° ′ ″			″	
X_1	68 32 58.303	1̄.9688260[illegible]	9	+0.087	+0.78
X_3	38 58 34.205	1̄.7986486[illegible]	26	+0.193	+5.02
X_6+X_7	106 12 49.736	1̄.9822[illegible]	−6	+0.336	−2.02
	Sum = S_l = 1̄.74984331			S'_l = + 3.78	

	X − e/3	Log Sine	δ	Test: Final Error	Test: Change in Log Sine
X_2+X_8	80 59 54.162	1̄.99461796	4	+0.161	+0.64
X_6	73 15 9.724	1̄.98117733	6	+0.110	+0.66
X_8	36 28 2.346	1̄.77405260	29	−0.059	−1.71
	Sum = S_r = 1̄.74984759			S'_r = − 0.41	

ϵ_4 = 10,000,000 × $(S_r - S_l)$ = − 4.2 $S'_r - S'_l$ = − 4.19

	x_1 u_1=0.05	x_2 u_2=0.05	x_3 u_3=0.06	x_4 u_4=0.06	x_5 u_5=0.07	x_6 u_6=0.09	x_7 u_7=0.07	x_8 u_8=0.06
Table I.—Numerical Statement of Co-efficients in Equations of Condition.								
A	+1	+1	+1	+1				
B			+1	+1	+1	+1		
C					+1	+1	+1	+1
D	−9	+4	−22			+12	+6	+29
	$-s_1$	$+s_{2,3}$	$+s_{2,3}-s_3$			$-s_{6,7}+s_6$	$-s_{6,7}$	$+s_8$
Table II.—Products of Co-efficients in Table I and the corresponding u.								
A.u	+0.05	+0.05	+0.06	+0.06				
B.u			+0.06	+0.06	+0.07	+0.09		
C.u					+0.07	+0.09	+0.07	+0.06
D.u	−0.45	+0.20	−1.32			+1.08	+0.42	+1.74
Table III.—Products of quantities in Table II and those in corresponding cols. of Table I.								
Au.A	+0.05	+0.05	+0.06	+0.06				
Au.B			+0.06	+0.06				
Au.D	−0.45	+0.20	−1.32					
Bu.B			+0.06	+0.06	+0.07	+0.09		
Bu.C					+0.07	+0.09		
Bu.D			−1.32			+1.08		
Cu.C					+0.07	+0.09	+0.07	+0.06
Cu.D						+1.08	+0.42	+1.74
Du.D	+4.05	+0.80	+29.04			+12.96	+2.52	+50.46

Table IV.—Equations between Factors.

Equ. No.	f_1	f_2	f_3	f_4	ε
1	+0.22	+0.12		−1.57	+0.198
2		+0.28	+0.16	−0.24	+0.493
3			+0.29	+3.24	+0.517
4				+99.83	−4.2
* The quantities below the diagonal are not to be recorded, being identical with those above.	−0.0655			+0.3564	−0.10800
			0.0000	0.0000	0.00000
				−11.2041	+1.41300
		+0.2145	+0.16	+0.6164	+0.38500
			+0.29	+3.24	+0.517
				+38.6259	−2.75700
			−0.1193	−0.4598	−0.25718
				−1.7713	−1.10636
			+0.1707	+2.7802	+0.22982
				+86.8546	−3.89336
				−45.2813	−3.74309
				+41.5733	−7.63645
				f_4 = − 0.18369	

Sum =	Co-eff. of	In Equ.
+ 0.22	f_1	1
+ 0.12	f_2	1
− 1.57	f_4	1
+ 0.28	f_2	2
+ 0.16	f_3	2
− 0.24	f_4	2
+ 0.29	f_3	3
+ 3.24	f_4	3
+ 99.83	f_4	4

+ 0.22 f_1 = − 0.12 f_2 + 1.57 f_4 + 0.198 =
= + 0.10958 − 0.28839 + 0.198 = + 0.01919 ; f_1 = + 0.08725
+ 0.2145 f_2 = − 0.16 − f_3 − 0.6164 f_4 + 0.38500 =
= − 0.04409 + 0.11322 + 0.38500 − 0.19537 ; f_2 = − 0.91315
+ 0.1707 f_3 = − 2.7802 f_4 + 0.22982 =
= + 0.51069 + 0.22982 = + 0.74051 ; f_3 = + 4.33800

Table V.—Final Determination of Errors.†

	x_1 u_1=0.05	x_2 u_2=0.05	x_3 u_3=0.06	x_4 u_4=0.06
f_1.A	+0.0872	+0.0872	+0.0872	+0.0872
f_2.B			−0.9132	−0.9132
f_3.C				
f_4.D	+1.6532	−0.7347	+4.0411	
Sum	+1.7404	−0.6475	+3.2151	−0.8260
Sum × u = x″	+0.0870	−0.0324	+0.1929	−0.0496

	x_5 u_5=0.07	x_6 u_6=0.09	x_7 u_7=0.07	x_8 u_8=0.06
f_1.A				
f_2.B	−0.9132	−0.9132		
f_3.C	+4.3381	+4.3381	+4.3381	+4.3381
f_4.D		−2.2042	−1.1021	−5.3269
Sum	+3.4249	+1.2207	+3.2360	−0.9888
Sum × u = x″	+0.2397	+0.1099	+0.2265	−0.0593

† Note.—The Errors obtained in this Table are to be applied with changed signs, as corrections, in the Triangle sheets.

Computed and Compared by ______________ 18___

P. 11. *Page 1.*

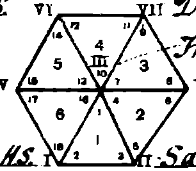
Survey of India Department.

NO.____ PARTY (_________________________________) SEASON 18_____-____.

Reduction of ________ Thob ________ *Polygon by the Method of Minimum Squares.*

Explanation of Symbols.

X_n	denotes	Concluded Angle.
x_n	,,	most Probable Error of X_n.
u_n	,,	Reciprocal Weight ,,
δ_n	,,	change of Log Sin X_n for 1″
ϵ	,,	Spherical Excess.
e	,,	right hand member, or Error, of equation.
f	,,	Factor (indeterminate).
[ ]	,,	Summation.



Statement of Data.

No.	X (° ′ ″)	u	No.	X (° ′ ″)	u	No.	X (° ′ ″)	u
1	73 29 40·52	0·06	7	40 23 3·39	0·04	13	65 7 39·43	0·08
2	48 42 31·60	0·06	8	58 28 49·46	0·03	14	57 6 10·78	0·04
3	57 47 50·94	0·08	9	81 8 9·46	0·05	15	57 46 12·47	0·04
4	54 5 20·92	0·05	10	54 17 19·28	0·07	16	72 36 56·03	0·07
5	78 52 36·91	0·07	11	53 27 48·55	0·04	17	59 31 54·48	0·04
6	47 2 5·03	0·03	12	72 14 54·72	0·05	18	47 51 12·26	0·05

Determination of Right hand Members, or Errors, of Equations.

Triangular Errors = Sum of Angles − (180° + ε).

Δ 1	Seconds of X	Final Errors tested	Δ 2	Seconds of X	Final Errors tested	Δ 3	Seconds of X	Final Errors tested
	″	″		″	″		″	″
X_1	40·52	− 0·049	X_4	20·92	− 0·101	X_7	3·39	− 0·200
X_2	31·60	+ 0·073	X_5	36·91	− 0·009	X_8	49·46	− 0·091
X_3	50·94	+ 0·072	X_6	5·03	− 0·011	X_9	9·46	− 0·158
Sum	3·06		Sum	2·86		Sum	2·31	
ε	2·964		ε	2·981		ε	2·759	
e_1 = +	0·096	+ 0·096	e_2 = −	0·121	− 0·121	e_3 = −	0·449	− 0·449

Δ 4	Seconds of X	Final Errors tested	Δ 5	Seconds of X	Final Errors tested	Δ 6	Seconds of X	Final Errors tested
	″	″		″	″		″	″
X_{10}	19·28	− 0·054	X_{13}	39·43	+ 0·089	X_{16}	56·03	− 0·110
X_{11}	48·55	+ 0·046	X_{14}	10·78	+ 0·124	X_{17}	54·48	+ 0·017
X_{12}	54·72	+ 0·042	X_{15}	12·47	+ 0·113	X_{18}	12·26	+ 0·005
Sum	2·55		Sum	2·68		Sum	2·77	
ε	2·516		ε	2·354		ε	2·858	
e_4 = +	0·034	+ 0·034	e_5 = +	0·326	+ 0·326	e_6 = −	0·088	− 0·088

Central Error = Sum of Angles at III − 360°

	Second of X	Final Errors tested
		″
X_1	40·52	− 0·049
X_4	20·92	− 0·101
X_7	3·39	− 0·200
X_{10}	19·28	− 0·054
X_{13}	39·43	+ 0·089
X_{16}	56·03	− 0·110
Sum	59·57	
e_7 = −	0·43	− 0·430

Side Error = 10,000,000 × {Sum of Log Sines on right − Sum of Log Sines on left}.

	$X - \frac{\epsilon}{3}$	Log Sine	δ	Test: Final Error	Test: Change in Log Sine
	° ′ ″			″	
X_3	48 42 30·612	$\bar{1}$·8758492,6	+ 19	+ 0·073	+ 1·39
X_5	78 52 35·916	$\bar{1}$·9917637,7	+ 4	− 0·009	− 0·04
X_8	58 28 48·540	$\bar{1}$·9306735,5	+ 12	− 0·091	− 1·08
X_{11}	53 27 47·712	$\bar{1}$·9048724,7	+ 15	+ 0·046	+ 0·69
X_{14}	57 6 9·996	$\bar{1}$·9240962,9	+ 13	+ 0·124	+ 1·61
X_{17}	59 31 53·527	$\bar{1}$·9354610,9	+ 13	+ 0·017	+ 0·22

Sum = S_l = $\bar{1}$·5628164,3 S'_l = + 2·79

e_8 = 10,000,000 × {S_r − S_l} = − 0·6

	$X - \frac{\epsilon}{3}$	Log Sine	δ	Test: Final Error	Test: Change in Log Sine
	° ′ ″			″	
X_2	57 47 49·952	$\bar{1}$·9274562,3	+ 14	+ 0·072	+ 1·01
X_6	47 2 4·037	$\bar{1}$·8643708,7	+ 19	− 0·011	− 0·21
X_9	81 8 8·540	$\bar{1}$·9947815,6	+ 3	− 0·158	− 0·47
X_{12}	72 14 53·881	$\bar{1}$·9788133,3	+ 6	+ 0·047	+ 0·28
X_{15}	57 46 11·685	$\bar{1}$·9273257,9	+ 13	+ 0·113	+ 1·47
X_{18}	47 51 11·308	$\bar{1}$·8700635,9	+ 19	+ 0·005	+ 0·10

Sum = S_r = $\bar{1}$·5628163,7 S'_r = + 2·19

$S'_r - S'_l$ = − 0·60

Table V.—Final Determination of Errors.†

	x_1 u_1 = 0·06	x_2 u_2 = 0·06	x_3 u_3 = 0·08	x_4 u_4 = 0·05	x_5 u_5 = 0·07	x_6 u_6 = 0·03	x_7 u_7 = 0·04	x_8 u_8 = 0·03	x_9 u_9 = 0·05
f_1 A	+ 1·0346	+ 1·0346	+ 1·0346						
f_2 B				− 0·1704	− 0·1704	− 0·1704			
f_3 C							− 3·1415	− 3·1415	− 3·1415
f_4 D									
f_5 E									
f_6 F									
f_7 G	− 1·8520			− 1·8520			− 1·8520		
f_8 H		+ 0·1862	− 0·1372		+ 0·0392	− 0·1862		+ 0·1176	− 0·0294
Sum	− 0·8174	+ 1·2208	+ 0·8974	− 2·0224	− 0·1312	− 0·3566	− 4·9935	− 3·0239	− 3·1709
Sum × u = x″	− 0·0490	+ 0·0732	+ 0·0718	− 0·1011	− 0·0092	− 0·0107	− 0·1997	− 0·0907	− 0·1585

	x_{10} u_{10} = 0·07	x_{11} u_{11} = 0·04	x_{12} u_{12} = 0·05	x_{13} u_{13} = 0·08	x_{14} u_{14} = 0·04	x_{15} u_{15} = 0·04	x_{16} u_{16} = 0·07	x_{17} u_{17} = 0·04	x_{18} u_{18} = 0·05
f_1 A									
f_2 B									
f_3 C									
f_4 D	+ 1·0044	+ 1·0044	+ 1·0044						
f_5 E				+ 2·4635	+ 2·4635	+ 2·2635			
f_6 F							+ 0·2866	+ 0·2866	+ 0·2866
f_7 G	− 1·8520			− 1·8520			− 1·8520		
f_8 H		+ 0·1470	− 0·0588		+ 0·1274	− 0·1274		+ 0·1274	− 0·1862
Sum	− 0·8476	+ 1·1514	+ 0·9456	+ 1·1115	+ 3·0909	+ 2·8361	− 1·5654	+ 0·4140	+ 0·1004
Sum × u = x″	− 0·0593	+ 0·0461	+ 0·0473	+ 0·0889	+ 0·1236	+ 0·1134	− 0·1096	+ 0·0166	+ 0·0050

† NOTE.—The Errors computed in this Table are to be applied with changed signs, as corrections, to the Triangle sheets.

Computed and Compared by __________________________ 18____.

Survey of India Department.

NO.____ PARTY (_____________________________) SEASON 18__ __-___.

Reduction of ______________________________ *Polygon by the Method of Minimum Squares.*

	x_1 $u_1=0.06$	x_2 $u_2=0.06$	x_3 $u_3=0.08$		x_4 $u_4=0.05$	x_5 $u_5=0.07$	x_6 $u_6=0.03$		x_7 $u_7=0.04$	x_8 $u_8=0.03$	x_9 $u_9=0.05$		x_{10} $u_{10}=0.07$	x_{11} $u_{11}=0.04$	x_{12} $u_{12}=0.05$		x_{13} $u_{13}=0.08$	x_{14} $u_{14}=0.04$	x_{15} $u_{15}=0.04$		x_{16} $u_{16}=0.07$	x_{17} $u_{17}=0.04$	x_{18} $u_{18}=0.05$
Table I.—Numerical Statement of Co-efficients in Equations of Condition.																							
A	+1	+1	+1	B	+1	+1	+1	C	+1	+1	+1	D	+1	+1	+1	E	+1	+1	+1	F	+1	+1	+1
G	+1				+1				+1				+1				+1				+1		
H		−19	+14			−4	+19			−12	+3			−15	+6			−13	+13			−13	+19
		$-s_2$	$+s_3$			$-s_5$	$+s_6$			$-s_8$	$+s_9$			$-s_{11}$	$+s_{12}$			$-s_{14}$	$+s_{15}$			$-s_{17}$	$+s_{18}$
Table II.—Products of Co-efficients in Table I and the corresponding u.																							
A.u	+0.06	+0.06	+0.08	B.u	+0.05	+0.07	+0.03	C.u	+0.04	+0.03	+0.05	D.u	+0.07	+0.04	+0.05	E.u	+0.08	+0.04	+0.04	F.u	+0.07	+0.04	+0.05
G.u	+0.06				+0.05				+0.04				+0.07				+0.08				+0.07		
H.u		−1.14	+1.12			−0.28	+0.57			−0.36	+0.15			−0.60	+0.30			−0.52	+0.52			−0.52	+0.95
Table III.—Products of the quantities in Table II and those in corresponding columns of Table I.																							
Au.A	+0.06	+0.06	+0.08	Bu.B	+0.05	+0.07	+0.03	Cu.C	+0.04	+0.03	+0.05	Du.D	+0.07	+0.04	+0.05	Eu.E	+0.08	+0.04	+0.04	Fu.F	+0.07	+0.04	+0.05
Au.G	+0.06			Bu.G	+0.05			Cu.G	+0.04			Du.G	+0.07			Eu.G	+0.08			Fu.G	+0.07		
Au.H		−1.14	+1.12	Bu.H		−0.28	+0.57	Cu.H		−0.36	+0.15	Du.H		−0.60	+0.30	Eu.H		−0.52	+0.52	Fu.H		−0.52	+0.95
Gu.G	+0.06				+0.05				+0.04				+0.07				+0.08				+0.07		
Hu.H		+21.66	+15.68			+1.12	+10.83			+4.32	+0.45			+9.00	+1.80			+6.76	+6.76			+6.76	+18.05

Summations from Table III

Co-eff. of Factor	In Eq.
[Au.A] = + 0.20 f_1	1
[Au.G] = + 0.06 f_g	1
[Au.H] = − 0.02 f_h	1
[Bu.B] = + 0.15 f_2	2
[Bu.G] = + 0.05 f_g	2
[Bu.H] = + 0.29 f_h	2
[Cu.C] = + 0.12 f_3	3
[Cu.G] = + 0.04 f_g	3
[Cu.H] = − 0.21 f_h	3
[Du.D] = + 0.16 f_4	4
[Du.G] = + 0.07 f_g	4
[Du.H] = − 0.30 f_h	4
[Eu.E] = + 0.16 f_5	5
[Eu.G] = + 0.08 f_g	5
[Eu.H] = − 0.00 f_h	5
[Fu.F] = + 0.16 f_6	6
[Fu.G] = + 0.07 f_g	6
[Fu.H] = + 0.43 f_h	6
[Gu.G] = + 0.37 f_g	7
[Hu.H] = + 103.19 f_h	8

Table IV.—Equations between Factors, and determination of the Values of *f*s.

Eq. No.		Co-efficients of f_1	f_2	f_3	f_4	f_5	f_6	f_g	f_h	e
1		+ 0.20						+ 0.06	− 0.02	+ 0.096
2			+ 0.15					+ 0.05	+ 0.29	− 0.121
3				+ 0.12				+ 0.04	− 0.21	− 0.449
4					+ 0.16			+ 0.07	− 0.30	+ 0.034
5			*			+ 0.16		+ 0.08	− 0.00	+ 0.326
6							+ 0.16	+ 0.07	+ 0.43	− 0.088
7								+ 0.37		− 0.43
8									+ 103.19	− 0.6
	From Equation 1							− 0.0180	+ 0.0060	− 0.02880
	" 2							− 0.0167	− 0.0967	+ 0.04033
	" 3							− 0.0133	+ 0.0700	+ 0.14967
	" 4							− 0.0306	+ 0.1313	− 0.01487
	" 5							− 0.0400	+ 0.0000	− 0.16300
	" 6							− 0.0306	− 0.1881	+ 0.03850
	Bring down Eqn. 7							+ 0.37		− 0.43
9	Sum							+ 0.2208	− 0.0775	− 0.40817
	From Equation 1								− 0.0020	+ 0.00960
	" 2								− 0.5607	+ 0.23393
	" 3								− 0.3675	− 0.78575
	" 4								− 0.5625	+ 0.06375
	" 5								− 0.0000	+ 0.00000
	" 6								− 1.1556	+ 0.23650
	Bring down Eqn. 8								+ 103.19	− 0.6
10	Sum								+ 100.5417	− 0.84197
11	From Equation 9								− 0.0272	− 0.14327
	Sum of 10 and 11								+ 100.5145	− 0.98524
									f_h = −	0.00980

* The quantities below the diagonal are not to be recorded, being identical with those above.

+ 0.2208 f_g = + 0.0775 f_h − 0.40817
= − 0.00076 − 0.40817 = − 0.40893
f_g = − 1.85204

+ 0.20 f_1 = − 0.06 f_g + 0.02 f_h + 0.096
= + 0.11112 − 0.00020 + 0.096 = + 0.20692
f_1 = + 1.03460

+ 0.15 f_2 = − 0.05 f_g − 0.29 f_h − 0.121
= + 0.09260 + 0.00284 − 0.121 = − 0.02556
f_2 = − 0.17040

+ 0.12 f_3 = − 0.04 f_g + 0.21 f_h − 0.449
= + 0.07408 − 0.00206 − 0.449 = − 0.37698
f_3 = − 3.14150

+ 0.16 f_4 = − 0.07 f_g + 0.30 f_h + 0.034
= + 0.12964 − 0.00294 + 0.034 = + 0.16070
f_4 = + 1.00438

+ 0.16 f_5 = − 0.08 f_g − 0.00 f_h + 0.326
= + 0.14816 − 0.00000 + 0.326 = + 0.47416
f_5 = + 2.96350

+ 0.16 f_6 = − 0.07 f_g − 0.43 f_h − 0.088
= + 0.12964 + 0.00421 − 0.088 = + 0.04585
f_6 = + 0.28656

Computed and Compared by ____________________ 18____

PART III.

Azimuths.

CHAPTER I. CIRCUMPOLAR-STAR OBSERVATIONS FOR AZIMUTH.

Object of measuring azimuths.

1. In order that any network of triangulation may be correctly orientated, it is necessary that at least one side of one of the triangles should have its azimuth rigorously determined by astronomical observations. Theoretically, if the azimuth of one side is known, all the others follow from it, by a simple computation based on assumed elements of the earth's figure in combination with the measured angles of the triangles. Practically, however, the difficulty comes in that the elements of the earth's figure are not yet known with absolute certainty, and also that the unavoidable errors in the measurement of the angles become involved in the computed azimuths. To meet this, it was originally intended that the computed azimuths should at certain stated distances be *corrected* by means of celestial observations, and the accumulation of error thus kept within narrow bounds. The modern view is, however, opposed to this, and the practice now obtaining is to observe astronomical azimuths at convenient intervals of about 2° apart, and, by comparing these with the azimuths brought up by computation from the origin at Kaliánpur*, to form equations whose solution will, when the whole scheme is complete, afford corrections to the adopted elements, and aid in pointing out the existence of local attractions or other anomalies.

General principles.

2. In the Third Edition of the Auxiliary Tables, pages 78 to 81, six methods of measuring an azimuth are given: of these, however, only one—the second somewhat modified—is ever used in the Great Trigonometrical Survey where a rigorous determination is required. Circumpolar stars, *i.e.*, those stars whose N.P.D. is less than the latitude of the station of observation, are, in their diurnal course, twice stationary in azimuth, once on the eastern, and once on the western side of the pole; at which times they are said

* The adopted initial azimuth is that of Súrentál at Kaliánpur taken at 190° 27′ 5″·10, *vide* page 141 of Volume II of the *Account of the Operations of the Great Trigonometrical Survey of India.*

to be *at elongation.* At these instants their true azimuths are easily computed, and they afford excellent objects for observation, being almost stationary in the same direction for a considerable time. The principle of the observation consists merely in measuring the angle in the horizontal plane between the star at elongation and a referring mark, whence the azimuth of the latter, and through it the azimuth of any side of the triangulation at that station, becomes known. The method of observation, the management of the levels, which is especially important, and other valuable details will be found at pages 38 *et seq.* of the Auxiliary Tables, to which the reader is referred. Further information on this subject will also be found in Chapter XII, Volume II of the *Account of the Operations of the Great Trigonometrical Survey of India.* Practically the time during which the observations may be made, is not limited to the exact moment of elongation; the limit of safety depending partly on the latitude, and partly on the star's polar distance. The measurements made before and after the elongation are corrected for the interval by a process explained in the next chapter, and thus become of equal value with those made at the moment of elongation in determining the final azimuth. The zeros and changes of face should be the same as those laid down for horizontal angles, *vide* Part II, Chapter I, para. 24 of this handbook, with this exception only, that four rounds instead of three should be taken on each face, divided up as exemplified on page 42 of the Auxiliary Tables.

Referring mark.

3. The referring mark should be at a distance of about 2 miles, so as to be well defined in the telescope with solar focus, and yet sufficiently close to ensure easy visibility. It is convenient also, though not indispensable, that it should be about on the same level as the station from which the observations are made, in order to avoid corrections for collimation and level, which would otherwise be necessary. The aperture of the lamp should be reduced to one inch or thereabouts.

Time-limits of observations.

4. Some criterion of this limit of safety is desirable in order that the observer may lay out his time to the best advantage. In imposing such a limit it may be as well to state that the formulæ now employed in the computation of azimuths are sufficiently rigorous to justify an observer in selecting a circumpolar star at almost any point of its diurnal course for determination of azimuth. It must be remembered on the other hand that a star

is only stationary in azimuth at elongation, and the farther it is from that point the quicker it moves (in azimuth), and the more difficult of accurate intersection it becomes in consequence. The following limitation may be accepted, *viz.*, that a circumpolar star should not be used for observations of azimuth when its change of azimuth exceeds 1″ in 10 seconds of time. The following table, which is founded on this limitation, gives the approximate value of the hour angle reckoned from time of elongation (either before or after), which should not be exceeded in observations of this kind.

N.P.D.	Latitude.						
	8°	12°	16°	20°	24°	28°	32°
	h m	*h m*	*h m*	*m h*	*h m*	*h m*	*h m*
1° 0′	2 20	2 12	2 5	1 57	1 50	1 43	1 36
1 15	1 49	1 43	1 38	1 32	1 27	1 21	1 16
1 30	1 30	1 25	1 21	1 16	1 12	1 7	1 3
1 45	1 17	1 13	1 9	1 5	1 1	0 57	0 54
2 0	1 7	1 3	1 0	0 57	0 53	0 50	0 47
2 30	0 53	0 50	0 48	0 45	0 43	0 40	0 37
3 0	0 44	0 42	0 40	0 38	0 35	0 33	0 31
4 0	0 33	0 31	0 30	0 28	0 27	0 25	0 23
5 0	0 26	0 25	0 24	0 23	0 21	0 20	0 19
6 0	0 22	0 21	0 20	0 19	0 18	0 17	0 16

The table is based on the following approximate formula, $\sin P = \frac{\delta A}{\delta P} \cdot \frac{\text{colat}}{\text{N.P.D.}}$, where P is the hour angle from culmination, and δA is the change of azimuth corresponding to a change δP in the hour angle. The value of the fraction $\frac{\delta A}{\delta P}$ is taken at ·007, corresponding to the change of 1″ of azimuth in 10 seconds of time, as explained above. It is by no means intended that the observer should use this table to extend his work to an undue amount on any one night with a view of thereby decreasing the number of nights' work required. It is a well known fact, though as yet quite unexplained, that even the best instruments will give different results at different times, or even on consecutive nights, when all the circumstances are *apparently* the same; and it is therefore always better, when a certain quantity of measures of one thing have to be accumulated, to spread them over several days, rather than to take them all in one. The chief use of the table is to enable the observer to see how long before or after elongation, it will be safe for him to work, supposing that some accidental cause, such as clouds, prevents him observing at or very near the most favourable position.

CHAPTER II. COMPUTATION OF CIRCUMPOLAR-STAR OBSERVATIONS.

1. The form provided for these calculations is *P. 32*. Specimens of the calculation are given at the end of this chapter.

Statement of Data.

2. Page 1 commences with a statement of data. First are entered the circumpolar stars, the one observed at Eastern Elongation after E. and that at Western Elongation after W. Below are entered the stars employed for finding time, together with their approximate declinations and zenith distances, provision being made for the case when the stars employed for time of western elongation are not the same as those for eastern elongation. On the right-hand side of the same statement are given the Azimuth of the Referring Mark as obtained through the triangulation, the Latitude and Longitude of the observing station obtained in the same way—the longitude being reduced to time and then to decimals of a day—the names and numbers of the Theodolite and Chronometer employed, the values of the level scales of the theodolite, and lastly the name of the observer. The rest of page 1 is taken up with the determination of correction and rate of the chronometer from the time-star observations. The correction is found for each night by ascertaining the chronometer times at which certain selected stars pass the meridian and comparing them with the Stars' Right Ascensions, and the rate is given by the successive differences of these comparisons from night to night.

Corrections for Instrumental Errors.

3. The first thing to be done in reducing the time-star observations, is to compute the corrections necessary for error of *Collimation*, error of Level or *Inclination* in the transit axis, and error of setting of the instrument in the meridian, otherwise called *Deviation*. The method of obtaining these corrections is already fully explained in the Auxiliary Tables,

Third Edition, pages 42 and 43. Provision is made in the form for the necessary calculations under the heading of "Transit Corrections for Instrumental Errors." The portion of the form at the bottom of page 1, headed "Deduced Chronometer Corrections and Rates," does not need explanation as the headings of the several columns are sufficiently explicit.

The Apparent Places of the Circumpolar Stars.

4. The next step is to find the apparent places of the circumpolar stars at elongation. If they are Nautical Almanac stars, page 2 of the form is not required, as the apparent places are given for Upper Transit at Greenwich, and the R.A. and N.P.D. can be entered at once on page 3 for each day of observation in the columns set apart for the purpose. The corrections to these values for difference of longitude and for time of elongation must next be computed and entered in the two following columns. Taking the change for one day and multiplying this by the longitude reduced to the decimal of a day (see page 1) gives the first correction. As stars transit in India before they transit at Greenwich, the correction is always a backward one. For the second correction it is first necessary to know the horary angle at elongation: this is computed on the lower portion of page 3 for the first date of observation, and for the present purpose will be sufficient. The angle which is obtained in arc must be reduced to time and to the decimal of a day and then multiplied by the daily changes in R.A. and N.P.D. to give the correction to elongation. If the elongation is eastern the correction will be a backward one, if western, a forward one. Columns 2, 3 and 4 of each compartment added together algebraically, give the star's apparent place at elongation.

Stars taken from Star Catalogues.

5. If now the circumpolar stars are not Nautical Almanac stars, but taken from a Star Catalogue, page 2 of the form must be employed for calculating their *apparent* places on the first and last days of observation. The catalogue gives the *mean* places of stars on the 1st January of its epoch. The corrections required to reduce to *mean* places on the 1st January of the year of observation are similar for R.A. and N.P.D., and are of the following form.

$$\left\{ p + \mu + \frac{s}{100} \cdot \frac{y}{2} \right\} y$$

in which p is the Annual Precession, μ the Annual Proper Motion, s the

Secular Variation, in R.A. and N.P.D. respectively, and *y* the number of years from the epoch of the catalogue to the 1st January of year of observation. If the star is near the pole, and the period elapsed from the epoch of the catalogue be considerable, a further correction is necessary, as stated in the foot-note to page 2 of the form (see page 62 of the Auxiliary Tables). The *apparent* places at Greenwich *mean midnight* are then obtained by applying the corrections

$$Ee + Ff + Gg + Hh + l + L - 300 \text{ to the R.A.,}$$

$$\text{and } Ee' + Ff' + Gg' + Hh' + l' + L - 300 \text{ to the N.P.D;}$$

e, f, &c., and e′, f′, &c., being furnished by the catalogue and E, F, &c., called Airy's Day Numbers, being taken from the Nautical Almanac for the first and last days of observation. Provision for these calculations is made on page 2 of the form. Having obtained the *apparent* places of the circumpolar stars for Greenwich *mean midnight* (Gr. M. M.) on the first and last days of observation, the next step is to reduce these to the times of Upper Transit at Greenwich. This calculation is made independently of the form. From page II of the Nautical Almanac for the required month take out the sidereal time of Greenwich mean midnight for the astronomical date of observation, *i.e.*, take the mean of the successive sidereal times of mean noon and subtract 12 hours, and compare it with the R.A. of the star.* Four cases will present themselves, *viz.*,

(1)	R.A. of star	*greater* than Sid. T.	at Gr. M. M. by	*less* than	12 hours.
(2)	,,	,,	,,	*more*	,,
(3)	,,	*less*	,,	*less*	,,
(4)	,,	,,	,,	*more*	,,

In case (1) the star transits at Greenwich on the date of observation after Gr. M. M. by a period of time represented by

$$\text{R.A. of star} - \text{Sid. Time at Gr. M. M.}$$

This reduced to the fraction of a day and multiplied by the daily change

* If the R.A. of the star is greater than the R.A. of the Sun at Gr. mean noon by less than $3^m\ 56^s{\cdot}5$ the star will transit twice during the date of observation, and care must be taken to compare with that transit which occurs nearest the elongation observed.

in R.A. or N.P.D. gives the correction to the time of transit at Greenwich and this correction is a *forward* one.

(2) The star has transited between Gr. M. N. and Gr. M. M. on the date of observation, but has not passed its lower culmination. The period from transit is represented by

Sid. Time at Gr. M. M. + 24 hours – R.A. of star,

and the correction is a *backward* one.

(3) The star has transited on the date of observation before Gr. M. M. by a period of time represented by

Sid. Time at Gr. M. M. – R.A. of star,

and the correction is a *backward* one.

(4) The star will transit after a period represented by

R.A. of star + 24 hours – Sid. Time at Gr. M. M.,

and the correction is a *forward* one. The difference in R.A. or N.P.D. between the first day and last day, divided by the number of days gives the daily change in each for entry in the heading of the columns on page 3, "R.A. at Upper Transit Greenwich," and "N.P.D. at Upper Transit Greenwich." Page 3 may now be entered and employed as described for Nautical Almanac stars.

Horary Angle and Azimuth, and their Factors.

6. After the *apparent* places of the circumpolar stars have been found at times of elongation, the remainder of the computation at the bottom of page 3 may be proceeded with. The horary angles and azimuths at elongation on the first days of observation may be computed, and then what are called the "Factors" for horary angle and azimuth. These factors are obtained to save the labour of computing the horary angle and azimuth independently each day with the corresponding N.P.D.

Deduction of Star's Azimuth &c.

7. Having obtained the horary and azimuth angles for the first day, it is only necessary to correct them for subsequent days by multiplying the changes in N.P.D. by the 'factors'. For this calculation employ the upper part of

page 4 of the form. Enter the table on page 4 headed "Deduction of Star's Azimuth &c.," and fill in the first three columns according to the dates of observation, and in the fourth column, omitting the first line corresponding to the date on which the N.P.D. has been actually calculated, fill in successively the accumulated change in N.P.D. from that date from page 3.

Sidereal Times of Elongation.

8. Now multiply each of these by the factor for the horary angle and enter in the fifth column. In the sixth column enter the horary angle for the first date—as computed on page 3—in degrees, minutes and seconds to one place of decimals, and by applying the corrections from the fifth column obtain the horary angles for the subsequent dates. In the seventh column change these angles into time. In the eighth column enter the star's R.A. at elongation from page 3 for the several dates. If the elongation is eastern subtract the horary angle from the R.A. at elongation adding 24 hours to the R.A. if necessary and if western add. The results are the sidereal times of elongation, and are to be entered in the ninth column to one place of decimals.

Azimuths at Elongation.

9. In the tenth column, omitting the first line, are entered the corrections to the azimuth of the first date obtained by multiplying the quantities in the fourth column by the factor for azimuth from page 3. In the last column is first entered the Azimuth at Elongation as computed for the first date and below the successive azimuths obtained by applying the corrections in the preceding column, keeping two places of decimals.

Formula for δA.

10. Up to this point the calculations have only been concerned with the star or stars at elongation. But they are observed many times both before and after elongation; and it is necessary to calculate, for each observation, a quantity δA, by which the azimuth of the star at the moment of observation, differs from its azimuth at elongation. The formula employed is

$$\delta A = \frac{2 \sin^2 \frac{1}{2}\delta P \operatorname{cosec} 1'' \tan A \cos^2 \text{N.P.D.}}{1 - 2 \sin^2 \text{N.P.D.} \sin^2 \frac{1}{2} \delta P \pm \cot P \sin \delta P},$$

in which A is the azimuth of the circumpolar star at elongation, P the corresponding hour angle and δP the interval from elongation of the star for which δA is required in seconds of arc. The last term of the denominator is + or − according as the star is *below* or *above* the position of elongation. Of the above formula the factor tan A. $\cos^2$ N.P.D. may be considered as constant for each night, and the successive values are computed in the table in the centre of page 4 of the form. The headings of the several columns of this table sufficiently explain themselves.

Calculation of δP.

11. After this table follow a series of compartments for computing δP for each observation on each night for a pair of zeros, Face Right and Face Left. The chief part of the calculation is necessitated by the Chronometer times of observation needing correction for error and rate of the chronometer.

Continued.

12. The first four columns need no remark except that the name of the star is to be entered in the first. In the heading of the fifth insert the 'Chronometer Time of Corrected Transit' of the time-star from page 1, or if more than one star has been observed for time, the mean time of transit of all the stars may be employed; but this should only be done when all the stars have been regularly observed. Below enter the individual chronometer times of the observations of the circumpolar star. The difference between these and the time in the heading of the column are the chronometer intervals from transit of the time-star, at which moment the chronometer correction is known. These are given in the form in seconds, and when multiplied by the rate per second, taken from page 1 and entered in the heading of the seventh column, give the corrections to be recorded in the seventh column for the rate of the chronometer during the intervals. In the heading of the next, or eighth column enter the correction to the chronometer at time of transit of the time-star from page 1, and below it the algebraical sum of this quantity and those in the preceding column. In the heading of the ninth column enter the sidereal time of elongation of the circumpolar star from page 4, for the particular date and below it the algebraical sum of the fifth and eighth columns. The difference of the quantity in the heading of the column, and each of those below is the interval from elongation, or δP, and is to be given in the form in minutes and seconds and also in minutes and decimals of a minute to two places.

Calculation of δA.

13. When the above process has been gone through for each pair of zeros in turn, the next part of the calculation is the finding of the corresponding values

of δA. This is commenced on page 7 under the heading "Computation of δA and Corrected Angle between Referring Mark and Star." First enter δP in minutes and decimals of a minute as previously computed on page 4, and with this as one argument and the N.P.D. of the circumpolar star as the other, enter Table XXV of the Auxiliary Tables, Part I, and take out the value of ($2 \sin^2$ N.P.D. $\sin^2 \frac{1}{2} \delta P$), subtract it from unity and enter the result in the form. Next with δP and the Hour Angle, P, as arguments, enter Part II of Table XXV, and take out ($\cot P$. $\sin \delta P$) : this quantity will be + when the altitude of the star is less than at elongation and − when greater. Take the algebraical sum of the two quantities and then the log and the co-log. Again refer to Table XXV and to Part III, and with δP as argument in minutes and seconds, from page 4, take out the quantity ($\log 2 \sin^2 \frac{1}{2} \delta P \operatorname{cosec} 1''$). Insert ($\log \tan A$ $\cos^2$ N.P.D.) from page 4 and add. The sum is equal to $\log \delta A$. Taking out the natural number, which is in seconds of arc, and adding to or subtracting from the observed angle, Referring Mark and Star, according to the position of the Referring Mark relatively to the star, gives the angle Referring Mark and Star at elongation, or as it is called in the form "Corrected Angle, Referring Mark and Star."

Instrumental Correction.

14. When this has been done for all the observations, the next step is to calculate any correction which may be due to the body of the instrument employed for the observations not being perfectly level as shewn by the level readings taken for the purpose.

Continued.

15. For the calculation of this correction the tangent of the circumpolar star's altitude at elongation is required. This is computed at the bottom of page 11 of the form, from the formula

$$\sin \text{Alt.} = \sec \text{N.P.D.} \sin \text{Lat.}$$

This gives the correct altitude, which has to be converted to apparent altitude by the addition of refraction. Taking the tangent of this angle and multiplying it by the value of 1 division of the scale of the Level on the body of the instrument, or adding their logs, as in the form, gives a factor for the level error to be employed on page 12.

Instrumental Correction.

16. Now enter page 12 and to the mean of all the east readings of the level on Face Right add the mean of all the east readings on Face Left, for each pair of zeros independently: do the same for the west readings, then ¼th of the difference of the two resulting quantities is the level correction, denoted in the form by N in divisions of the level. Multiplying by the factor on the preceding page gives the correction in seconds of arc to be applied to the Angles Referring Mark and Star at elongation. The sign of the correction is found by the rule at the bottom of the page in the form.* The rule given on page 44 of the Auxiliary Tables applies to the angle Referring Mark and North Point. The correction will have the same sign for the observations on the same pairs of zeros, Face Right and Face Left.

Continued.

17. The lower half of page 12 may now be filled up. First abstract all the results "Corrected Angle Referring Mark and Star" from pages 7 to 11. Take out the means of the results on each face of each pair of zeros separately and then the mean of Face Right and Face Left. The sign of this quantity will be + if the R.M. is east of the star and − if west. Apply the level corrections from the upper half of the page and there result as many values of the angle Referring Mark and Star at Elongation as there are pairs of zeros. Now apply the Corrected Angle to the Star's Azimuth at Elongation from the south, which is obtained thus:—if the elongation is east add 180° to the azimuths computed on page 4 for the respective dates, but if west subtract the azimuths from 180° and there result as many values of the Azimuth of the Referring Mark. Take the mean of the values of Azimuth by Western Elongation and also that by Eastern Elongation and then the mean of both. This angle is then applied to the angle between one of the surrounding stations and the Referring Mark for comparison with the value brought up by triangulation.

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* The following consideration may help in the application of the sign:—If the E. end readings are the greater the apparent, or instrumental, zenith is west of the true zenith, and a plane through the instrumental zenith, the instrument and star will cut the horizon to the east of the point to which the star would be referred from the true zenith. If the Referring Mark is to the east of the star the angle Referring Mark and Star is consequently too small and the correction is therefore additive. Similar reasoning applies to other positions of the Referring Mark and the apparent zenith.

NO. ...... PARTY ( ) SEASON 18 - .

Computation of Circumpolar Star Observations for Azimuth at Toungzûn H.S.

Circumpolar Stars. E. No. 1302 Gr. 9-Yr. W. Polaris

Transit Stars.		Approx. Dec. N. or S. ° ′	Z. D. ° ′
No. 893 Gr. 9-Yr.	E.E.	29 - 54 S.	46 - 20
Sirius	W.E.	16 - 34 S.	33 - 0

Azimuth of Referring Mark (by Triangulation) 31°-16′-25″·85

Latitude of Station of Obsn. 16-25-50·13

Longitude " 97-42-46·62 = 6h-30m-51s·11 = 0d·271

Instrument Barrow's 24-in: Theod: No. 2 Chron. Sidereal T. No. 3402 by Frodm.

1 Divn. of Transit Axis Level = l = 0″·875

1 " Body Level, l_1 = 1″·031 ; l_2 =

Observer Major G. Strahan R.E.

Transit Corrections for Instrumental Errors.

Date 1884	18th Mar.	18th Mar.	19th Mar.	19th Mar.								
Star	Sirius	No. 893	Sirius	No. 893								
Collimation	+ 6·12	− 4·14	+ 2·96	− 2·28								
	+ 4·92	+ 1·40	+ 1·76	+ 1·20								
	− 0·72	+ 1·44	+ 0·10	+ 0·46								
	− 4·14	+ 4·90	− 2·28	− 4·68								
	+ 1·40	− 5·10	+ 1·20	− 1·08								
Sum	+ 7·58	− 1·50	+ 3·74	− 6·38								
⅕ Mean = a	+ 0·8	− 0·2	+ 0·4	− 0·6								
	div.	div.	div.	div.	div.	div.	div.	div.	div.	div.	div.	div.
Inclination	−27·5	−46·0	−42·0	−53·0								
	+141·5	+139·1	+109·6	+124·9								
Sum	+114·0	+93·1	+67·6	+71·9								
¼ Mean = i	+28·50	+23·28	+16·90	+17·98								
i × l	+24·9	+20·4	+14·8	+15·7								
Cos Z.D.*	0·839	0·691	0·839	0·691								
Product = β	+20·9	+14·1	+12·4	+10·8								
Deviation	− 7·52	− 7·95	−15·78	−21·70								
Sin Z.D.*	0·545	0·723	0·545	0·723								
Product = γ	− 4·1	− 5·7	− 8·6	−15·7								
a + β + γ	+17·6	+ 8·2	+ 4·2	− 5·5								
1 / 15 Sin N.P.D.*	0·07[illegible]	0·077	0·070	0·077								
Product = Corrn.	+ 1·23	+ 0·63	+ 0·29	− 0·42								

NOTE.—Keep a, i × l, β and γ to 1 place, and the correction to the Observed Time of Transit to 2 places of decimals.
* Take from Table XXIV of Auxiliary Tables, 3rd Edition, 1887.

Rules for the signs of Corrections, the Star being above the Pole.

Collimation + when F. L. reads highest with circles reading from left to right and the Transit is observed with the Face to the East. Inclination + when West pivot of telescope axis is highest. Deviation + for Stars of North aspect and − for Stars of South aspect when reading of Azimuth circle is too high with instruments reading from left to right.

Deduced Chronometer Corrections and Rates.

Transit Star	Date	Chronometer Time of Observed Transit	Correction from above	Chronometer Time of Corrected Transit = T	Star's R.A.	Chronometer Correction = R.A. − T	Rate Correction per day	Mean Rate Correction per second
	1884	h m s	s	h m s	h m s	h m s	m s	
Sirius	18th Mar.	6-39-30·	+ 1·23	6-39-31·23	6-40- 3·32	+ 0- 0-32·09	−	
No. 893	" "	9- 4-32·	+ 0·63	9- 4-32·63	9- 5- 3·66	+ 31·03	−	
Sirius	19th "	6-39-50·	+ 0·29	6-39-50·29	6-40- 3·30	+ 13·01	− 0-19·08	
No. 893	" "	9- 4-54·	− 0·42	9- 4-53·58	9- 5- 3·64	+ 10·06	− 0-20·97	− 0·000236

Computed and Compared by .... 18 .

Survey of India Department.

NO.____ PARTY (________________ SEASON 18__-__.

Computation of Circumpolar Star Observations for Azimuth at Toungziin H.S.

Deduction of Apparent places of Circumpolar Stars at Greenwich mean midnight on 18th March 1884 *and* 22nd March 1884

Star No. 1302 Gr. 9-Yr. Catalogue

For R.A.

e	$\bar{1}$·96649	f	0·01127	g	1·39205	h	$\bar{1}$·96922
E	0·79576	F	1·40522	G	0·19307	H	1·51724
Sum	0·76225		1·41649		1·58512		1·48646

e.E	5·784	1st January 1872 Mean R.A.	14h 9m 22·991s
f.F	26·091	Corrn. to 1st January 1884*	− 4·105
g.G	38·470	1st January 1884 Mean R.A.	14 9 18·886
h.H	30·652	Corrn. to first date of Obsn.	+ 2·827
I	108·314	Apparent R.A.	14 9 21·713
L	93·516		
Sum	302·827		
	− 300·000		
Corrn.	+ 2·827		

For N.P.D.

e′	$\bar{1}$·76886	f′	0·30682	g′	1·62265	h′	$\bar{1}$·82281
E	0·79576	F	1·40522	G	0·19307	H	1·51724
Sum	0·56462		1·71204		1·81572		1·34005

e′.E	3·670	1st January 1872 Mean N.P.D.	11° 51′ 4″·09
f′.F	51·528	Corrn. to 1st January 1884	+ 3 22·97
g′.G	65·421	1st January 1884 Mean N.P.D.	11 54 27·06
h′.H	21·880	Corrn. to first date of Obsn.	+ 13·71
I′	77·691	Apparent N.P.D.	11 54 40·77
L	93·616		
Sum	313·706		
	− 300·000		
Corrn.	+ 13·706		

For R.A.

e	$\bar{1}$·96649	f	0·01127	g	1·39205	h	$\bar{1}$·96922
E	0·79703	F	1·38033	G	0·19482	H	1·51745
Sum	0·76352		1·39160		1·58687		1·48667

e.E	5·801	1st January 1884 Mean R.A.	14 9 18·886
f.F	24·638	Corrn. to last date of Obsn.	+ 3·061
g.G	38·625	Apparent R.A.	14 9 21·947
h.H	30·667		
I	108·314		
L	95·016		
Sum	303·061		
	− 300·000		
Corrn.	+ 3·061		

For N.P.D.

e′	$\bar{1}$·76886	f′	0·30682	g′	1·62265	h′	$\bar{1}$·82281
E	0·79703	F	1·38033	G	0·19482	H	1·51745
Sum	0·56589		1·68715		1·81747		1·34026

e′.E	3·680	1st January 1884 Mean N.P.D.	11 54 27·06
f′.F	48·658	Corrn. to last date of Obsn.	+ 12·62
g′.G	65·686	Apparent N.P.D.	11 54 39·68
h′.H	21·841		
I′	77·691		
L	95·016		
Sum	312·622		
	− 300·000		
Corrn.	+ 12·622		

Star________

For R.A.

e	.	f	.	g	.	h	.
E	.	F	.	G	.	H	.
Sum	.						

e.E	.	1st January 18 _ Mean R.A.	
f.F	.	Corrn. to 1st January 18 _	
g.G	.	1st January 18 _ Mean R.A.	
h.H	.	Corrn. to first date of Obsn.	
I	.	Apparent R.A.	
L	.		
Sum			
	− 300·000		
Corrn.	.		

For N.P.D.

e′	.	f′	.	g′	.	h′	.
E	.	F	.	G	.	H	.
Sum	.						

e′.E	.	1st January 18 _ Mean N.P.D.	
f′.F	.	Corrn. to 1st January 18 _	
g′.G	.	1st January 18 _ Mean N.P.D.	
h′.H	.	Corrn. to first date of Obsn.	
I′	.	Apparent N.P.D.	
L			
Sum			
	− 300·000		
Corrn.	.		

For R.A.

e	.	f	.	g	.	h	.
E	.	F	.	G	.	H	.
Sum	.						

e.E	.	1st January 18 Mean R.A.	
f.F	.	Corrn. to last date of Obsn.	
g.G	.	Apparent R.A.	
h.H	.		
I	.		
L	.		
Sum			
	− 300·000		
Corrn.			

For N.P.D.

e′	.	f′	.	g′	.	h′	.
E	.	F	.	G	.	H	.
Sum	.						

e′.E	.	1st January 18 Mean N.P.D.	
f′.F	.	Corrn. to last date of Obsn.	
g′.G	.	Apparent N.P.D.	
h′.H			
I′			
L			
Sum			
	− 300·000		
Corrn.			

* Note.—If p = Annual Precession, μ = Annual Proper Motion, s = Secular Variation, y = Number of years between epoch of Catalogue & 1st January of year of Observation, the Correction to the R.A. or N.P.D. given in the Catalogue (as the case may be) to obtain the place of the Star on the 1st January of the year of Observation is $\left\{ p + \mu + \frac{s}{100} \cdot \frac{1}{2} y \right\} y$, to which if the interval between the epoch of the Catalogue and the year of Observation is large and the Star is near the pole, a further correction is necessary, viz., $+ \frac{y^3}{6}\left(\frac{d^3 \text{R.A.}}{dy^3} \text{ or } - \frac{d^3 \text{Dec.}}{dy^3}\right)$, see pp. 63 and (117) of Table LIV of the Auxiliary Tables, 3rd Edition, 1867.

Computed and Compared by ________________ 18____

NO. ____ PARTY (____________) SEASON 18____

Computation of Circumpolar Star Observations for Azimuth at Toungziun H.S.

Deduction of Star's place at times of Maximum Elongation on days of Observation.

Star No. 1302 Gr. 9-Yr. Cat.

Date 1884	R.A. at Upper Transit Greenwich (Diff. for 1 Day -+ 0.058)	Corrections for Longitude	Corrections to Elongation	R.A. at Eastern Elongation	R.A. at Western Elongation
	h m s	s	s	h m s	h m s
Mar. 18	14 9 21.72	− 0.02	− 0.01	14 9 21.69	
19	21.78	− .02	− .01	21.75	
20	21.84	− .02	− .01	21.81	
21	21.90	− .02	− .01	21.87	
22	21.95	− .02	− .01	21.92	

Star ____________

Date 18__	R.A. at Upper Transit Greenwich (Diff. for 1 Day)	Corrections for Longitude	Corrections to Elongation	R.A. at Eastern Elongation	R.A. at Western Elongation
	h m s	s	s	h m s	h m s

Date 1884	N.P.D. at Upper Transit Greenwich (Diff. for 1 Day -- 0.270)	Corrections for Longitude	Corrections to Elongation	N.P.D. at Eastern Elongation	N.P.D. at Western Elongation
	° ′ ″	″	″	° ′ ″	° ′ ″
Mar. 18	11 54 40.74	+ 0.07	+ 0.07	11 54 40.88	
19	40.47	+ .07	+ .07	40.61	
20	40.20	+ .07	+ .07	40.34	
21	39.93	+ .07	+ .07	40.07	
22	39.66	+ .07	+ .07	39.80	

Date 18__	N.P.D. at Upper Transit Greenwich (Diff. for 1 Day)	Corrections for Longitude	Corrections to Elongation	N.P.D. at Eastern Elongation	N.P.D. at Western Elongation
	° ′ ″	″	″	° ′ ″	° ′ ″

Computation of Factors for Horary Angle and Azimuth.

For Star No. 1302

Horary Angle and Azimuth on 18th March 1884 at E. Elongation

	° ′ ″	Horary Angle.	Azimuth.
Lat. of Station	16 25 50.13	Tan 1̄.4696693	Sec 0.0181076
Star's N.P.D.	11 54 40.88	Tan 1̄.3241591	Sin 1̄.3147057
		Cos 2̄.7938284	Sin 1̄.3328133
		86 26 0.92	12 25 35.11

* Factor for Horary Angle.		* Factor for Azimuth.	
Sec² N.P.D.	0.0189068	Cos N.P.D.	1̄.9905466
Tan Latitude	1̄.4696693	Sec Latitude	0.0181076
Cosec Hor. Angle	0.0008419	Sec Azimuth	0.0102952
Sum	1̄.4894180	Sum	0.0189494
Nat. No. = −	0.309	Nat. No. = +	1.045

For Star ____________

Horary Angle and Azimuth on ______ 18__ at ____ Elongation

	° ′ ″	Horary Angle.	Azimuth.
Lat. of Station		Tan	Sec
Star's N.P.D.		Tan	Sin
		Cos	Sin
		° ′ ″	° ′ ″

* Factor for Horary Angle.		* Factor for Azimuth.	
Sec² N.P.D.		Cos N.P.D.	
Tan Latitude		Sec Latitude	
Cosec Hor. Angle		Sec Azimuth	
Sum		Sum	
Nat. No. = −		Nat. No. = +	

* Note.—The sign of the Factor for Horary Angle is always minus, and that of Factor for Azimuth always plus.

Computed and Compared by ______________________ 18____.

Survey of India Department.

NO.____ PARTY (________________________) SEASON 18___-___.

Computation of Circumpolar Star Observations for Azimuth at Toungziun H.S.

Deduction of Star's Azimuth and the Sidereal time at Maximum Elongation on days of Observation.

Period: Date 1884.	Period: Elongation	Star	Change in N.P.D. = a	Correction to Horary Angle = a × Factor computed on p. 8	Horary Angle at Maximum: In Space	Horary Angle at Maximum: In Time	Star's R.A. at Maximum	Sidereal Time of Maximum	Correction to Azimuth = a × Factor computed on p. 8	Azimuth at Maximum
			"	"	° ′ ″	h m s	h m s	h m s	"	° ′ ″
March 18	E.	1302			86-26 0·92	5-45-44·06	14- 9-21·69	8-23-37·63		12-25-35·11
" 19			− 0·27	+ 0·08	1·00	44·07	21·75	37·68	− 0·28	34·83

If δA = the difference of Azimuth of a Star at Maximum Elongation and at a time δP before or after Elongation, $\delta A = \frac{(2 \sin^2 \frac{1}{2} \delta P \cdot \text{Cosec } 1'') (\text{Tan A Cos}^2 \text{N.P.D.})}{1 - 2 \sin^2 \text{N.P.D.} \sin^2 \frac{1}{2} \delta P \pm \text{Cot P} \sin \delta P}$ in which A is the Azimuth of the Star at Elongation, and P the Hour Angle in space.

NOTE.—The last term of the denominator is − or + according as the Star is *above* or *below* the position of Maximum Elongation.

Computation of Constant Tan Azimuth × Cos² N.P.D. for each day of Observation.

Date 1884	Elongation	Star	Log. Tan Azimuth	Log. Cos² N.P.D.	Sum = Log Constant	Date 18__	Elongation	Star	Log. Tan Azimuth	Log. Cos² N.P.D.	Sum = Log Constant
Mar. 18	E.	1302	$\bar{1}$·34311	$\bar{1}$·98109	$\bar{1}$·32420						
19			11	09	20						
20			10	09	19						

Correction of Observed Times, and Deduction of δP.

Date and Elongation 1884.	No. of Observation	Face	Zero	Observed Times*: Time of Transit − 7 52 1·93	Interval from Transit	Correction for Rate: Rate per Second = −0·000236	Total Correction*: Chronometer Correction = + 0 0 31·6	Corrected Times*: Sidereal Time of Maximum − 8 23 37·6	δP † in Time: In Minutes and Seconds	δP † in Time: In Minutes and Decimals	Remarks.
				h m s	s	s	h m s	h m s	m s	m	
18th Mar. Eastern	1	R	0	7-59-10	+ 428	− 0·1	+ 0 0 31·5	7-59-41·5	23-56·1	23·94	
	2	"	–	8- 2-47	645	·2	31·4	8- 3-18·4	20-19·2	20·32	
Star No. 1302	3	L	180	14-18	1336	·3	31·3	14-49·3	8-48·3	8·81	
	4	–	–	17-30	1528	·4	31·2	18- 1·2	5-36·4	5·61	
	5	–	–	26-24	2062	·5	31·1	26-55·1	3-17·5	3·29	
	6	–	–	29- 4	2222	·5	31·1	29-35·1	5-57·5	5·96	
	7	R	0	39- 1	2819	·7	30·9	39-31·9	15-54·3	15·90	
	8	–	–	41-56	2994	·7	30·9	42-26·9	18-49·3	18·82	
	9										
	10										

* NOTE.—The quantities in these columns should be kept to only 1 place of decimals.

† NOTE.—The reduction of δP into space is not required, as the terms in which functions of P and δP occur have been tabulated in the 3rd Edition of the Auxiliary Tables (No. XXV, Parts I, II and III) with δP in *time* as the argument.

Computed and Compared by ________________________ 18____.

NO.—— PARTY (————————————) SEASON 18—-—.

Computation of Circumpolar Star Observations for Azimuth at Toungzin H.S.

Computation of δA and Corrected Angle between Referring Mark and Star.

Date———— Star———— Elongation————

Elements of Computation	Observation 1st F.—— zero——	2nd F.——	3rd F.——	4th F.——	5th F.——	6th F.——	7th F.——	8th F.——
δP in minutes and decimals								
1 − 2 sin² N.P.D. sin² ½ δP (1)								
* Cot P. sin δP (2)								
Sum								
Log.								
Co-log.								
Log. 2 sin² ½ δP cosec 1″ (3)								
Log. tan A cos² N.P.D.								
Sum = log. δA								
δA								
Observed ∠, R.M. and Star								
Corrected ∠ Do.								

Date 18: March Star No. 1302 Elongation E.

Elements of Computation	Observation F. R zero 0°	F. R 0°	F. L 180°	F. L 180°	F. L 180°	F. L 180°	F. R 0°	F. R 0°
δP in minutes and decimals	23.94	20.32	8.31	5.61	3.29	5.96	15.90	18.82
1 − 2 sin² N.P.D. sin² ½ δP (1)	0.99976	0.99983	0.99997	0.99999	1.00000	0.99999	0.99990	0.99986
* Cot P. sin δP (2)	+0.00649	+0.00552	+0.00240	+0.00152	−0.00089	−0.00162	−0.00432	−0.00511
Sum	1.00625	1.00535	1.00237	1.00151	0.99911	0.99837	0.99558	0.9947[illegible]
Log.	0.00271	0.00232	0.00103	0.00066	$\bar{1}$.99961	$\bar{1}$.99929	$\bar{1}$.99808	$\bar{1}$.99771
Co-log.	$\bar{1}$.99729	$\bar{1}$.99768	$\bar{1}$.99897	$\bar{1}$.99934	0.00039	0.00071	0.00192	0.00229
Log. 2 sin² ½ δP cosec 1″ (3)	3.05074	2.90861	2.18267	1.79049	1.32759	1.84304	2.69590	2.84204
Log. tan A cos² N.P.D.	$\bar{1}$.32420	$\bar{1}$.32420	$\bar{1}$.32420	$\bar{1}$.32420	$\bar{1}$.32420	$\bar{1}$.32420	$\bar{1}$.32420	$\bar{1}$.32420
Sum = log. δA	2.37223	2.23049	1.50574	1.11403	0.65218	1.16795	2.02202	2.16853
δA	+ 3-55.68	2-50.02	0-32.04	0-13.00	0-4.49	0-14.72	1-45.20	2-27.[illegible]
Observed ∠, R.M. and Star	161-5-26.03	6-26.60	8-42.08	8-59.54	9-6.62	8-54.90	7-32.74	6-58.[illegible]
Corrected ∠ Do.	161-9-21.71	16.62	14.12	12.54	11.11	9.62	17.94	26.[illegible]

Deduction of Correction for Dislevelment of Body of Instrument.

Correction = d″ × Tan Altitude of Star at Elongation, where d = Dislevelment in Divisions multiplied by Value of 1 Division of Level Scale.

	For Star No. 1302 E.E.		For Star———— W.E.	
	° ′ ″		° ′ ″	
N.P.D. of Circumpolar Star	(11 54 41)	Log Sec 0.0094534	(- -)	Log Sec
Latitude of Stn. of Observation	(16 25 50.13)	Log Sin $\bar{1}$.4515617	(- -)	Log Sin
	° ′ ″		° ′ ″	
Altitude	16-48-10	Log Sin Alt. = Sum $\bar{1}$.4610151	- -	Log Sin Alt. = Sum
Correction for Refraction	+ 3-11		+ - -	
Apparent Altitude	16-51-21	Log Tan $\bar{1}$.48142	- -	Log Tan
1 Division Level Scale	= 1.031 ;	Log 0.01326	= ;	Log
	Log Factor	= Sum $\bar{1}$.49468	Log Factor	= Sum

* Cot P sin δP is + when the star is below and − when above the position of maximum elongation. For (1) (2) (3) see Parts I, II, III of Table XXV of the Auxiliary Tables, 3rd Edition, 1887.

Survey of India Department.

NO.____ PARTY (________________________) SEASON 18____.

Computation of Circumpolar Star Observations for Azimuth at Toungzün H.S.

Deduction of Correction for Dislevelment of Body of Instrument, and Resulting Azimuth of R.M.

Eastern Elongation.

Date.	E. End	W. End	E. End	W. End	E. End	W. End	E. End	W. End	E. End	W. End	E. End	W. End
Sum of Level readings on F.L.	272.8	244.1										
Mean of above $= M_l$	68.2	61.03										
Sum of Level readings on F.R.	210.4	305.7										
Mean of above $= M_r$	52.6	76.42										
Sum of M_l and $M_r = \Sigma$	120.8	137.45										
½ Diff. between Σ for E. end & Σ for W. end $=$ N	4.16											
Log N	0.61909											
Log Correction $=$ Log Factor $+$ Log N	0.11377											
*Correction	1.30											

Western Elongation.

Date.	E. End	W. End	E. End	W. End	E. End	W. End	E. End	W. End	E. End	W. End	E. End	W. End
Sum of Level readings on F.L.												
Mean of above $= M_l$												
Sum of Level readings on F.R.												
Mean of above $= M_r$												
Sum of M_l and $M_r = \Sigma$												
½ Diff. between Σ for E. end & Σ for W. end $=$ N												
Log N												
Log Correction $=$ Log Factor $+$ Log N												
*Correction												

Eastern Elongation.

Face L.	Zero and Date					
	180° 18th Mar.	°	°	°	°	°
Observed Angle R.M. and Star corrected to Elongation	" 14.12	"	"	"	"	"
	12.54					
	11.11					
	9.62					
Mean	11.85					

Face R.	Zero and Date					
	0° 18th Mar.	°	°	°	°	°
Observed Angle R.M. and Star corrected to Elongation	" 21.71	"	"	"	"	"
	16.62					
	17.94					
	26.31					
Mean	20.66					

Mean of both Faces	− 161° 9′ 16.26″					
Level Corrn.	− 1.30					
Corrected Mean	− 161 9 17.55					
Azth. of Star	192 25 35.11					
Azth. of R.M.	31 16 17.56					

Western Elongation.

Face L.	Zero and Date					
	°	°	°	°	°	°
Observed Angle R.M. and Star corrected to Elongation	"	"	"	"	"	"
Mean						

Face R.	Zero and Date					
	°	°	°	°	°	°
Observed Angle R.M. and Star corrected to Elongation	"	"	"	"	"	"
Mean						

Mean of both Faces	−					
Level Corrn.	−					
Corrected Mean	−					
Azth. of Star						
Azth. of R.M.						

Resulting Azimuth of Referring Mark (Konlah H.S.) … by Eastern Elongation 31° 16′ 18.542″
by Western " 18.622
Mean 31 16 18.582

Angle between ________ and Referring Mark ________ − ________

Azimuth of ________ Do. ________ by Observation ________

* Note.—When the sum of the E. End readings is *greater* than that of the W. End and the R.M. is *east/west* of the star, this is *additive to/subtractive from* the angle, R.M. and star, and *vice versa* when the sum of the W. End readings is the greater.

Computed and Compared by ________________________ 18____.

PART IV.

Electro-Telegraphic Longitudes.

CHAPTER I. OBSERVATIONS FOR DIFFERENCE OF LONGITUDE.

Introductory.

1. All the known methods of determining absolute longitude by direct observation of celestial bodies are deficient in the accuracy that is indispensable for the requirements of Geodesy, and are therefore never resorted to in the Trigonometrical Branch for such a purpose. By the term "determining absolute longitude" is here meant the ascertaining of both local and Greenwich time by direct celestial observations at any one station. A knowledge of the *difference* of longitude of two selected stations within reasonable distance of each other is however far easier of attainment, and supplies equally well the data required for geodetical purposes.

2. Even this process was troublesome prior to the invention of the Electric Telegraph; but in the present day by its aid results are obtained of an order of accuracy far beyond what had been previously thought possible. It must be borne in mind that difference of longitude is merely the difference of local times (either solar or sidereal) at any given instant; if therefore an observer at one station can by any means ascertain the time at another place, which corresponds to a given time at his own station, he is at once in possession of the difference of longitude.

Early methods of determining differences of longitude.

3. Before the invention of the Electric Telegraph it was customary to make instantaneous signals by the explosion of rockets, or of small quantities of gunpowder or by other similar devices, so arranged that they could be seen from both the stations whose difference of longitude was to be determined. The observers at both stations noted their own local times of the occurrence of the signal, and hence a comparison of the local times, and consequently a value of the difference of longitude was obtained. It was assumed that the transmission of light was instantaneous, and that the signal was seen precisely at the same moment at both stations, a

perfectly legitimate assumption when the enormous velocity of light is considered.

Introduction of the Electric Telegraph.

4. This method and that of transporting chronometers were for a long time the most reliable; but they have been entirely superseded by the telegraphic method, wherever a telegraph wire is available for the purpose, as the latter furnishes a far more accurate and convenient mode of comparing the time at distant stations: suppose, for the sake of example, an observer at the eastern station, when his local time is exactly noon, sends an arbitrary signal through the wire to an observer at the western station, who, on receiving the signal, notes his local time to be $11^h\ 45^m$: it is obvious that, neglecting the time of transmission of the signal along the wire, the difference of longitude is 15 minutes. This is the principle that underlies the various methods of finding differences of longitude by the Electric Telegraph, but in practice considerable modifications of such a simple and obvious plan as this are necessary: the rationale however of all is the same, *viz.*, the power of communicating (almost) instantaneously with a distant station.

Instruments required.

5. In the first place each observer must be provided with a transit instrument and an astronomical clock for finding and keeping his local time with all possible accuracy. These, including the chronographs on which the transits are recorded by electricity, and the collimators, are described in great detail in the Introductory Chapters to Volumes IX and X of the "Account of the Operations of the Great Trigonometrical Survey of India" which should be in the possession of every one intending to take part in these operations.

Masonry pillars for the various instruments.

6. The mounting and adjustment of the transit instruments will be first described. Sufficient stability for these instruments can only be secured by mounting them on carefully isolated brick pillars. A brief description of these, as well as of the pillars required for the clock and the collimators, and of the portable observatories in which the observations are made, will be found in Sections 1 to 5 of Chapter II of Volume X. The plan and sections of the pillars with full details and dimensions are given in a plate facing this page for convenience of reference. The pillars must be carefully built according to the dimensions in the drawings, and attention must be given to the bond and quality of the mortar.

Clock Pillar

Fig: 1
Side Elevation

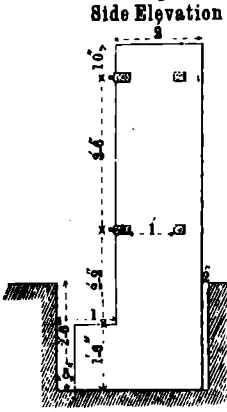
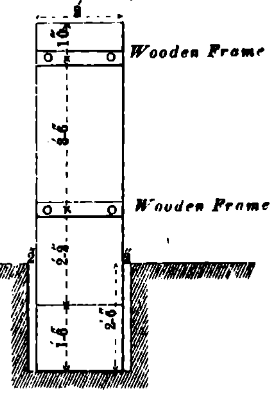
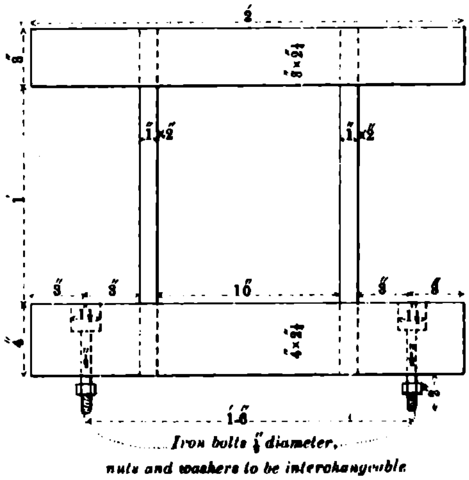


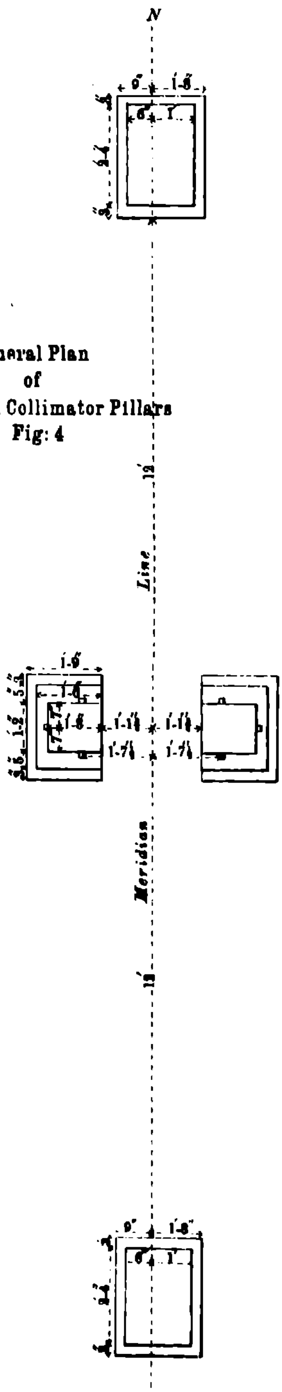
Fig: 2
Front Elevation

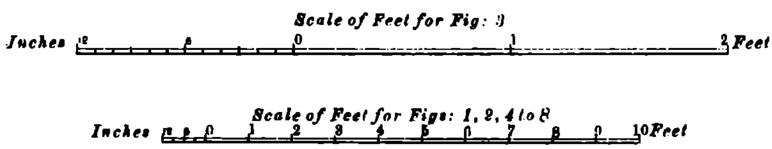


Enlarged Plan of wooden frames in Clock Pillar
Fig: 3



General Plan
of
Transit and Collimator Pillars
Fig: 4





Transit Pillar

Fig: 5
Front Elevation

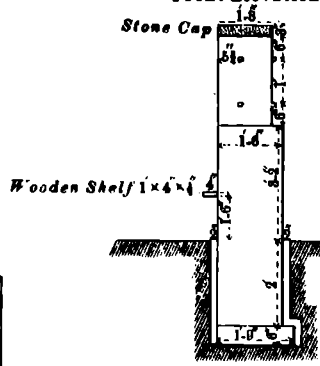
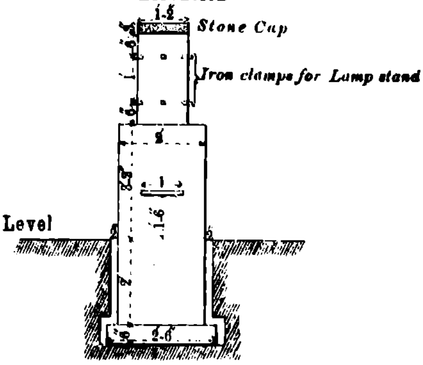


Fig: 6
Side Elevation



Collimator Pillars

Fig: 7
Side Elevation

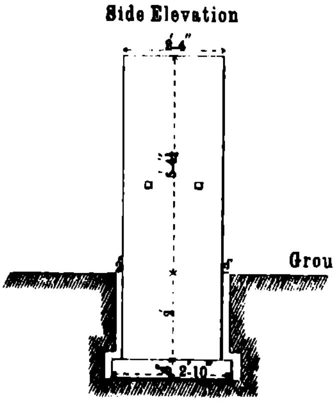
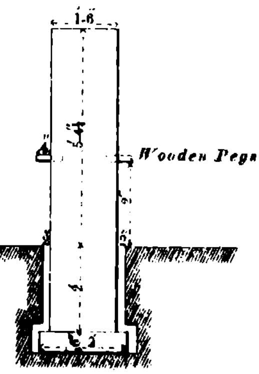


Fig: 8
Front Elevation



Continued.

7. The depth of foundations shown in the drawings will generally be sufficient, but more should be given if necessary. Great care should be taken that the inner faces of the transit piers are exactly parallel to the meridian line, laid down *vide* para. 10, and equidistant from it. The central lines of the collimator pillars should be 3 inches to the east side of the meridian line, for the better accommodation of the tripod stands on which the collimators are mounted.

Continued.

8. The insulating space round the piers should be about 2 inches in breadth; and in excavating for the foundation, the greater space required for the lowest courses may be obtained by undermining, if the soil will admit of it, care being taken to preserve the edges as sharp as possible. The iron clamps for the lamp stand are formed of flat iron about 1½″ broad and ½″ thick, built firmly into the transit piers. Each pair is meant to support an upright iron bar 1″·05 broad, and 0″·4 thick by allowing it to pass through holes of about that size cut in each clamp ½″ clear of the brickwork. There are six pairs in all, *viz.*, one pair on the northern and one on the southern face of each pier, one on the eastern side of the east pier and one on the western side of the west pier. The holes in each pair must be carefully plumbed to ensure the iron bar for the lamp stand being supported in a vertical position. The two northern as well as the two southern pairs should be 3′ 3″ apart from centre to centre. During the building the rough level must be occasionally applied, and the piers brought up evenly, so that when finished the stone caps may be level.

Continued.

9. When the piers are finished the ground between them may be excavated, and then the whole filled up with perfectly dry sand. The excavation is finally covered over with a board made to fit it, a round hole the size of the mercury trough being cut therein and closed with a moveable cap. The construction of these pillars is generally entrusted to a sub-surveyor who precedes the observers by a week or two, so that the pillars may have time to settle and harden before being brought into use.

Determination of meridian line.

10. Before commencing their construction the sub-surveyor will have to lay out a meridian line which must be true within a few minutes of arc. As the pillars are almost always built in the vicinity of a Telegraph Office, this process

is very simply performed, since nearly all the Telegraphic Offices of India are now supplied every day at 4 P.M. with the exact Madras time by electric signal. About the time the daily signal is expected from Madras, the observer must repair to the office and note the exact time by his chronometer when the signal is received. Hence he knows the error on Madras time, and consequently (as his longitude is very approximately known) the error on local time. Now by a well known computation he can ascertain the mean local time of the transit of Polaris (which is the best star to employ for the purpose), and hence, by applying his chronometer error, the chronometer time of its transit. Armed with this knowledge the observer is in a position, with the help of a theodolite, to ascertain the direction of the meridian very closely. Take the following example.

Example of method of finding approximate meridian.

11. An observer in longitude $5^h\ 13^m\ 46^s$ east of Greenwich and $7^m\ 13^s$ west of Madras notes that on November 1, 1889, his chronometer, when the Madras signal is received, stands at $3^h\ 28^m\ 4^s$: find the chronometer time when Polaris is on the observer's meridian.

	h	m	s
At 4^h Madras time the observer's true time in W. Longitude $7^m\ 13^s$ is	3	52	47
Chronometer time	3	28	4
Therefore his chronometer is slow		24	43
Sidereal time Gr. Mean noon Nov. 1, 1889 ...	14	43	51
Correction for longitude of observer		−	52
Sidereal time at local mean noon	14	42	59
Right ascension of Polaris	25	19	13
At Culmination, sid. interval after mean noon ...	10	36	14
Do. do. reduced to mean interval	10	34	30
Chronometer slow		24	43
Chronometer time of upper culmination	10	9	47 P.M.

If then the observer, at chronometer time $10^h\ 9^m\ 47^s$ (having previously levelled and adjusted his theodolite), sets the middle wire on Polaris he will have obtained the direction of the meridian with sufficient accuracy. The chronometer rate is here neglected, it is generally in practice sufficiently well known to allow of its application in determining the time of transit; the above example assumes that the observer is using a mean time chronometer. Suppose a sidereal chronometer to be employed, and the time shown by it on receiving the 4 o'clock signal from Madras to be $18^h\ 3^m\ 8^s$ then, the other data remaining the same, the computation of chronometer time would be as follows:—

	h	m	s
Sidereal time local mean noon (as above) ...	14	42	59
„ equivalent of $3^h\ 52^m\ 47^s$ mean time ...	3	53	25
Sidereal time of receiving signal	18	36	24
Chronometer time „ „	18	3	8
Chronometer slow		33	16
Right ascension of Polaris	1	19	13
Chronometer time of upper culmination ...	0	45	57

Comparison with other methods.

12. A still more accurate result could be obtained by observing Polaris or some other circumpolar star at elongation; but in most cases a sub-surveyor's knowledge of Astronomy would not be sufficient to enable him to make the necessary computations. If a sidereal chronometer be at hand and the sub-surveyor is able to obtain and regulate his time, Table LIII of the Auxiliary Tables might prove serviceable. But at all events the method described above has the advantage of extreme simplicity and is quite precise enough, as even at culmination the azimuth of Polaris alters only about 1 minute of arc in 3 minutes of time in Indian latitudes. The meridian line being once obtained, it should be marked by pegs at some distance north and south to enable the builders to refer to it when necessary during the construction of the pillars.

Details of pillars &c.

13. The building of the pillars should be taken up in the following order:—

1. Clock Pillar.
2. Transit Pillars.
3. Collimator „

The clock-room must not be less than 10 feet square, and conveniently close to the Telegraph Office if not in the building itself. The details of the clock pillar are given in Figures 1, 2 and 3.

Within 50 feet of the clock-room the station site should be selected, with an open view on the meridian, clear of buildings and trees to within an elevation of 8° to the north and 15° to the south.* The details of the transit and collimator pillars are given in Figures 4, 5, 6, 7 and 8.

No part of the pillars should be plastered except the upper surfaces of the collimator pillars: when they are completed the insulating spaces should be filled with dry sand. A straight line touching the upper surfaces of the two collimator pillars should be horizontal, and pass 2¼ inches above the level of the upper surface of the stone caps on the transit piers. This is important to ensure the mutual visibility of the two collimators through the hole in the cube of the transit telescope.

Connexion with Triangulation.

14. It hardly ever happens that the transit pillars occupy the exact site of a Trigonometrical Station, therefore it will be necessary to determine their geodetic position by executing a traverse or series of triangles to connect them with one or more of the Principal Stations of the Triangulation. This part of the work also generally falls to the share of an assistant or sub-surveyor. The point midway between the two pillars is taken as the exact site of the longitude station.

Setting up the clock.

15. It is advisable on arriving at a longitude station to set up and adjust the clock first, and to obtain an approximate rate by means of a chronometer of known rate which should be always carried by a longitude party. The error of the clock can be obtained very approximately from the Telegraph Office by the 4 o'clock signal from Madras. The chief precaution to be taken in fixing the clock is the insuring its verticality. This may be done either by a rough spirit-level on the frame, or preferably by a plumb-line applied to one of the vertical edges of the clock case. Evenness of beat, which is judged by the ear only, is obtained by a regulating screw on the arm which conveys the motion of the pendulum to the clock train.

Electrical connexion of clock.

16. The clock is so contrived that, by attaching to the two binding screws on the case, wires from the two poles of a galvanic battery, a continuous current passes through, broken at the commencement of each second by means of a small tilt hammer raised by one of the clock wheels. It is very necessary

* These figures apply to the greater part of India. A good rule for the purpose is that the meridian on the north side should be unobstructed to a zenith distance of 4° more than the co-latitude.

to see that the contacts of this hammer are perfectly clean, and that it is sufficiently raised at each second to break the current.

For raising or lowering the pendulum bob and thus increasing or decreasing the clock rate, a screw at the bottom is provided with a graduated head, each division corresponding to a change of rate of one second *per diem*.

Mounting and adjusting of the collimators and transit instrument.

17. The collimators and transit instrument may now be mounted and adjusted, and in doing this the small transit (alluded to in Volume X, Chapter I, at the beginning of Section 3) which is made to fit the same Ys as the large one will be found of great service in adjusting the Ys by hand, both in level and azimuth, sufficiently near to be within the range of the adjusting screws of the large telescope. A description of these instruments is given in such detail in the Introductory Chapters of Volume X that it is needless to repeat it here, and it will be only necessary to describe briefly the adjustments that must be performed on the transit telescope, before an exact determination of the local time can be obtained by its means. These are six in number :—

18. 1*st*—Adjustment of the eye-piece to obtain a clear view of the wires :—

Adjustment for clear vision of wires.

This is effected by merely pushing in or drawing out the eye-glass in its socket till the eye is satisfied. This adjustment varies for different eyes.

19. 2*nd*—Adjustment of the wires to the solar focus of the object glass :—

Adjustment to solar focus.

This is effected both in the collimators and in the transit telescope, by two antagonizing screws which move the eye tube in and out. When a star or any very distant object is well and clearly defined, the screws must be clamped once for all and not meddled with subsequently, as any change in this adjustment alters slightly the value of the wire intervals.

20. 3*rd*—Adjustment of the line of collimation :—

Collimation adjustment.

The line of collimation may be defined to be the line joining the centre of the wires, and the centre of the object-glass, and unless this line is placed so as to cut the transit axis at right angles, a *correction for collimation* to the recorded time of transit becomes necessary. The movement of the line of collimation is effected by turning the micrometer head which urges a slide carrying the wires to the right or left, and its position with regard to the transit axis is ascertained by reference to the collimators. The simplest way of realizing the principle of a collimator is to consider the cross in its focus to be an infinitely distant object. Now, if we look through the south collimator at the north one, and by means of the proper adjusting screw superpose the images of the two crosses, those two images are to all intents and purposes two infinitely distant signals at the extremities of a diameter of the celestial sphere. Let the centre wire of the large transit be now placed on the cross of the north collimator and then suppose the telescope to be revolved on the transit axis through 180°, it is obvious that if the sight line is perpendicular to the axis it will now accurately intersect the south cross; if not, half the difference is the error of collimation which may be corrected by the proper screws, or preferably may be computed from the micrometer readings of the collimators.

Continued.

21. This is the principle, but in practice the crosses of the north and south collimator are not exactly superimposed, but are separated by a small angular interval which is measured by the micrometer attached to the south collimator, and allowed for in the computation; the necessary intersections of the north and south crosses in the transit telescope are made by the micrometer, and the micrometer reading of the true line of collimation, when perpendicular to the transit axis, is deduced. The method of determining the actual value of the collimation constant c* will be explained in the next chapter. The reader may also consult with advantage Section 2 of Chapter V of Volume X. Note, that in fitting on the cap protecting the micrometer head, care should be taken not to disturb the reading to which the micrometer is set, usually called C_s, and that when removing the cap, this reading should be checked before moving the head.

Adjustment of level.

22. *4th*—Adjustment of the horizontality of the transit axis:—

In small instruments this is generally performed by a striding level the feet of which stand on the pivots of the axis; but in transit instruments of the size employed on the longitude work a more delicate method is resorted to, which depends

* c is the difference between the angle that the line joining the centre of the wires with the centre of the object-glass makes with the rotation axis, and 90°, and is measured in divisions of the transit telescope micrometer, one division = $0^{s}\cdot0224$ or $0''\cdot336$.

for its accuracy on the horizontality of the surface assumed by mercury when at rest: a particular eye-piece called after its inventor "Bohnenberger" is required which enables light to be thrown down the telescope tube from the wires towards the object-glass, while the observer's eye is at the instrument; by this device it is possible to see at the same time the wires themselves and their image reflected from a trough of mercury to which the telescope is directed. Now suppose the central wire to be truly collimated, and then, by means of the levelling screws the axis to be so adjusted that the central wire *coincides with its own reflection,* it is evident that the line of collimation is vertical, and being also perpendicular to the transit axis by the previous adjustment in collimation, the latter must be horizontal. The actual method in use, which consists, not in truly levelling the axis, but in measuring by the micrometer its inclination, is described in Section 4 of Chapter V of Volume X.

23. 5*th*—Adjustment of the setting circles:—

Adjustment of the setting circles.

These circles can be used to read either declinations or zenith distances; the latter are much the more convenient for a portable instrument, for if the former are adopted a new adjustment is necessary at every station, whereas in the latter case one adjustment once for all suffices. The best means of securing this is to set the circles to read 0° when the horizontal wires and their image in mercury coincide, and firmly clamp them in that position, making the final fine adjustment of the pointers by the small levels attached to them.

24. 6*th*—Approximate adjustment of the transit instrument to the meridian:—

Adjustment in the meridian.

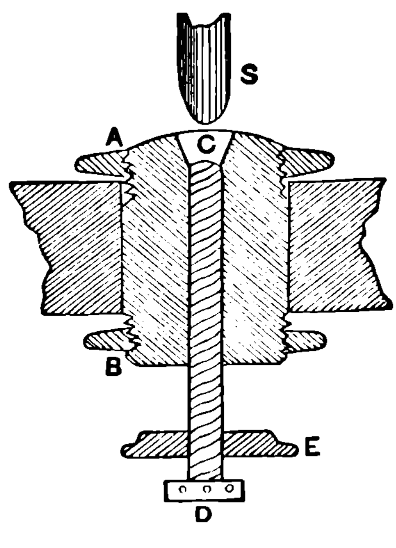
There are several well known methods of performing this adjustment, which are treated of in all text books of Astronomy. The most convenient procedure, and the one almost always adopted in the longitude parties, is by the transit of Polaris. Sidereal time having been obtained accurately, either by computation from the mean time supplied to the Telegraph Office, as explained in para. 11 of this Chapter, or by transit of a zenith star, it is only necessary to shift the whole transit telescope and axis, by means of the azimuthal screw provided for this purpose, so that when the sidereal time is equal to the right ascension of Polaris, that star may be seen on the central wire (collimated). It is immaterial whether the tribrach fitted with the azimuthal screw is placed on the east or west pillar, but it should be noted in the observatory memo. book which pillar is used. A rough adjustment by means of the small transit is supposed to have been first made, otherwise the run of the azimuthal screw may not be sufficiently long to admit of the adjustment being completed. The line of collimation will then describe approximately the meridian of the place. The transit instrument should have been previously levelled; but if it requires to be moved much in azimuth, or if the upper surfaces of the piers are not fairly level, it may be found necessary to repeat the operation.

Farther remarks on adjustments.

25. In all the three adjustments for collimation, level and azimuthal deviation, no attempt is made in practice to render them quite exact; it is far more convenient, and accordant with almost universal usage, to reduce the errors to minute quantities, and to apply corrections for them, *vide* Sections 1 to 5 of Chapter V of Volume X.

Adjustment of the Chronographs.

26. The Chronographs and their adjustments are fully described in Sections 4 and 5 of Chapter I of Volume X, and generally speaking, if all the teeth of the wheels are kept clean by occasional brushing with a stiff brush, and the adjustment of the fan is carefully made, these machines work satisfactorily. This latter adjustment is, however, a very delicate one, and in order to understand thoroughly how to make it, a knowledge of the mechanism of the bearing is necessary; the following diagram (not drawn to scale) sufficiently illustrates it.



Continued.

27. The cup C in which the spindle S works is to be screwed up by the screws A and B until only the slightest possible play of the spindle is left, and then the lightest touch on the central steel screw D completes the adjustment. The screw E serves to clamp the screw D. In nine cases out of ten if the chronograph is giving trouble the fault lies at this point.

Employment of the Telegraph.

28. It has been found by experience that the most advantageous way of utilizing the telegraph is by sending through it the clock beats at either station, the same beats being thus recorded on the chronographs at both stations. Suppose the eastern clock is first made to record its beats on both chronographs, then a certain selected list of stars is observed and recorded at the east station by the east clock; and after an interval of time equal to the difference of longitude of the two stations, the same stars culminate and are recorded at the western station, still *by the east clock.* The length of time occupied by the stars in passing from the meridian of the east station to that of the west is thus known, subject only to errors in the clock rate (not in the clock correction) and to the error caused by neglecting the time occupied by the signals in passing between the two stations.

Alternate employment of east and west clocks.

29. The errors and irregularities in the clock rate are necessarily very small, as the difference of longitude rarely exceeds 15 or 20 minutes of time, and a long programme of stars is selected, thus minimizing the error due to this cause: the length of time occupied by the signals between the two stations is cancelled very approximately by employing alternately the east clock, and west clock to transmit the beats; for it is obvious that as the current passes in opposite directions in the two cases, the resulting longitude is as much too great when the latter clock is used, as it is too small when the former one is employed. Thus, if Δ LE represent the difference of longitude by the east clock and Δ LW by the west clock, $\frac{1}{2}(\Delta LE + \Delta LW)$ will be the true difference of longitude, and $\frac{1}{2}(\Delta LW - \Delta LE)$ the retardation of the electric current in passing along the line and through the instruments.

Specimen of skeleton Programme.

30. The following programme (in skeleton) has been found the most convenient after many trials, and should not be departed from without sanction :—

I. Both observers adjust their telescopes in collimation and level; it is supposed that the azimuthal deviation has been previously reduced to a very small quantity.

II. The clock at the eastern station is made to graduate both chronographs, and the transits of eight stars (the same at both stations,) four to the north of the zenith and four to the south, are recorded by each observer on his own chronograph; the leading pen marking the clock seconds and the following pen the transits over the several wires, this arrangement of the pens being usually called $Q +$. *Vide* Section 5, Chapter III, Volume X.

III. When these eight stars have passed both stations, after an interval of about four minutes, another similar group of eight stars is recorded on both chronographs as before, the only difference being that the leading pen now records the transits over the wires and the following pen the clock beats, this arrangement being usually called $Q -$. There must now follow an interval somewhat greater than the difference of the longitudes of the two stations, at the end of which

IV. The west clock being made to graduate both chronographs, the transits of eight stars at the two stations are recorded $Q +$.

V. Another group of eight stars is observed $Q -$.

VI. Both observers remeasure their collimation and level corrections. This process may also be performed with advantage between III and IV if the adjustments seem at all unsteady, the interval being made a little longer for this purpose if necessary. The zenith distance of these stars should not exceed 25° except under urgent necessity. It will be found convenient if one, at least, out of the sixteen stars observed with each clock, be a Nautical Almanac star, as a very approximate clock correction can be found at once by means of it.

Pen equation.

31. Before commencing the observations of transits and again after completing them, the value of the pen equation should be recorded for at least two minutes on the chronograph drum. This is effected by the employment of certain pegs in connexion with the commutator board by which the direction of the electric current is regulated and which cause the observer's clock to actuate both pens; and thus the distance between them may be measured with the scale used in reading off the transits, which is described in the next chapter. For a description of the commutator board and the pegs appertaining to it, *vide* Section 6, Chapter III, Volume X.

32. At some time during this programme, data for computing accurately the azimuthal deviation of the transit instrument should be secured, either by observing the transits of two circumpolar stars (the nearer to the pole the better) at opposite culminations, or by transits of a high and low star: the former method being considered preferable. It is usual also for each observer to compare the two clocks at some time during the night's work. This may be done by putting in pegs 4, 6, 8, 10, 19, 22, 23, and subsequently 2, 10, 8, 18, 22, 23. In the first case the observer's clock will mark seconds with A pen, and the distant clock with B pen, and in the second the duties of the pens will be reversed.

Transits of circumpolar stars for deviation.

33. During the first night's work it is convenient to ascertain the minute of the distant clock corresponding to the minutes of the observer's clock; this is done as follows:—The observer calls his colleague and telegraphs the word "minute". The latter then sends his clock signals through at least one minute break, and cuts it off a few seconds after the break, and immediately telegraphs the number of that minute. The observer by noting his own clock time when the break of the distant clock occurs secures the corresponding minutes of the two clocks. Thus suppose an observer when receiving his colleague's clock, notes the minute break at $4^h\ 19^m\ 27^s$ by his own clock; and then is told by his colleague that the minute is 28, he will be in a position to fill up the minutes in the chronograph sheet by the simplest possible little computation. It is necessary for both observers to do this.

Means of ascertaining the minute of the distant clock.

34. It has been found that practised observers have no difficulty in securing transits of stars at intervals of 1 or 1¼ minutes, and in making the necessary adjustments between groups I and II and between IV and V in about 3 or 3½ minutes. Calculated on this supposition it will be seen that the duration of the programme will ordinarily not exceed 2½ hours, including the collimation and level adjustments; but inexperienced observers should certainly allow somewhat more time.

Length of Programme.

35. The wires in the transit telescopes are 25 in number (occasionally 35, but the outside groups are in this case never used) divided into 5 groups of 5 each, the equatorial intervals between the wires and between the groups being about

Concerning the wires of the transit telescopes.

$2\frac{1}{4}$ and $4\frac{1}{2}$ seconds of time respectively for an equatorial star. It has been found by experience that very little advantage is gained by noting transits over more than 11 wires. The central 11 are generally used, *viz.*, from H to R (or R to H according to the position of the illuminated pivot), the wires being called by the letters of the alphabet from A to Y. It is advantageous to use the same wires in both positions of the telescope, as any errors caused by incorrect wire intervals are thereby cancelled.

Duties of Observatory assistant or recorder.

36. A well trained observatory recorder should always be attached to a longitude party. He is generally made responsible, in conjunction with the tindal of the party, for the proper packing and loading of the instruments when travelling. He should know sufficient about the setting them up to save the executive officer a considerable amount of trouble. He must be able to bring up stars' places from the catalogues approximately to date of observing, and sufficiently well trained in computations to carry on the reductions of the transits under supervision. His duties in the observatory are chiefly confined to calling out the zenith distances of the stars (north or south as the case may be) to the observer and giving him warning as the star enters the field of the telescope. He has a chronometer before him set to sidereal time, to enable him to do this; it is found that 1 minute's warning is sufficient, as the star should be then well within the field and about 40 seconds from the first wire to be observed. He also records in a book kept for the purpose, the names of the wires observed, and any other details that the observer thinks worth entering for future guidance. It has been found quite practicable for a sub-surveyor to do the recording work, as well as the accounts and correspondence of the party without being overworked.

Thorough knowledge of commutator board essential.

37. Before attempting the measurement of an arc of longitude the observers must be thoroughly acquainted with the details of their commutator board, and know almost instinctively, or at all events quite by rote, the pegs to put in to produce certain effects, *vide* Section 6, Chapter III, Volume X. To have to think out the proper situation of the pegs during a programme, or even to have to refer to written rules, is objectionable, and may possibly lead to stars being missed, or at all events to hurry and nervousness which are fatal to good observing. The adjustment of the pens and relays must be thoroughly mastered, for if anything goes wrong with the chronographs during the passage of a group of stars, the time for

remedying it is embraced within a few seconds, otherwise stars are missed and the record rendered incomplete. The feeling that this is the case, and that the other observer is noting the transits of stars which will all have to be rejected for want of the corresponding observations at his own station, is most embarrassing to a young observer, and frequently leads him to make a casual turn of any screw that happens to be nearest to him in the hope that the fault may be thus remedied; finding this ineffective he becomes bewildered and nervous, and probably loses the whole group.

General causes of failure.

38. A few of the most usual sources of failure are here given.

(*a*) Clock pen ceases to record seconds:—

If the observer's own clock is in use, the pen may cease recording either owing to its clinging to the armature, or to its failure to be attracted by it. In the first case, the spring is not strong enough to push the armature away from the electro-magnet when the current ceases, or else the clock mechanism fails to break the circuit. In the second, either the spring is too strong and the attraction of the electro-magnet to the armature is not sufficient to overcome its tension, or else the clock circuit is broken. Failures of this kind are almost always due to a derangement of the armatures and springs and not to the clock circuit; the former should therefore be first examined. If the other station clock is in use, failures of seconds may be caused in the same way, but it is in this case more often the receiving relay *F* (*vide* Plate III, Volume X) which is in fault. The adjustment of this relay, when the current from the distant station is very weak, becomes a somewhat delicate matter, and as the current constantly varies in strength the adjustment needs altering from time to time; it should be set so as to move with a very weak current, but if set too finely it is apt not to fly back at each beat: the correct setting can only be gained by experience. This relay acts directly upon the pen batteries, and the length of the signals depends upon the length of time during which the tongue of the relay *F* is jammed up against its stops.

(*b*) Observer's pen ceases to act:—

This is probably due to derangements of the armature and springs as above, but it may also be caused either by the tappet failing to break the circuit, or by the circuit being permanently interrupted owing to a broken or defective wire or dirty connection somewhere: contact with other wires is a fruitful source of mishaps in this case.

(*c*) Distant signals cease altogether:—

This is generally irremediable and may be caused either by the distant observer having omitted to pull out peg 23, or by a break in the line, or some other station having cut in. There is no remedy but to wait till the signals recommence.

Cautions.

39. Always be careful when sending clock signals to withdraw peg 23. It is obvious that as long as it remains in, the clock signals, instead of passing out to the line wire, will find their way to the nearest earth through the receiving relay *F*, the observer meanwhile being quite unaware that his colleague is receiving no signals at all. Note that the tongues of relays when no current is passing should be jammed against the dummy stop. All connections must be kept very clean, and accidental contacts carefully avoided. After the night's work is over the line wire should be detached from the signalling key, and secured from contact with any part of the apparatus, otherwise there may be a hindrance to the ordinary telegraphic communication the next day. The line battery, and also the pen and sounder batteries should be "short circuited," by putting the zinc and copper poles in metallic communication with each other, if they are working feebly, otherwise there is no necessity for this precaution.

Knowledge of signalling necessary.

40. It will be found most desirable that observers should have a fair acquaintance with the Morse Alphabet, (a copy of which will be found at page 85 of the Appendix to Auxiliary Tables) and be able to send messages at the rate of 6 or 7 words per minute. It has been usual to obtain the services of a Telegraphic signaller to make the necessary arrangements for joining up the wire at the beginning of a night's work, so as to be continuous between the two longitude stations, *i.e.* without the intervention of any relay, and this plan will be found the most convenient, as it is hardly likely that a Survey Officer will be sufficiently skilful at signalling or well enough up in the routine of the Telegraph Department to accomplish this for himself. But it is at the close of a night's work

that a certain amount of skill in sending and receiving messages becomes important, as it enables the two officers concerned to communicate the results of the night's work, and to make any arrangements for the next evening without having recourse to "urgent" messages the next day, which have to be paid for and are somewhat costly. The following simple code has been found useful for communicating the results of the work and is recommended for adoption. The observer at the eastern station, having called the attention of the western observer, sends the following signal —— —— —— —— to signify that he has observed all the clock stars on the Programme, and having received the signal – – 'understood,' sends —— —— to signify that the azimuth stars have also been observed, at least to such an extent that the azimuthal deviation for that particular night can be obtained. If a group is incomplete he sends a – for every unobserved star and a —— for each one observed. Thus —— – – —— —— —— – – signifies that the 1st, 4th, 5th and 6th in that group have been observed and the others missed. If the azimuth stars are lost it is signalled thus – – – – – –.

It should be remembered that stars observed at one station only are absolutely useless, and therefore if one observer knows that the other for some reason is unable to observe a certain group it is quite useless for him to do so.

41. With such a programme as that sketched out in para. 30, six nights' observations, if about two-thirds of the stars are secured on an average each night, are sufficient at one station, provided there is no great disparity between the numbers of stars N and S, or between the numbers of observations with E or W clock respectively. The routine for pivot changes is generally as follows:—

Number of nights' observations, and pivot changes necessary.

	Eastern Observer.	Western Observer.
1st night	*I. P. E.*	*I. P. E.*
2nd "	*I. P. W.*	*I. P. E.*
3rd "	*I. P. W.*	*I. P. W.*
4th "	*I. P. E.*	*I. P. W.*
5th "	repeat one of the above combinations.	
6th "	" another " "	

selecting for repetition on the last two nights those combinations which for any reason appear to have been least satisfactory.

This succession of pivots has been arrived at as the most suitable, after many experiments. The transit instruments have given rise to much discussion on this subject, as it has been found that change of pivots gives rise to greater discrepancies in arcs of longitude than was thought probable, or can at present be accounted for. This matter is treated of in considerable detail in the Appendix to Volume X, to which the reader is referred, with a recommendation that the outlines of the subject should be mastered by any officer undertaking Electro-Telegraphic longitude work.

Personal equation.

42. It is advisable that the relative personal equation of the observers should be determined at least three times during the season, *viz.*, once before the measurement of the first arc, again in the middle of the season, and lastly after the completion of the last arc. It is not intended that these times should be in all cases rigidly adhered to if found inconvenient, but they are so obviously the best that an effort should be made towards this end. The method of measuring the equation is fully detailed in Chapter V, Volume IX and in Chapter IV, Volume X. It is best to distribute the measures over two nights rather than do the whole together, as the comparison of the two will furnish some sort of test of trustworthiness. The transits of 40 stars, *i.e.*, 20 N and 20 S of the zenith can be conveniently observed each night. The Idiometer, described at page 35 of Volume IX, designed by Lieut-Colonel (now Major-General) Maxwell Campbell, R.E., for the purpose of measuring the *absolute* personal equation of each observer, though an exceedingly ingenious instrument, was found from some unknown causes not to agree very satisfactorily in the results it gave with those derived from divided transits, and its use has therefore been abandoned for some years past.

Star Catalogues.

43. A few words are necessary as to the Star Catalogues to be employed on this work. Bearing in mind the fact that the determination of the difference of time at the two stations reduces itself in practice to the length of time occupied by a heavenly body in passing from the meridian of the eastern station to that of the western, it is obvious that great accuracy in the knowledge of star places is quite unnecessary; it suffices if their right ascensions are known within 20 or 30 seconds of time and their declinations within 3 or 4 minutes of arc. Hence the use of the B.A.C. Catalogue, one of the most extended in existence though somewhat inaccurate, is legitimate, and it may be employed with great advantage.

Besides the Nautical Almanac and the Connaissance des Temps, the Greenwich Catalogues of 1864, 1868 and 1872, and the Washington Catalogue should also be at hand, as they may occasionally be required when more correct places are necessary. Table LIV of the 3rd Edition of the Auxiliary Tables of the Survey of India contains a very useful list of circumpolar stars for determining azimuthal deviation which may be used with safety, so long as both observers employ the same stars: the quantities e, f, g, h, l will however have to be computed as they are not given in the table. Their places should always be brought up to the day of observation rigorously. For stars within 3° of the pole, observations of transits on three or five wires are sufficient. It will be found convenient if both stars of a pair are observed with same clock, and one pair with the eastern and one with the western clock.

N. Collimator may serve for meridian mark on emergency.

44. If by any mischance the azimuth stars are missed, the azimuthal deviation can generally be deduced for one or even two nights with sufficient accuracy from the reading of the north collimator, its position with regard to the meridian being determined from the previous night's observations; as explained in para. 12 of the next chapter. This collimator is by its construction exceedingly stable, and its cross rarely moves more than 4 or 5 divisions of the transit micrometer from night to night and frequently much less: this movement corresponds to about 1·7 seconds of arc which is immaterial as the clock stars are always selected within a few degrees of the zenith, where the correction due to such a small error in the deviation correction is practically rejectaneous.

Other methods of deducing deviation.

45. It is not always necessary that a pair of circumpolar stars at opposite culminations should be observed. In low latitudes this method may be found impracticable, as lower culminations would then occur at inconveniently low altitudes. It is then better to employ a north circumpolar star at upper culmination and a star as far south as possible. Azimuthal deviation in this case, as in the former one, has the effect of accelerating the transit of one star and retarding that of the other, thereby giving a large divisor in the formula for determining it, which is the object aimed at for securing a trustworthy value. In this system also it is most desirable that both observers should use the same stars, unless they are so well known that no uncertainty attaches to their catalogued places.

CHAPTER II. COMPUTATION OF DIFFERENCE OF LONGITUDE.

General arrangement of computations.

1. It will be convenient to divide the various operations, by which the final arc of longitude is obtained from the raw material furnished by the chronographic records, into the following thirteen heads, *viz.* :—

(1) Writing up the headings of the sheets, and marking them off.

(2) Reading off the star-signals, and entering the times in the transit sheets.

(3) Reduction to centre wire, and constants for collimation, &c.

(4) Abstract of level, and collimation corrections.

(5) Determination of the value of pen equation.

(6) Reduction of circumpolar star-places to date of observation.

(7) Computation of star-constants, A.B.C. for circumpolar stars.

(8) „ of correction for azimuthal deviation.

(9) „ of corrected times of transit of longitude stars, and clock-rates.

(10) Deduction of clock-rate corrections.

(11) Abstract of personal equation, and deduction of its final value.

(12) „ of transits with each clock, and deduction of $\Delta L \pm \rho$.

(13) Final deduction of ΔL and ρ.

Each of these will now be explained in detail. It is not indispensable to observe this order of succession in the computations, but it will be found in the main convenient to conform to it as far as possible.

Writing up headings, &c.

2. The first process—writing up the headings of the sheets, and marking them off—is usually undertaken on the day following the night of observation, while all the details are still fresh in the observer's mind. On each sheet should be noted in large clear round hand the following particulars :—

(*a*) The name of the arc, *i.e.*, of the two stations at its extremities, the eastern one being always put first.

(*b*) The station of observation.

(*c*) The name of the observer.

(*d*) The telescope used.

(*e*) Its position as regards the illuminated pivot.

(*f*) The clock used.

(*g*) The chronograph used.

(*h*) The date.

(*i*) The number of the sheet: each arc should commence with new numbering.

To "mark off" the sheet, the name of each star should be written on the record, near the place where the warning signal is found. The characteristic letters of the first and last wires observed are then inserted, and again the name of the star after the last wire. The clock minutes, and here and there the clock hour, must also be entered, as they will be found indispensable when the sheet has to be read off. The hour and minute of the other observer's clock must be obtained on the first night by the process explained in the preceding chapter: an approximate knowledge of the rate will enable the observer to fill in these details for subsequent nights. At the foot of the sheet is given an extract from the collimation and level form for the night, showing the values of b and c to be used in the computation. The sheet being then signed by the observer, is complete as a record. The reader should however be here cautioned that a somewhat different procedure with regard to the collimation and level corrections has been recently approved which is explained in para. 8 of this chapter; and it will be unnecessary in future to enter more than the values of C_0, C_s and M at the foot of the chronograph sheets.

Preservation of chronograph sheets.

3. It should be remembered that these sheets are in effect the field books of the party, and must be preserved with the greatest care. After they have been subjected to the next process—reading off—they should be pasted into a file book similar to those used for filing letters, but of sufficient size to render the folding of the sheet unnecessary. At the end of this chapter will be found a specimen of a chronograph sheet from actual practice on the arc Mangalore–Bombay in 1888. It has been selected casually from the records of the astronomical parties, and contains a few omissions and imperfections such as are almost invariably met with in an ordinary night's work.

Reading off the sheets.

4. The next step is the reading off the sheets, and entering the clock times of star-transits over the wires in the form provided. The reading off is performed by means of a scale of peculiar construction. It is engraved on some transparent material, talc or glass, its length being equal to the distance between two consecutive marks representing seconds, made automatically on the chronographic sheet by the clock. As the lengths of the seconds are liable to small irregularities, the division lines of the scale are made closer at one end than at the other, so that by sliding the scale up and down a position may be gained, in which its length accurately coincides with that of a second on the sheet. The scale is divided into tenths, and half tenths are read by estimation. At the end of this chapter will be found a set of forms adapted to the whole system of longitude computation. That in which the times of transits read off, as described above, are entered, is distinguished as No. 1. To give the complete computation of one whole arc would have unduly extended these forms: they are merely intended to furnish the reader with specimens, such as have been found in practice to be convenient for the purpose. They illustrate, with the exception of the last, (No. 10), the arc Mangalore–Bombay. In Form No. 10 the arc Nagarkoil–Mangalore has been selected for illustration in consequence of its being a more complete measurement than Mangalore–Bombay, and consequently affording a better type of the method employed in deducing the final value. It is quite unnecessary (except in the case of circumpolar stars observed for azimuth, whose treatment is somewhat different throughout) to record the minutes and seconds of every wire observed; it suffices if this is done for the first and last wires only; for the intermediate ones only the decimals of the second to the nearest half tenth should be entered in the form. The hour and minute of transit over the first and centre wires are entered at the top of the form, the latter being repeated below when the mean of all the wires is deduced. It is very desirable that all the sheets appertaining to one arc *at both stations* should be read off by one and the same computer, otherwise a personality may be introduced, which would have to be determined experimentally, and applied as a correction. When two computers are employed on reading off the sheets, it will be found more expeditious that one should read and the other record the readings, than that each should work independently. It will be observed that two vertical columns are appropriated to the reduction of each star; the first having been filled up as described above, the use of the second has now to be explained.

Reduction to centre wire.

5. The time of transit over each wire must be reduced to the centre one by means of a correction (additive for wires before the middle one and subtractive for those after it) which, for any particular star and wire, is the wire interval from the centre in equatorial seconds multiplied by the secant of the star's declination. This computation, like many others in longitude work, involves a vast quantity of arithmetic, but can be considerably abridged by the use of a judiciously constructed table. Several methods have been tried at different times, but the one now in use, has, with the help of a table given after Form 1 at the end of this chapter, proved to be the most expeditious, and is easily understood by native computers. For its employment Crelle's Tables of multiplication and division are necessary; and here it may be as well to state once for all, that these Tables are now used throughout the longitude calculations, to the almost entire exclusion of logarithms. Except in the case of close circumpolar stars for which special methods are required, the declination secants, which are taken out to four decimal places, lie between the limits of 1·0 and 1·4. The table is therefore prepared with six vertical columns, the first contains the distinguishing letter appropriated to each wire, the second is headed 1·0, and contains the value of the wire intervals multiplied by 1·0, the third is headed 1·1 and contains the same intervals multiplied by that quantity, and so on up to 1·4. The last horizontal line in the table containing the values of *m* is explained below. Crelle's Tables are subsequently employed in multiplying the last three figures of the secant by the first three of the wire interval, the product being added to the proper number in the reduction table. Thus suppose the equatorial interval of wire *B* to be 29·07 and the nat. secant of the star observed to be 1·2417, the computation would stand thus

By the reduction table	29·07 × 1·2	= 34·88
By Crelle's Tables	*29·1 × ·0417	= 1·21
	Total reduction for wire *B*	= 36·09

The wire intervals shown in the table were those appertaining to Telescope No. 2 during the season 1887-88: a new table would of course be necessary, if these intervals are found to vary from time to time, as in all probability they will, or if another telescope be employed. Any

* The omission of the second decimal place in the value of the wire interval when multiplying by the last three figures of the secant, can in no case affect the second place of decimals in the total reduction to centre for any wire.

moderately good computer soon learns to perform this calculation mentally without writing down a single figure, and it is thus very quickly got through. The reduction to the centre for *all* the wires is shown in the specimen form, but in practice only those should be computed which are required : these are generally the middle eleven wires. From Form 2, in which the results are entered, the figures in the second of the two columns appropriated to each star in Form 1 are now obtained by the addition or subtraction of the reduction corrections, and their mean taken out at the bottom. The last three horizontal lines of Form 2 are filled in with the constants for collimation, level, and deviation respectively for each star. It will be noticed that the collimation constant C $= m \sec \delta$, and it is in order to facilitate the computation of this quantity that m has been inserted in the last line of the Reduction Table, as it can be multiplied by sec δ in an exactly similar manner to that employed in the case of the wire intervals.

Discrepant observations, and their treatment.

6. Discrepancies from the mean exceeding ·3 of a second are not frequent; when they occur the chronograph sheet should be examined to find out if any misreading has passed unnoticed. A doubt may arise as to whether a discrepancy is sufficiently large to render the observation worthless, and to justify its rejection *in toto*. A criterion has been worked out and incorporated in the following little table as a guide to the observer. It is founded on the same principles as the well known criterion by Professor Peirce, and its construction is explained in the General Report of the Operations of the Survey of India Department for the year 1885-86. Appendix, page lxvi. It has no pretensions to theoretical accuracy but suffices for the object in view. Thus if eleven wires have been observed and the star's declination is 25°, an observation differing $0^s \cdot 24$ from the mean should be rejected.

Criterion for rejecting doubtful Observations.

Declination	0° to 10°	10° to 20°	20° to 30°	30° to 40°	40° to 43°
No. of wires Observed					
5	·11	·13	·15	·16	·16
6	·13	·15	·17	·18	·19
7	·14	·17	·19	·20	·21
8	·15	·18	·20	·22	·22
9	·16	·20	·22	·24	·24
10	·17	·21	·23	·25	·25
11	·18	·22	·24	·26	·27

Procedure when the chronograph rate is irregular.

7. It not unfrequently happens that the chronograph becomes irregular in its rate, and, owing to increased friction or other causes, goes so slowly that the ordinary scale cannot be applied to the chronographic record. In this case the true distance between the recording pens must be taken off with a pair of compasses, and applied (either backwards or forwards according to which pen is leading) to the star-signals on the record, and the pricks read off by actual measurement from the preceding and succeeding second-signals; in which case the space left in Form 1, for "Correction for *Q*" will be left vacant, or filled in with o. The true distance between the recording pens can always be obtained from the run of pen equation at the beginning and end of each night's work, when both pens are actuated by the observer's clock. Form 1 cannot at this stage be further proceeded with, but will be referred to again.

Determination of collimation and level constants.

8. The improved method of deducing the collimation and level corrections alluded to in para. 2, requires some little explanation. The previous practice had been that the value of c, as determined by observations to the collimators each night, should be used for the correction of the transits of that night, separate values being employed *I.P.E.* and *I.P.W.* Occasionally, however, considerable discrepancies occurred between these values, and an examination into the matter showed that it would be far more satisfactory theoretically to take a mean C_0 for a whole arc, rather than attempt to get nightly values: it certainly has proved more satisfactory practically also, as the test by circuits, when the arcs are computed with a mean C_0, unmistakeably shows. The procedure now adopted will be understood from the Abstract of Collimation and Level, Form 3. C_0 is to be determined twice (or more) each night, preferably once at the beginning of the work, and again towards its close, and entered in the proper column of the form. C_s the micrometer head reading during the observation of transits, is also inserted for each night, and also M which is the reading of the micrometer head, when the real wires and their image reflected from mercury coincide. Up to this point no change has been made in the process which is described in Sections 2, 3 and 4, Chapter V, Volume X. The next step is to take out the mean of all the values of C_0 in position *I.P.E.* and also of those in position *I.P.W.*; the mean of these two quantities gives the final value of C_0, the difference between which and C_s gives the

values of c_1 for each night; and so long as C_s remains the same, so long will c_1 and c (which is equal to c_1 less the diurnal aberration) remain constant. Note that when the telescope is *I.P.E.* $c_1 = C_0 - C_s$ and for *I.P.W.* $c_1 = C_s - C_0$. By a similar process the values of b are found: each determination being kept separate, and used for the groups to which it appertains: it is usual to determine b twice in the night's work, at the same time as C_0.

Measurement of Pen equation.

9. The next stage of computation is the determination of the value of the Pen Equation. This is measured directly from the chronograph sheets by the reading scale, and if the pens have remained unaltered during the night's work, a mean is taken for the whole. It is usual to make 20 measurements on the run of pen equation at the beginning, and 20 at the end, distributing them as evenly as possible along the whole length. There is no special form used for this process.

Computation of azimuthal deviation.

10. We now come to the three forms 4, 5 and 6 employed in computing the azimuthal deviation of the sight line of the telescope by means of the circumpolar star observations. Form 4 is used for bringing up to date the places of the circumpolar stars observed for deviation. It is desirable that in computing these only trustworthy catalogues should be employed. For remarks on the catalogues to be employed the reader is referred to para. 43 of the preceding chapter. When the star whose place is required is found in two or three of these catalogues, it is customary to deduce its place on the first of January of the year of observation from each of them, and take the mean as the adopted value. Its place on the day of observation is brought up from the 1st of January by means of Airy's Day Numbers or by the Independent Quantities.* The places at Greenwich mean midnight are considered good enough, as any error incurred by neglecting a few hours one way or the other, is probably smaller than the inaccuracy of their observed places, especially as small errors in the computed places can have no appreciable effect on the final results, so long as the same stars are used by both observers.

* The Form is adapted to Airy's Day Numbers, but very little change will be required for the Independent Quantities. These latter are called in the Nautical Almanac "Quantities for correcting the Places of Stars."

11. It will generally suffice if the place is computed for the first and last night's observation on each arc, the intermediate ones being determined by interpolation: occasionally, however, if the measurement of the arc is much protracted by bad weather, or other causes, additional intermediate values should be computed as a guide to the interpolation; but this must be left to the computer's discretion, as no general rule can be given. In Form 5 are computed (for circumpolar stars) the star constants, $C = m \sec \delta$ for collimation, $B = m \sec \delta \cos \zeta$ for level, and $A = m \sec \delta \sin \zeta$ for azimuth. These are the same constants as are computed at the bottom of Form 2 for clock stars; but as circumpolar stars require more accuracy in their treatment, Form 5 must be used for them. For computing these the latitude of the station, and the declination of the stars observed, should be known to the nearest second. The form, with the help of the precepts regarding signs, &c., at the foot of it, is sufficiently simple to render any further explanation unnecessary: the number of decimals retained should be regulated according to the specimen given.

Continued.

12. We must now return to Form 1, and taking up first the circumpolar stars only, the corrections Cc, Bb, and Q, must be filled in with their proper signs, and added algebraically to the mean of the wires. The times thus obtained are copied into Form 6, and the deviation constant a is deduced according to the rules given in detail at pages 24, 25, 26 of Volume X. It is usual to observe two pairs of circumpolar stars each night, the first of which appertains to the first two groups, and the second to the last two groups. Unless there has been some disturbance of the telescope during the progress of the work, these two values of a should not differ by more than 10 divisions. If a high and low star are used for determining the deviation instead of a pair of circumpolars the method of computation is identically the same. It has been mentioned in para. 44 of the preceding chapter that in the event of the azimuth stars being lost on any night, the position of the telescope with regard to the meridian can be deduced from that of the North Collimator, by assuming its stability for the time elapsed. The principle of thus obtaining the deviation is as follows:—

Continued.

Let C_0 be the micrometer reading when the sight-line is perpendicular to the rotation axis.

Let D be the micrometer reading when the sight-line intersects the North Collimator.

Also let a and k represent respectively the deviations of the telescope and north collimator from true north, measured in divisions of the micrometer (positive to the west of north). These quantities are connected by the following formula:—

$$k = a \pm C_0 \mp D$$

the upper signs being for position *I.P.W.* and the lower for *I.P.E.* Hence it is obvious that k may be computed if a is known or a may be computed if k is known (C_0 and D are always obtainable from the collimation records). If therefore a cannot be obtained on any particular night owing to the azimuth stars being lost, it is obvious that a knowledge of k from the observations of a previous (or subsequent) night, will enable us to supply the omission by the solution of the equation

$$a = k \mp C_0 \pm D.$$

On several arcs nightly values of k have been taken out, and have rarely been found to differ 5 divisions on consecutive nights; whereas a is subject to considerably larger fluctuations, especially after change of pivots.

Completion of Form 1.

13. We can now again return to Form 1 and with the known values c, b, and a, form the corrections Cc, Bb, Aa as well as Q, which must all be carefully entered with the proper signs.* The quantities $C = m \sec \delta$, $B = C \cos \zeta$ and $A = C \sin \zeta$ are obtained from the last three lines of the table for reduction to centre wire. These four corrections with their proper signs being now added algebraically to the mean of the wires, the sum is entered in the line "corrected"; and the last two lines are also at the same time (after the first day) completed. Having proceeded thus far, the observer will have the satisfaction of noticing, from the agreement of his clock-rate correction as deduced from different stars, what confidence can be placed in his work.

Deduction of clock-rate corrections.

14. The next step is to deduce the actual corrections for clock-rate for each night to be applied to the transit observations by means of Form 7. The filling up of this form seems to be too obvious to need explanation; its object

* The signs of c and b are determined in Form 3 and of a in Form 6. The signs of C, B and A are given in the foot-note to Form 5.

is merely to compute, by means of the known clock-rate correction for 24 hours, (or multiples of 24 hours if any nights have been missed) the proportional correction for the time elapsed during the passage of a star from the meridian of the eastern to that of the western observer: if the computer is in any doubt an explanation will be found on page 31 of Volume X.

Computation of personal equation.

15. Personal equation next claims the attention of the computer. The stars observed in *divided transits* for this purpose must be entered and computed in Form 1 exactly like the longitude stars as far as the line "mean"; beyond this it is unnecessary to carry the calculation. Each star will of course require two pairs of columns, one for each observer. The results are then abstracted into Form 8. As the relative personal equation of the observers is generally measured three times during the field season, the selection of the exact value to be used for each arc must be partly left to their judgment. In the case of the arc Mangalore–Bombay, as the equation was measured within a few days of completion of the arc the value then obtained was used on that arc; a mean of the two days, April 12 and April 13, on which it was determined, being taken. Throughout the whole discussion of personal equation, stars of north and south aspect must be kept distinct, and two equations recognized, applicable to stars of N. and S. aspect respectively.

Approximate value of ΔL.

16. We are now in a position to fill up Form 9 and deduce the approximate values of the arc; they are still necessarily only approximate, for those values obtained by transits with the eastern clock are too small by the quantity ρ, (the amount of retardation of the electric current through the line wire and relays) those with the western clock being too great by the same amount. In filling up this form the most recent practice, found by experience to be the most convenient, is to enter the stars in groups of north and south aspect. The aim in selecting a programme, is to obtain four groups of longitude stars, in each of which are found 4 stars of north and 4 of south aspect; these being separated in Form 9 give rise to 8 groups, alternately north and south. The mean of the results by each group is first taken out in column 15, and then the corrections for clock-rate and personal equation are entered in columns 16 and 17; the algebraical addition of the three columns then affords the final value of $\Delta L \pm \rho$ for that particular group.

Final value of ΔL.

17. With the means of these results we now enter Form 10, and the question arises, how is the best final mean to be obtained from them. If no stars in the programme were missed, the number of north stars equal to the number of south, the number of observations with the eastern clock equal to the number with the western clock, and the pivot changes quite symmetrical, it seems obvious that the proper course would be simply to take the arithmetical mean of the whole as the best result; but when these conditions do not obtain, it becomes a matter of some difficulty to decide on the best method of proceeding. Various systems have been tried at different times, but the latest and most approved is as follows. The upper part of Form 10 gives the values of $\Delta L \pm \rho$ for each day, keeping separate the results by north and south stars with each clock, the pivot position of each telescope being entered for each day in the first column of the form. In the next columns are shown the results by each clock, north and south stars being lumped together; in the next are given the results by north and south stars separately, the mean of two clocks being taken. The next column shows the final result for each day, being the mean of columns 7 and 8 or 9 and 10. The last column ρ is half the difference between columns 7 and 8. In the latter part of the form these results are further digested. It is obvious that only four combinations of pivot positions are possible, *viz*:—

Telescope No. 1	Telescope No. 2
I.P.E.	*I.P.E.*
I.P.W.	*I.P.E.*
I.P.W.	*I.P.W.*
I.P.E.	*I.P.W.*

These four are entered in the first column. The next ten are similar to those in the upper part of the form, differing only inasmuch as they contain (or may contain) the means of the results of several days, on which the pivot positions have been the same. The mean of the figures in the last column is the finally accepted value of the arc.

Method of measuring wire intervals.

18. Wire intervals should always be determined at the beginning of the season; as, in case of an accident occurring to them, the intervals being unknown, all previous work of that season would be more or less uncertain and unsatisfactory: it is as well that a second determination be made

towards the close, to ascertain whether any noticeable change has occurred. Experience shows that such changes are generally very minute; still it is more satisfactory to re-determine the intervals, in case the sudden jars and shocks, to which these instruments are constantly exposed in transit, should have altered them. They are most easily determined by noting the length of time occupied by a close polar—and therefore slow moving—star in passing over them; the intervals being all reckoned from the central wire. The equatorial interval is obtained simply by multiplying the time occupied in the transit, by the cosine of the star's declination; except in the case of very close circumpolar stars, when a small correction is necessary owing to the star not passing in a straight line from wire to wire, but in a slightly curved circular path. The interval obtained in the simple way must then be decreased, in the proportion that the sine of the angle described by the star in its diurnal course bears to the length of the corresponding arc. This correction is very small except in the case of stars within 2° of the pole, and even then it may be considered rejectaneous for intervals of less than 10 seconds: if required, it may be easily computed with the aid of the following table which is taken with slight modifications from page 148, Volume II of Chauvenet's Practical and Spherical Astronomy, second edition:—

log t	log k	log t	log k
1·778	10·00000	2·982	9·99965
2·079	9·99999	3·008	9·99960
2.255	9·99999	3·033	9·99955
2·380	9·99998	3·056	9·99950
2·477	9·99997	3·079	9·99945
2·556	9·99995	3·100	2·99939
2·623	9·99993	3·120	9·99933
2·681	9·99991	3·139	9·99927
2·732	9·99989	3·158	9·99920
2·778	9·99986	3·175	9·99914
2·819	9·99983	3·192	9·99907
2·857	9·99980	3·209	9·99899
2·892	9·99977	3·224	9·99892
2·924	9·99973	3·239	9·99884
2·954	9·99969	3·254	9·99876

The first column contains the logarithm of the number of seconds, t, occupied by a star in transiting the wire interval, and the second contains the value of $\log \frac{\sin \theta}{\text{arc } \theta}$, θ being the diurnal arc described by the star during that time. The following example will sufficiently explain its use. Suppose a star whose declination is 88° 45′ occupies 1623 seconds in passing from any selected wire to the central one, required the true equatorial interval.

Log	1623	=	3·21032	
Log cos	88° 45′	=	8·33875	
Log k from the Table		=	9·99899	
Log sum		=	1·54806	. . . $35^{s}\cdot32$.

At least eight or ten different stars not exceeding 6° N. P. Distance, should be observed for wire intervals; the effect however of erroneous values, should such exist, may be completely cancelled in observing longitude stars, by the simple precaution of using the same wires in both positions of the telescope, *viz.*, *I.P.E.* and *I.P.W.*

Number of decimals to be retained.

19. The number of decimal places to be retained in the various stages of the computation merits careful attention. In reading off the chronograph sheets record to the nearest half tenth, the reduction to centre wire is to be recorded to nearest hundredth. There is of course no object to be gained theoretically by computing the latter of these quantities more rigorously than the former, it is simply a matter of convenience as the method of calculating the reduction explained in para. 4 is certainly easier to work in its present form, than if it were imperative to enter the last figure as 0 or 5. The mean of the wires is to be entered to the nearest hundredth, A, B, C are to be computed to four places of decimals, a, b, c to one place, and Q to two places. In the Abstract of Transits, Form 9, Stars' declinations are to be entered to the nearest minute. Times of transit and corrections thereto are to be entered to two places of decimals.

Continued.

20. In taking the means of the groups in Form 9, and thenceforward to the final results, including clock-rate and personal equation, the third place of decimals is to be retained. Micrometer heads are always to be read by estimation to tenths of the division engraved on the head, and it will suffice if the means of two or more readings are retained to the same order of accuracy, *viz.*, tenths of a division. In the case of close circumpolar stars, whose transits are used for determining Azimuthal deviation, it

is sufficient if the times of transit over each wire, and the reduction to centre wire are given to the nearest tenth of a second. In taking the mean of the wires the hundredths should be retained.

Programmes.

21. The following programmes are given as specimens. It will be found convenient to paste the programme on a sheet of millboard about the size of foolscap paper, and hang it up where it can be conveniently referred to, in the clock-room. The first programme is for the eastern and the second for the western station.

ARC MANGALORE-BOMBAY, STATION MANGALORE		$\lambda = 12° 52'$ $\Delta L = 8^m\ 7^s$
Local Sidereal Times		Chronograph pegs to be used
11-25	λ Draconis and 1070 Gr. 72 for Azimuth	4, 6, 8, 10, 15, 25,
11-36	Call Western station	23, 24
11-38 to 11-51	1st Group, Q +, with East Clock	1, 8, 13, 10, 15, 25
12-4 to 12-17	2nd Group, Q −, " "	1, 10, 12, 8, 17, 20, 25
12-26	Cut off Clock and receive West Clock	
12-36 to 12-55	3rd Group, Q +, with West Clock	8, 18, 22, 23, 10, 15, 25
13-1 to 13-17	4th Group, Q −, " "	10, 19, 22, 23, 8, 17, 20, 25
13-27	Call Western station and send results	23, 24

ARC MANGALORE-BOMBAY, STATION BOMBAY.		$\lambda = 18° 54'$ $\Delta L = 8^m\ 7^s$
Local Sidereal Time		Chronograph pegs to be used
11-25	λ Draconis and 1070 Gr. 72 for Azimuth	4, 6, 8, 10, 15, 25
11-28	Receive call from Eastern station	23, 24
11-38 to 11-51	1st Group, Q +, with East Clock	8, 18, 22, 23, 10, 15, 25
12-4 to 12-17	2nd Group, Q −, " "	10, 19, 22, 23, 8, 17, 20, 25
12-18	East Clock stops send West Clock at once	1, 9
12-36 to 12-55	3rd Group, Q +, with West Clock	1, 8, 13, 10, 15, 25
13-1 to 13-17	4th Group, Q −, " "	1, 10, 12, 8, 17, 20, 25
13-19	Receive results from Eastern station and send ditto.	23, 24

Star lists.

22. For the above programme the stars were those given in the subjoined list, which is made out in the form in which it has been found convenient for the observatory recorder to have before him. The numbers refer to the British

Association Catalogue unless followed by the abbreviation Gr. 72 which signifies the Greenwich Catalogue of 1872.

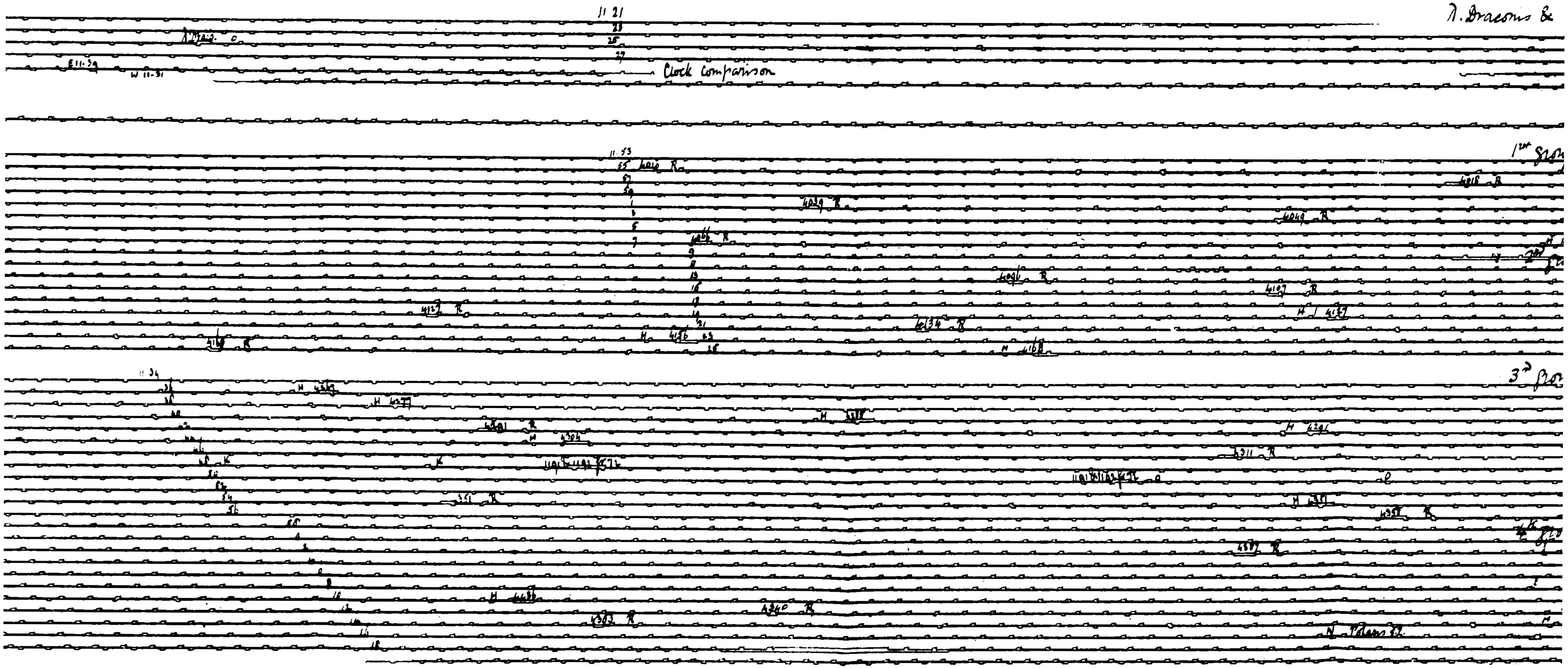
	Name of Star.	Mag.	Approximate R. A.			N. P. D.		Zenith Distance.*		North or South
			h	m	s	°	′	°	′	
1st Group	λ Draconis	3-4	11	24	47	20	3	51	3	N
	1070 Gr. 72	4		27	29	121	14	50	8	S
	4010	6-7		46	33	51	29	19	37	N
	4018	7		48	0	48	28	22	38	N
	4030	7		49	44	94	31	23	25	S
	4039	7		52	31	85	54	14	48	S
	4049	5½		54	14	85	43	14	37	S
	4056	6		56	1	67	17	3	49	N
	4066	6		58	33	67	55	3	11	N
	4072	4		59	31	80	39	9	33	S
2nd Group	4096	6	12	4	22	83	34	12	28	S
	4107	6		6	12	63	30	7	36	N
	4114	6		7	46	79	7	8	1	S
	4127	5		10	42	65	26	5	40	N
	4134	7		12	26	93	20	22	14	S
	4141	6		13	38	66	21	4	45	N
	4156	5		15	2	71	35	0	29	S
	4168	6		16	52	84	4	12	58	S
3rd Group	4267	6		35	55	78	57	7	51	S
	4277	6		37	53	90	57	19	51	S
	4285	6		39	43	50	7	20	59	N
	4291	6		41	19	83	26	12	20	S
	4304	6		43	49	61	50	9	16	N
	4311	6		44	53	51	52	19	14	N
	1191-92 Gr. 72	5-6		48	8	5	58	65	8	N
	4351	4½		53	24	71	59	0	53	S
	4358	6½		54	51	92	46	21	40	S
4th Group	4387	5	13	0	53	68	15	2	51	N
	4394	5½		2	41	98	23	27	17	S
	4406	4½		4	36	71	53	0	47	S
	4421	4-5		6	40	61	33	9	33	N
	4431	7		8	17	87	57	16	51	S
	4436	7		9	50	86	21	15	15	S
	4440	6		11	14	80	0	8	54	S
	4453	6		13	17	55	19	15	47	N
	Polaris S.P.	1½		17	1	1	17	72	23	N

* This form was made out for the observer at Bombay, the zenith distances therefore refer to that station.

General remarks on programmes.

23. This programme has been selected at random from the records, and is one that was carried out in the season 1887-88 by Colonels Strahan and Heaviside. The intervals between the stars and between the groups will be found too small for inexperienced hands, but quite practicable for experienced observers. The facsimile of the Chronograph record given at the end of the chapter is that of one night's work on this particular programme, and will be found of great assistance to the novice as a standard of reference on the proper method of writing up the record, &c.

Arc Mangalore Bombay - Station Bombay - Observer Strahan - Telescope No. 2 . I. P. E. C



		C_0	C_s	M	c_1	c	L	
Cr	11.10	2499.8	2500.0	2498.0	−0.2	−1.1	+1.4	for first two groups
	12 25	2499.0		2499.2	−1.0	−1.9	+0.2	for last " "
	Mean	2499.4						

Mean −1.5 for all groups

Form No. 1. 8th April 1888, Observer Colonel G. Strahan, Transit Telescope No. 2, I.P.E. East Clock, Station Bombay, Arc Mangalore–Bombay.

Star ...	4267		4277		4285	
Declination and Aspect*	+ 11° 3′	S	− 0° 57′	S	+ 39° 53′	N
Observed Times of Wires and Reduction to Central Wire	h m 12 35	h m 12 36	h m 12 37	h m 12 38	h m 12 39	h m 12 39
	H 55·50	9·19	H 52·90	6·34	H 37·60	55·11
	·83	·23	·13	·32	·55	·14
	·20	·27	·35	·25	·45	·05
	·70	·19	·90	·31	·35	·09
	·00	·25	·20	·41	·25	·13
	·27	·27	·43	·43	·10	·10
	·57	·24	·73	·44	·15	·16
	·83	·19	·97	·42	·10	·17
	·55	·33	·40	·35	·90	·10
	·75	·29	·63	·38	·93	·26
	R 23·07	9·33	R 19·90	6·42	R 12·80	55·24
Mean	h m 12 36	s 9·25	h m 12 38	s 6·37	h m 12 39	s 55·14
Corrections: C c (c = −2·9)	...	− ·06	...	− ·06	...	− ·08
B b (b = −0·6)	...	− ·01	...	− ·01	...	− ·02
A *a* (*a* = +8·3)	...	+ ·03	...	+ ·06	...	− ·09
Q (Q = +1·61)	...	+ 1·61	...	+ 1·61	...	+ 1·61
Corrected Mean	12 36	10·82	12 38	7·97	12 39	56·56
Do. of Day Preceding	...	6·20	...	3·44	...	52·03
Apparent Clock Rate Correction	...	− 4·62	...	− 4·53	...	− 4·53

* For explanation of the term "aspect" consult Volume X, Chapter IV, Section I.

Transcribed by ____________ Computed and Compared by ____________

Table for Reduction to Centre Wire, Telescope No. 2. 1887-88.

Wire	Equatorial Intervals = I	I × 1·1	I × 1·2	I × 1·3	I × 1·4
A	31·36	34·50	37·63	40·77	43·90
B	29·07	31·98	34·88	37·79	40·70
C	26·88	29·57	32·26	34·94	37·63
D	24·50	26·95	29·40	31·85	34·30
E	22·34	24·57	26·81	29·04	31·28
F	17·89	19·68	21·48	23·26	25·05
G	15·63	17·19	18·76	20·32	21·88
H	13·44	14·78	16·13	17·47	18·81
I	11·19	12·31	13·43	14·55	15·67
J	8·90	9·79	10·68	11·57	12·46
K	4·41	4·85	5·29	5·73	6·17
L	2·21	2·43	2·65	2·87	3·09
M	...	...	...	...	...
N	2·29	2·52	2·75	2·98	3·21
O	4·55	5·01	5·46	5·92	6·37
P	9·05	9·96	10·86	11·77	12·67
Q	11·25	12·38	13·50	14·63	15·75
R	13·48	14·83	16·18	17·52	18·87
S	15·74	17·31	18·89	20·46	22·04
T	18·01	19·81	21·61	23·41	25·21
U	22·47	24·72	26·96	29·21	31·46
V	24·68	27 15	29·62	32·08	34·55
W	26·94	29·63	32·33	35·02	37·72
X	29·17	32·09	35·00	37·92	40·84
Y	31·43	34·57	37·72	40·86	44·00
*m**	0·0224	0·0246	0·0269	0·0291	0·0314

* *m* = value of Telescope Micrometer.

Form No. 2. ***Reduction to Centre Wire, and Constants for Collimation, Level, and Azimuth,*** C.B.A., ***for Telescope No. 2, at Bombay, Lat.*** $= 18° 54'$.

Star ...	4267	4277	4285			
Declination $= \delta$ ...	$+11° 3'$	$-0° 57'$	$+39° 53'$			
Zenith Distance $= \zeta$* ...	7 51	19 51	20 59			
Nat. sec δ ...	1·0189	1·0001	1·3032			
Nat. cos ζ ...	·9906	·9406	·9337			
Nat. sin ζ ...	·1366	·3396	·3581			
A ...	31·95	31·36	40·87			
B ...	29·62	29·07	37·88			
C ...	27·39	26·88	35·03			
D ...	24·96	24·50	31·93			
E ...	22·76	22·34	29·11			
F ...	18·23	17·89	23·32			
G ...	15·92	15·63	20·37			
H ...	13·69	13·44	17·51			
I ...	11·40	11·19	14·59			
J ...	9·07	8·90	11·60			
K ...	4·49	4·41	5·74			
L ...	2·25	2·21	2·88			
M ...	...	...	...			
N ...	2·33	2·29	2·99			
O ...	4·64	4·55	5·93			
P ...	9·22	9·05	11·80			
Q ...	11·46	11·25	14·67			
R ...	13·74	13·48	17·56			
S ...	16·04	15·74	20·51			
T ...	18·35	18·01	23·47			
U ...	22·89	22·47	29·28			
V ...	25·15	24·68	32·16			
W ...	27·45	26·94	35·11			
X ...	29·72	29·17	38·01			
Y ...	32·02	31·43	40·96			
$m \sec \delta = C$ ...	0·0228	0·0224	0·0292			
$C \cos \zeta = B$ ...	0·0226	0·0211	0·0273			
$C \sin \zeta = A$ ...	0·0031	0·0076	0·0105			

* If declination (δ) is North and greater than (λ) the Lat. $\zeta = \delta - \lambda$.
" " less " $\zeta = \lambda - \delta$.
" South $\zeta = \lambda + \delta$.
In the first case the star is north of the zenith, and south in the last two.

Form No. 3.

Abstract of Determination of Collimation and Level Correction Constants.

Astronomical Date	Station and Telescope	Instrumental Position	Collimation				Level		Mean C_0	Station and Telescope	Instrumental Position	Collimation				Level		Mean C_0
			C_0	C_s	c_1	c	M	b				C_0	C_s	c_1	c	M	b	
			d	*d*	*d*	*d*	*d*	*d*				*d*	*d*	*d*	*d*	*d*	*d*	
1888 April 4	MANGALORE (Telescope No. 1)	*I. P. W.*	1742·7	1745·0	+3·2	+2·3	1742·2		*I. P. W.* = 1742·2 *I. P. E.* = 1741·3 General Mean = 1741·8	BOMBAY (Telescope No. 2)	*I. P. W.*	2497·8				2495·2		*I. P. E.* = 2499·4 *I. P. W.* = 2496·6 General Mean = 2498·0
								−0·2					2498·0	0·0	−0·9		−3·4	
							1741·0					2496·8				2493·9		
„ 7		*I. P. W.*	1741·3				1742·6				*I. P. W.*	2495·9				2500·8		
				1745·0	+3·2	+2·3		+0·9					2497·0	−1·0	−1·9		+2·7	
			1742·1				1742·8					2495·9				2500·5		
„ 8		*I. P. E.*	1741·4				1739·2				*I. P. E.*	2499·8				2498·0		
				1740·0	+1·8	+0·9		+2·6					2500·0	−2·0	−2·9		−0·6	
			1741·1				1739·2					2499·0				2499·2		

Name of Star ... ... 1191, Gr. 72.
Date for which required ... April 8, 1888.
Epoch of Catalogue ... ... Jan. 1, 1872.
y = Number of years elapsed to Jan. 1 of year of observation = 16.

	R.A.		N.P.D.	
		s		"
Secular variation		+ 0·221		− 0·02
Do. × y		+ 3·536		− 0·32
Do. × y ÷ 200 ...		+ 0·018		0·00
Annual precession		+ 0·371		+ 19·62
Proper motion		− 0·006		− 0·02
Sum		+ 0·383		+ 19·60
Sum × y		+ 6·128		+ 5 13·6
Place at epoch of Catalogue ...		12^h 48^m 4·787		5° 53′ 10·1
Reduced place January 1, 1888 ...		12 48 10·915		5 58 23·7
e	9·75118		9·97695	
E	0·86503		0·86503	
	0·61621	4·132	0·84198	6·9
f	0·02716		0·33705	
F	1·25887		1·25887	
	1·28603	19·321	1·59592	39·4
g	1·40432		1·64948	
G	0·06985		0·06985	
	1·47417	29·797	1·71933	52·4
h	9·75376		9·99639	
H	1·47667		1·47667	
	1·23043	16·999	1·47306	29·7
l		124·665		53·6
L		114·104		114·1
Sum ...		+ 309·018		+ 296·1
Constant ...		− 300·000		− 300·0
Proper motion for fraction of year ...		− 0·002		0
Reduction to day of observation ...		+ 9·016		− 3·9
Reduced place		12^h 48^m 19·931		5° 58′ 19·8

Form No. 5. *Computation of Star-Constants* A. B. C. *for reduction of transits of Circumpolar Stars.*

Name of Star ...	1191 Gr. 72		Polaris (sub-polo)	
Date ...	April 8, 1888		April 8, 1888	
	E. Mangalore	W. Bombay	E. Mangalore	W. Bombay
	° ′ ″	° ′ ″	° ′ ″	° ′ ″
Lat = λ ...	12 52 14	18 53 49	12 52 14	18 53 49
Declination = δ or δ'*	84 1 40	84 1 40	91 17 22	91 17 22
$Z = \lambda - \delta$ or $\lambda - \delta'$*	−71 9 26	−65 7 51	−78 25 8	−72 23 33
Constant log† ...	$\bar{2}\cdot35025$	$\bar{2}\cdot35025$	$\bar{2}\cdot35025$	$\bar{2}\cdot35025$
log sec δ or δ'* ...	$0\cdot98277$	$0\cdot98277$	$1\cdot64776$ *n*	$1\cdot64776$ *n*
Sum = log C (1) ...	$\bar{1}\cdot33302$	$\bar{1}\cdot33302$	$\bar{1}\cdot99801$ *n*	$\bar{1}\cdot99801$ *n*
log sin Z (2) ...	$\bar{1}\cdot97608$ *n*	$\bar{1}\cdot95774$ *n*	$\bar{1}\cdot99107$ *n*	$\bar{1}\cdot97916$ *n*
log cos Z (3) ...	$\bar{1}\cdot50917$	$\bar{1}\cdot62382$	$\bar{1}\cdot30267$	$\bar{1}\cdot48072$
log A = (1) + (2) ...	$\bar{1}\cdot30910$ *n*	$\bar{1}\cdot29076$ *n*	$\bar{1}\cdot98908$	$\bar{1}\cdot97717$
log B = (1) + (3) ...	$\bar{2}\cdot84219$	$\bar{2}\cdot95684$	$\bar{1}\cdot30068$ *n*	$\bar{1}\cdot47873$ *n*
A ...	−0·2038	−0·1953	+0·9752	+0·9488
B ...	+0·0695	+0·0905	−0·1998	−0·3011
C ...	+0·2153	+0·2153	−0·9954	−0·9954

*$\delta' = (180 - \delta)$ is to be used instead of δ for stars below the Pole.

† This is the logarithm of *m*, the value of the micrometer screw of the Telescope taken at $0^{r}\cdot0224$. The letter *n* after a logarithm signifies that it is the logarithm of a negative quantity.

N.B.—Attend to signs throughout, South Declination is −. As a check on the proper application of the signs, note that for stars between

North horizon and pole,	A is + , B − , C − .
Pole and zenith,	A is − , B + , C + .
Zenith and south horizon,	A is + , B + , C + .

Form No. 6. *Deduction of Deviation Correction, a, from Star Observations.*

Arc	Station	Astro-nomical date	Instrumental Position	Clock in use	Star's Name	Culmination	No. of Wires Observed	Deviation Constant A	Observed Time of Transit	Correction for Collimation	Correction for Level	Correction for Pen Equation Q	Correction for Clock Rate	Seconds of Corrected Time of Transit	Right Ascension (increased by 12 hours for lower Culminations)	Apparent Clock Correction	Deduced Value of Deviation Correction a
									h m s	s	s	s	s	s	h m s	s	d
Mangalore–Bombay	Bombay	April 4	*I. P. W.*	W	λ Draconis	U	8	−0·0508	11 24 43·72	−0·06	−0·15	+1·67		45·18	11 24 47·08	+ 1·90	+38·1
		,,	,,	,,	1070 Gr. 72	,,	11	+0·0201	11 27 24·93	−0·03	−0·06	+1·67		26·51	11 27 31·11	+ 4·60	
		,,	,,	,,	1191 ,,	,,	5	−0·1957	12 48 21·61	−0·19	−0·31	+1·67		22·78	12 48 20·00	− 2·78	
		,,	,,	,,	1192 ,,	,,	5	−0·1955	12 48 29·63	−0·19	−0·31	+1·67		30·80	12 48 27·68	− 3·12	
		,,	,,	,,	Polaris	L	2	+0·9488	13 16 22·81	+0·90	+1·02	−1·67	−0·09	22·97	13 16 59·67	+36·70	+34·7
		April 7	*I. P. W.*	W	λ Draconis	U	8	−0·0508	11 24 56·78	−0·07	+0·12	+1·66		58·49	11 24 47·02	−11·47	+44·6
		,,	,,	,,	1070 Gr. 72	,,	11	+0·0201	11 27 37·73	−0·03	+0·05	+1·66		39·41	11 27 31·10	− 8·31	
		,,	,,	,,	1191 ,,	,,	5	−0·1957	12 48 35·50	−0·21	+0·25	+1·66		37·20	12 48 19·95	−17·25	
		,,	,,	,,	1192 ,,	,,	5	−0·1955	12 48 42·52	−0·21	+0·25	+1·66		44·22	12 48 27·62	−16·60	
		,,	,,	,,	Polaris	L	2	+0·9488	13 16 32·91	+1·00	−0·83	−1·66	−0·09	31·33	13 16 59·31	+27·98	+39·2
		April 8	*I. P. E.*	W	λ Draconis	U	8	−0·0508	11 24 59·96	−0·20	−0·03	+1·61		61·34	11 24 46·99	−14·35	
		,,	,,	,,	1070 Gr. 72	,,	11	+0·0201	11 27 43·03	−0·08	−0·01	+1·61		44·55	11 27 31·09	−13·46	+12·6
		,,	,,	,,	1191 ,,	,,	5	−0·1957	12 48 34·54	−0·62	−0·05	+1·61		35·48	12 48 19·93	−15·55	
		,,	,,	,,	1192 ,,	,,	5	−0·1955	12 48 42·49	−0·62	−0·05	+1·61		43·43	12 48 27·60	−15·83	
		,,	,,	,,	Polaris	L	2	+0·9488	13 17 9·20	+2·89	+0·18	−1·61	−0·09	10·57	13 16 59·26	−11·31	+ 3·9

Form No. 7.

Deduction of Clock Rate Corrections.

α = Correction for intervals between nights of observations.

β = Hourly corrections for nights of observations, interpolated by means of α.

Arc	Approximate Difference of Longitude	Intervals between nights of observations	α at E Station for		α at W Station for		Astronomical Dates of Observations	β for		Correction to Observed Difference of Times of Transit for	
			E Clock	W Clock	E Clock	W Clock		E Clock	W Clock	E Clock	W Clock
			s	s	s	s		s	s	s	s
Mangalore-Bombay	$8^h\ 8^m\ 7^s = 0^h.135$	1888 April 4 to 7	+ 1·29	−13·03	+ 1·21	−12·99	1888 April 4	+ 0·017	− 0·181	+ 0·002	− 0·024
		„ 7 to 8	+ 0·61	− 4·45	+ 0·44	− 4·66	„ 7	+ 0·020	− 0·185	+ 0·003	− 0·025
							„ 8	+ 0·022	− 0·190	+ 0·003	− 0·026

Form No. 8. ***Abstract of Observed Values of Personal Equation between Colonels Strahan and Heaviside.***

Observed with Telescope No. 2								
By Stars of	At Bombay Lat. = 18° 54′							
	April 12, 1888				April 13, 1888			
	Star	Declination °	′	Equation S − H s	Star	Declination °	′	Equation S − H s
North Aspect	3710	28	34	− 0·02	3710	28	34	− 0·02
	3735	26	5	− 0·01	3735	26	5	− 0·05
	3751	26	6	− 0·01	3751	26	6	− 0·09
	3776	20	48	− 0·08	3776	20	59	− 0·06
	3797	26	9	− 0·04	3797	26	9	− 0·03
	3809	25	16	− 0·05	3809	25	16	− 0·02
	3842	23	42	− 0·08	3851	32	10	− 0·04
	3851	32	10	− 0·03	3915	19	2	− 0·08
	3915	19	2	− 0·13	3937	28	24	− 0·04
	3937	28	24	− 0·08	3952	44	15	+ 0·04
	3952	44	15	− 0·03	3964	21	59	− 0·05
	3964	21	59	− 0·05	3990	20	51	− 0·03
	3990	20	51	− 0·01	3998	35	33	− 0·01
	3998	35	33	− 0·08	4010	38	31	− 0·02
	4010	38	31	− 0·02	4018	41	45	+ 0·05
	Mean $S_N - H_N$ = − 0·048				Mean $S_N - H_N$ = − 0·030			
South Aspect	3720	4	11	− 0·04	3696	7	8	− 0·03
	3761	12	18	− 0·04	3720	4	11	+ 0·02
	3785	4	14	− 0·01	3761	12	18	+ 0·02
	3824	15	0	− 0·08	3785	4	14	− 0·03
	3832	0	32	− 0·07	3824	15	0	− 0·07
	3862	6	39	− 0·07	3832	0	32	− 0·03
	3873	0	45	0·00	3862	6	39	0·00
	3886	17	4	− 0·13	3873	0	45	+ 0·04
	3900	3	28	0·00	3886	17	4	− 0·11
	3932	17	25	− 0·05	3900	3	28	+ 0·04
	3971	5	22	− 0·01	3932	17	25	− 0·05
	3975	− 6	3	− 0·04	3971	5	22	+ 0·04
	4030	− 4	31	− 0·05	3975	− 6	3	− 0·02
	4039	4	6	+ 0·03	3979	9	5	+ 0·03
	4049	4	17	− 0·01	4030	− 4	31	− 0·01
	4063	− 4	51	+ 0·03	4039	4	6	− 0·01
	4077	− 2	30	− 0·03	4049	4	17	− 0·09
	Mean $S_S - H_S$ = − 0·034				Mean $S_S - H_S$ = − 0·015			

Form No. 9. ***Observations of Transits with E Clock, and Deduction of the Apparent Difference of Longitudes, ΔL − ρ.****

MANGALORE (E) *Lat.* 12° 52′ 14″·14, *Long.* 74^h 53^m $9^s·89$: AND BOMBAY (W) *Lat.* 18° 53′ 49″·49, *Long.* 72^h 51^m $16^s·27$.

Astronomical Date	Star		Transits Observed at E — *By Heaviside, with Telescope No. 1*					Transits Observed at W — *By Strahan, with Telescope No. 2*					Difference of Corrected Times (W − E)		Correction for Rate of E Clock	Cor. for Per. Equations $H_N - S_N = +0^s·039$, $H_S - S_S = +0·025$	ΔL − ρ
	B.A.C. Number	Declination	Star's Aspect	Instrumental Position and Correction Constants	Mean Observed Time	Total Correction	Seconds of Corrected Time	Star's Aspect	Instrumental Position and Correction Constants	Mean Observed Time	Total Correction	Seconds of Corrected Time	By each Star	Mean of Group			
1888		° ′			h m s	s	s			h m s	s	s	m s				
Apr. 8	4018	+41 32	N	*I. P. E.*	11 48 3·42	+1·90	5·32	N	*I. P. E.*	11 56 11·06	+1·42	12·48	8 7·16	m s 8 7·160	+0·003	+0·039	8 7·202
	4056	+22 43	N	*d*	56 1·95	+1·84	3·79	N	*d*	12 4 9·50	+1·49	10·99	7·20				
	4066	+22 5	N	c + 0·9 b + 2·6 *a* − 6·0	58 34·40	+1·83	36·23	N	c − 2·9 b − 0·6 *a* + 8·3	6 41·85	+1·50	43·35	7·12				
„ 8	4049	+ 4 17	S	s	11 54 14·66	+1·77	16·43	S	s	12 2 22·05	+1·58	23·63	8 7·20	m s 8 7·185	+0·003	+0·025	8 7·213
	4072	+ 9 21	S	*Q* +1·71	59 32·21	+1·80	34·01	S	*Q* +1·61	7 39·61	+1·57	41·18	7·17				
„ 8	4107	+26 30	N	−1·71	12 6 15·53	−1·57	13·96	N	−1·61	12 14 22·94	−1·74	21·20	8 7·24	m s 8 7·257	+0·003	+0·039	8 7·299
	4127	+24 34	N		10 45·84	−1·58	44·26	N		18 53·25	−1·73	51·52	7·26				
	4156	+18 25	N		15 8·83	−1·60	7·23	N		23 16·21	−1·71	14·50	7·27				
„ 8	4096	+ 6 26	S		12 4 26·30	−1·65	24·65	S		12 12 33·59	−1·65	31·94	8 7·29	m s 8 7·287	+0·003	+0·025	8 7·315
	4134	− 3 20	S		12 29·91	−1·67	28·24	S		20 37·16	−1·62	35·54	7·30				
	4168	+ 5 56	S		16 55·74	−1·63	54·11	S		25 3·03	−1·65	1·38	7·27				

NOTE.—Transcribing Equation *nil*, all records having been transcribed by the same person.

* ρ is the retardation of an electric signal between the stations.

Form No. 10. ARC NAGARKOIL–MANGALORE. *Deduction of ΔL the difference of Longitude and ρ the retardation of Signals.*

(Telescope No. 2) at NAGARKOIL. (Telescope No. 1) at MANGALORE.

Telescope and Pivot Position	Astronomical Date	E. Clock		W. Clock		All Stars		Both Clocks		Both Clocks	ρ
		N. Stars	S. Stars	N. Stars	S. Stars	E. Clock	W. Clock	N. Stars	S. Stars	All Stars	
	1888	m s	m s	m s	m s	m s	m s	m s	m s	m s	s
Tel. No. 1, *I. P. E.* " 2, *I. P. E.*	Feb. 18	10 21·121	10 21·093	10 21·195	10 21·187	10 21·107	10 21·191	10 21·158	10 21·140	10 21·149	0·042
Tel. No. 1, *I. P. E.* " 2, *I. P. W.*	Feb. 19	10 21·026	10 21·028	10 21·094	10 21·101	10 21·027	10 21·097	10 21·060	10 21·065	10 21·062	0·035
Tel. No. 1, *I. P. W.* " 2, *I. P. W.*	Feb. 20	...	...	10 21·189	10 21·205	...	10 21·197	...	...	10 21·197	...
Tel. No. 1, *I. P. W.* " 2, *I. P. E.*	Feb. 21	...	...	10 21·252	10 21·252	...	10 21·252	...	...	10 21·252	...
Tel. No. 1, *I. P. W.* " 2, *I. P. E.*	Feb. 22	10 21·136	10 21·186	10 21·256	10 21·259	10 21·161	10 21·257	10 21·196	10 21·223	10 21·209	0·048
Tel. No. 1, *I. P. W.* " 2, *I. P. W.*	Feb. 24	10 21·090	10 21·112	10 21·180	10 21·185	10 21·101	10 21·183	10 21·135	10 21·149	10 21·142	0·041

Form No. 10—(Continued). ARC NAGARKOIL–MANGALORE. *Synopsis showing the results in the various Pivot positions and the Final Value of the Arc.*

Telescope and Pivot Position	Astronomical Date	E. Clock		W. Clock		All Stars		Both Clocks		Both Clocks
		N. Stars	S. Stars	N. Stars	S. Stars	E. Clock	W. Clock	N. Stars	S. Stars	All Stars
	1888	m s	m s	m s	m s	m s	m s	m s	m s	m s
Tel. No. 1, *I.P.E.* „ 2, *I.P.E.*	Feb. 18	10 21·121	10 21·093	10 21·195	10 21·187	10 21·107	10 21·191	10 21·158	10 21·140	10 21·149
Tel. No. 1, *I.P.E.* „ 2, *I.P.W.*	Feb. 19	10 21·026	10 21·028	10 21·094	10 21·101	10 21·027	10 21·097	10 21·060	10 21·065	10 21·062
Tel. No. 1, *I.P.W.* „ 2, *I.P.W.*	Feb. 20 & 24	10 21·090	10 21·112	10 21·185	10 21·195	10 21·101	10 21·190	10 21·138	10 21·154	10 21·146
Tel. No. 1, *I.P.W.* „ 2, *I.P.E.*	Feb. 21 & 22	10 21·136	10 21·187	10 21·254	10 21·256	10 21·161	10 21·255	10 21·195	10 21·222	10 21·208
	Means ...	10 21·093	10 21·105	10 21·182	10 21·185	10 21·099	10 21·183	10 21·139	10 21·145	10 21·141

PART V.

Astronomical Latitudes.

CHAPTER I. GENERAL OBSERVATIONS.

Introductory remarks.

1. When surveying is carried on with great accuracy and over large areas, it becomes necessary to take into account the deviation of the figure of the earth from the spherical form, and consequently one of the primary objects of a trigonometrical survey—and one that should be never lost sight of—is the determination of the absolute lengths of the equatorial and polar axes of the globe. It has been proved that the earth is only approximately spherical, but whether it is spheroidal or ellipsoidal or of some other form still has yet to be decided. If it is found to be either spheroidal or ellipsoidal the eccentricity of the meridian ellipse—and in the latter case that of the equator also—will have to be determined with the greatest precision. Hitherto in the calculations of the Great Trigonometrical Survey of India the earth has been assumed to be a spheroid, and the values of the axes that have been employed in the formulæ, by which the latitudes, longitudes and azimuths of the stations have been computed from the triangulation, are those determined by Colonel Everest in 1830 and known as "Everest's Constants, 1st Set." These values were deduced from a comparatively small amount of reliable data and are now believed to be sensibly erroneous.

Were the earth a perfect sphere composed of homogeneous matter, its size could be easily found by measuring the distance between two points on the same meridian, and finding their difference of latitude: this geodetic operation, which is usually called the "measurement of an arc of meridian," gives the number of feet on the circumference of a great circle of the globe, that subtend a known angle at the centre. Were the earth a perfect homogeneous spheroid, two "arcs of meridian" would have to be measured in different latitudes to establish its size and form, and were it a perfect homogeneous ellipsoid three such arcs would be necessary, two at least of which must be separated by a known difference of longitude.

Simple, however, as these methods appear in theory, the determination of the figure in practice is greatly complicated by a force known as local attraction, which is due to irregularities on the external surface and to variations of density of the material composing the crust, and which causes the direction of gravity to deviate from the normal to the mathematical form.* Owing to its presence very discordant values are obtained from different combinations of measured arcs for the ellipticity of the surface, and in order to eliminate error in the final result the stations of comparison have to be largely multiplied, arcs of parallel have to be measured in addition to arcs of meridian, and the astronomical azimuth determined at several of the latitude stations. The differences between the observed and geodetic latitudes, longitudes and azimuths can then be treated by the method of minimum squares, and the figure to which the mathematical surface of the earth most nearly conforms can be determined.

The following distinctive definitions are now given to emphasize and explain the special effect that local attraction has on direct determinations of latitude, as it is with them alone that this part of the Handbook has to deal:—

The *true* horizon is that great circle of the celestial sphere, whose plane touches the earth at the observer.

The *true* zenith is the elevated pole of the *true* horizon.

The *true* or *geodetic* latitude of a place is the angle that the normal to the mathematical surface makes with the plane of the equator; it cannot be determined by direct observation, as it is dependent on the position of the true zenith, and no means exist of finding the latter.

The *observed* or *astronomical* latitude is the angle made with the plane of the equator by the direction of gravity, as indicated by the plumb-line.

Were it not for local attraction† the plumb-line would coincide with the normal, and the difference between the geodetic and observed latitude would be solely due to errors in the adopted value of the ellipticity.

* By the "mathematical" form is meant the geometrical solid, whether spheroid or ellipsoid, to which the figure of the earth most nearly conforms.

† Local attraction affects every observed latitude directly and every geodetic latitude indirectly in having caused error in the adopted values of the initial azimuth and latitude at Kaliánpúr, on which all the Great Trigonometrical Survey computations are based.

Throughout this part of the Handbook wherever the word "latitude" occurs, the astronomical latitude is to be understood: similarly by the term "zenith" is meant that point in which the direction of the plumb-line infinitely produced above the horizon meets the celestial sphere, and by the term "horizon" (or the plane from which altitudes are measured) is meant the great circle perpendicular to the direction of the plumb-line.

Method of determination.

2. It follows directly from the definitions of the words employed that the latitude is equal both to the declination of the zenith and to the altitude of the elevated pole, and every known method of determination is based on one or the other of these equalities.

In India the determination of latitudes for geodetic purposes is at present effected with Strange's Zenith Sectors by measuring the zenith distances of stars on the meridian,* and very excellent results have been obtained from this method. There are several instances where 30 stars have been observed successively in one night and the resulting values of latitude have all fallen within a range of 2″. At many of Colonel Herschel's stations the probable error of a single determination was less than $\pm 0''\cdot 3$, and at Kátpálaiyam where he observed 50 stars, the probable error of the final value of latitude was $\pm 0\cdot''042$. No observer however runs the slightest chance of obtaining such results as these, unless in choosing his stars, in handling his instrument, in taking his observations and in computing out his results he pays the closest attention to every detail.

Selection of latitude stations.

3. When an officer is deputed to take latitude observations for geodetic purposes the series of principal triangulation on which he is to operate is notified, and the first duty that devolves on him is the selection of the particular stations of that series. The choice of the latitude stations should be governed by the following rules:—

(*a*) Every latitude station must be a principal station of the specified series of triangulation.

(*b*) Every principal station at which an astronomical azimuth has been observed for geodetic purposes must be a latitude station.

* The meridional zenith distance of a star is the difference between its declination and that of the zenith, and therefore the latitude is equal to the declination of the star ± its zenith distance, the upper sign being used for south stars the lower for north.

(c) On meridional series, latitude stations should be $\frac{1}{2}^{\circ}$ in latitude apart, and on longitudinal series the same amount in longitude.

(d) Stations are to be avoided at which a deflection of the plumb-line is to be anticipated in the direction of the meridian, a deflection in the prime vertical being of no consequence.

Strength of an Astronomical Party.

4. The strength of an Astronomical Party when employed on latitude operations is as follows:—

(1) The observer, who is generally a Deputy Superintendent.

(2) An Assistant-Surveyor or Sub-Surveyor whose duty is to build the pillars and who is always some ten days in advance of the main-body of the party.

(3) An observatory recorder, who also acts the part of writer.

(4) Eight burkundazes, who act as sentries over the treasure, instruments, baggage, &c.

(5) Twenty kahars or bearers, who have to carry the zenith sector on the march. During one field season the zenith sector was carried about in a hand-cart provided with springs: this method is inexpensive and answered admirably, but it is only feasible when the operations lie in a flat country, and when all the stations to be visited are plain stations.

(6) Forty khalasies. Thirty of these remain with the main-body: during a halt they are employed as dâk bearers and on ordinary observatory work, whilst on the march they carry the barometers, chronometers, &c., in doolies and pitch the tents. The remaining ten khalasies accompany the Assistant-Surveyor in charge of the advanced party and assist him in building the pillars and in laying out the meridian lines.

(7) It is advisable in many districts to engage a mason permanently for the season to accompany the advanced party.

(8) Sixteen country carts are required for the baggage of the main-body, and two for that of the advanced party.

CHAPTER II. DUTIES IN THE FIELD.

1. The advanced party should take the field ten days before the main body: the duties of the assistant in charge are:—(1) To select a site for the latitude pillar in the vicinity of the trigonometrical station; (2) To lay down the meridian by means of azimuth observations; (3) To build the latitude and collimator pillars.

Duties of the assistant in charge of the advanced party.

The position for the latitude station should be chosen where a space 15 feet square can be levelled for the observatory tent, where the collimator pillar can be built without much expense, and where the pole star can be seen at any hour of the night clear above all trees and buildings: in selecting it the advantage of minimising the effect of local attraction should not be lost sight of. In order to avoid the necessity of having to connect the latitude and trigonometrical stations by triangulation, the latitude station should be built if not over the trigonometrical station itself at any rate in its prime vertical, the direction of which is determined and laid down by means of the azimuth observations. If the trigonometrical station itself is not in a suitable position, and some site has to be found on the prime vertical, the *nearest* favorable place should be chosen, as the greater the distance the more effect will an error in the azimuth determination have in placing the latitude station off the prime vertical. If the error in the azimuth is 1′, the latitude of the trigonometrical station will differ from that of the latitude station, when 100 yards distant, by 0″·085, when 500 yards distant by 0″·43: an error in the azimuth of 2′ will double these differences. In Indian latitudes the prime vertical of a station and its latitude parallel are at the end of the first mile not one foot apart.

2. The assistant, who is provided with a small theodolite, has to mark out on the ground the direction of the meridian of the latitude station: this is necessary to enable him to find the correct position for the collimator pillar, or if collimators are dispensed with, to serve as a guide to the observer in choosing a natural meridian mark: it is also of use in laying down the direction of the prime vertical, when this is required. In order to mark out the direction of the meridian, the assistant must determine the azimuth of a referring mark from the trigonometrical station: if one of the neighbouring trigonometrical stations can be made easily

Azimuth observation by the assistant.

visible, it will serve as the referring mark and its azimuth can be taken out from the G. T. S. Synoptical Volume of the Series.

If no trigonometrical station can be seen without ray-cutting, the azimuth of a lamp set up a mile off in any direction must be determined by astronomical observations. The most convenient method of doing this, when the time is not known is, to observe the angle between the referring mark and a star and the star's altitude simultaneously: the star should be near the prime vertical and about 30° above the horizon. This process should be gone through with two different stars, one east the other west, and the results from the two sets of observations should not differ by more than 1′.

A simple method of checking the azimuth and one that should always be followed is to find the correct time by observing the altitudes of three or four stars and to then measure the horizontal angle between the referring mark and the pole star, a few minutes before and after the time that the latter is calculated to transit. The time may also be found by noting the time of transit of any known star near the zenith over the centre wire of the theodolite previously placed near the meridian.

The telescope of the theodolite should now, by means of the known azimuth of the referring mark, be pointed in the direction of the meridian, and a peg driven into the ground in the exact alignment about half a mile to the north of the station: as a single pointing of a telescope is affected by numerous sources of error, the angle between the peg and the referring mark should then be measured on one pair of zeros, and the position of the peg corrected in accordance with the result. The peg having eventually been placed exactly due north of the station, three more should be driven in to the east, south, and west by moving the telescope through arcs of 90°, 180° and 270° respectively: the positions of these latter should then be carefully checked by measuring a round of angles on both faces.

It is of the utmost importance that the latitude pillar be erected at some point on the line joining the east and west pegs, and its position in the true alignment should be carefully checked from both ends. When the actual site of the latitude station has been decided on, the direction of its meridian should be marked out on the ground with pegs as before: this direction cuts the known prime vertical line at right angles and can be at once obtained with a theodolite.

Construction of the latitude pillar.

3. The latitude pillar should be built of burnt bricks in mortar, attention being paid to the bond. The foundation should be 2 feet square and from 18 inches to 2 feet below the ground level: it should stand in a pit $2\frac{1}{2}$ feet square faced with brick; the sand for filling the intervening space of 3 inches all round should be collected and stored *in sitû* by the assistant, but not placed round the pillar until the observer has inspected the foundation. Above the ground level the pillar should be made circular in horizontal section and 2 feet in diameter: its height without the stone cap should be from $2\frac{1}{4}$ feet to $2\frac{3}{4}$ feet according to the height of the observer. As the stone cap is of special construction, one is carried about with the main body of the party and used successively, for every station.

Collimator pillar.

4. A collimator pillar should be built on the meridian line some 15 or 20 feet north or south of the latitude pillar: its horizontal section should be rectangular in shape and 3 feet by $2\frac{1}{2}$ feet in size, the shorter sides being parallel to the meridian line and equidistant from it, and its height should be greater than that of the brickwork (not including the stone cap of the latitude pillar) by 2 feet 7 inches. The foundations of the collimator pillar must depend on the nature of the soil and the weight of the pillar. A stone cap is unnecessary. A meridian mark consisting of a thin but well-defined line one foot long, must be drawn vertically down the centre of the side that faces the latitude station.

The collimator pillar is intended to carry two collimators, by means of which the deviation of the instrument from the meridian can be seen at any moment. The zenith sector however has no tendency to move in azimuth, and when once it has been adjusted to the meridian nothing short of a severe jar will disturb it. There is consequently no necessity to test the adjustment in azimuth by repeated references to collimators, and the possession of these instruments is rather a needless luxury. It is almost always feasible, by means of the zenith sector when first adjusted in azimuth, to find a substitute for them in some natural feature such as a distant tree or peak, which may be used as a meridian mark every evening at sunset, and as the actual deviation error of the instrument during work is determined from star observations nightly, and may be safely considered constant throughout a whole programme, the meridian mark need not be observed again.

As the greater number of latitude stations are near hill tops, the collimator pillars have often to be of considerable height, and a large saving of expense would be effected by their abolition: it might happen however that the latitude station was so shut in by trees that no distant meridian mark was obtainable, and in this case collimators would be indispensable. The decision therefore of whether to build a collimator pillar or not must rest with the assistant.

The natural meridian mark when employed should be as distant as possible, for there will be a parallax in the apparent position of any terrestrial point as observed with the telescope, since the line of collimation is 9·6 inches laterally distant from the meridian passing through the centre of the latitude pillar.

Preliminary work for the observer.

5. The permanent establishment of the Astronomical Parties is only maintained at a strength sufficient for electro-telegraph longitude work, and is considerably below that required for latitude operations. At the commencement of a "latitude" field season all the kahars, bullocks and carts and a large number of khalasies have to be raised and engaged for six months. It is therefore essential to start operations from some large city on the railway. Whilst the new hands are being engaged, a latitude pillar should be built at any suitable spot and the zenith sector* set up and overhauled. It would be useless to take a regular set of observations unless a principal station were in the vicinity, but the instrument should be thoroughly cleaned, tested and put into working order, before the party leaves for the field, as no artificer capable of executing the simplest repair is likely to be met with in the districts. The permanent adjustments also should be made, and the values of the instrumental constants determined before any regular observations are taken.

Permanent adjustments.

6. When the zenith sector has been brought into working order, the following adjustments, which will be found to remain invariable throughout the season, should be made once for all:—

(i) Stellar focus.

(ii) Horizontality of the horizontal wire.

(iii) Collimation in azimuth.

(iv) The zero of the setting-circle.

* For a description of this instrument see Volume XI of the "Account of the Operations of the Great Trigonometrical Survey of India."

7. This adjustment consists in placing the wires of the telescope in the principal focus of the object-glass, and its correctness can be tested as follows: obtain distinct vision of the wires by pushing in and drawing out the eye-piece from its cell, and direct the telescope upon some distant object; now move the eye first to one side and then to the other: if the image of the object retains a fixed position with regard to the wires, the latter are truly in the focus of the object-glass. If, however, on moving the eye to one side, the image of the object appears to move with the eye, then the principal focus of the telescope lies *beyond* the wires, and the object-glass and wires must be approximated. If on the other hand the image is between the eye and wires, it will move in the contrary direction to the eye.

Stellar focus.

The distance between the object-glass and wires can be altered by means of two antagonizing screws that work in the direction of the optical axis against a brass nut.

8. The horizontal wire can be made horizontal by means of two antagonizing screws that work against the same brass nut alluded to in the preceding paragraph, but in a direction at right angles to the screws mentioned there. By working against the nut they cause the eye-end to revolve in the telescope. To verify the horizontality of the horizontal wire, level the instrument and having set the telescope to a distant steady defined object near the horizon, or to a collimator, move it in azimuth so that the object may appear to move along the wire from one extremity of the field to the other. If the object does not remain intersected at all parts of the field, the error ought to be rectified by moving the wire-plate in the appropriate direction.

Horizontality of horizontal wire.

9. The adjustment for collimation in azimuth consists in making the line of collimation intersect the horizontal axis of rotation at right angles approximately; and can best be made by Gauss's method. Two small theodolites serve as collimators and are set up on their stands, one to the north and the other to the south, their crosses being superimposed: the wire-plate of the telescope is then moved laterally by means of two antagonistic screws placed behind the micrometer box, until the centre vertical wire intersects the

Collimation in azimuth.

crosses of both theodolites, when collimation error has been approximately eliminated.

This approximate adjustment of the line of collimation in azimuth when once made will remain undisturbed throughout the season: there will of course be a small remaining collimation error, which will probably vary within narrow limits from station to station; *this has to be determined nightly from star observations.*

It should be noted that the zenith sector in its working position obstructs the view of one theodolite through the other, and that it has to be revolved through 45° in azimuth and pointed some 15° from the zenith before this view can be obtained.

The zero of the setting-circle.

10. When the telescope is pointed in any direction, and the bubble of the setting-circle is at the centre of its run, the vernier of the setting-circle should read zenith distances to within half a minute: unless it does this, there will be difficulties in identifying the correct star, when two or more are in the field together.

The unit of measurement into which the field of view of the telescope is divided is one division of the micrometer, which works the horizontal wire. The head of this micrometer is divided into 100 divisions, and a comb with 100 teeth runs vertically down one side of the field: one revolution of the micrometer-head causes the horizontal wire to pass exactly over one whole tooth, the breadth of which is accordingly equal to 100 divisions. The whole field of view is therefore divided up into 10,000 divisions. To prevent confusion in counting the teeth, a small round hole has been made in the comb at the 10th tooth from one end, two such holes at the 20th, three such at the 30th, and so on up to the other extremity of the comb, except that at the 50th tooth and after, one larger hole has been made to represent five of the small ones. These marks at every 10th tooth divide the field of view into sections of 1,000 divisions each, and as the mark that denotes the 50th tooth or 5,000 divisions is near the centre of the field, it has been assumed as *the zero of vertical measurements.* When the horizontal wire is exactly in the centre of this mark, and the micrometer-head is reading 0·0 division, the point of intersection of the horizontal wire and the centre vertical wire is in the line of collimation.

To make the setting-circle read correctly, the following steps should be gone through in the order here given :—

(i) Place the horizontal wire by turning the micrometer-screw exactly in the centre of the 5,000 division mark on the comb, and then make the micrometer-head, which is only kept in position by friction, read 0·0 division by revolving it on its axis.

(ii) Observe the zenith distance, as recorded on the limb of the setting-circle, of any distant terrestrial object in both telescopic positions,* intersecting it by means of the tangent screw that belongs to the vernier of the setting-circle and not touching the micrometer-screw.

(iii) Take out the mean of the two zenith distances, and set the vernier of the setting-circle to read it in either telescopic position; bring the object again into the field.

(iv) Now tap the telescope until the object is intersected by the horizontal wire, taking care not to touch the tangent-screw of the setting-circle.

(v) Bring the bubble to the centre of its run by raising or lowering one end of the level itself by means of the capstan-headed screws provided for the purpose: the setting-circle will be then adjusted to read zenith distances.

Determination of the instrumental constants.

11. Before leaving for the field, the values of the following instrumental constants should be determined :—

(i) The equatorial intervals of the vertical wires.

(ii) The divisions of the head of the eye-piece micrometer.

(iii) The divisions of the scales attached to the levels.

* In a theodolite the two telescopic positions are distinguished by the position of the vertical circle and are known as "face-right" and "face-left": in a transit instrument they are denoted *I.P.E.* and *I.P.W.* according to the position of the illuminated pivot. In a zenith sector the telescope is at one end of the horizontal axis and the two positions have been named T.E. and T.W. according as to whether the telescope is at the east or west end of the horizontal axis.

If the determination of these constants is postponed till the middle or end of the season, the risk is run of some accident happening to the wires, levels, or micrometer which might render all previous observations useless.

The equatorial intervals of the vertical wires.

12. There are seven vertical wires known respectively as I, II, III, IV, V, VI and VII and so numbered that in the position telescope east, a zenith star transits the wires in the order I to VII, whilst in the position telescope west, wire VII is the first wire reached and wire I the last. In both telescopic positions wire IV is the centre wire, from which the intervals of all the others are measured. Wire I is liable to be confused with wire VII, as they are symmetrically situated with regard to wire IV, only on different sides of it, so that in one telescopic position wire I is to the apparent left and wire VII to the apparent right, whilst in the other the contrary holds. Similarly wires II and VI may be mistaken for each other, as also wires III and V. When an observer is timing the transit of a star, and has to inform the recorder of the particular wire crossed, he may easily fall into error, for in naming the wire he has to first note the telescopic position of the instrument and to then recollect the order of the wires in that position. He is more likely to confuse the wires in a zenith sector than in a transit instrument, as the telescopic position of the former is changed at every star. He will avoid, however, all possibility of mistake if he recollects that wire I is situated *nearest* to the comb.

When the transit of a star is being timed, the telescopic position must be recorded, or otherwise the observation will be useless. Suppose an equatorial star transits wire V at $7^h\ 31^m\ 11^s$ and that the equatorial interval of wire V is 7^s: then if the instrumental position was "telescope east," the star would have crossed the line of collimation at $7^h\ 31^m\ 4^s$, whereas if the instrumental position had been "*west*" the correct time of transit would have been $7^h\ 31^m\ 18^s$.

The equatorial wire intervals are required to be known to the nearest tenth of a second of time. They can best be determined by noting the time of transit of slow-moving circumpolar stars over each of the vertical wires. The cosecant of the declination of the star, multiplied by the observed interval between the times of transit over any wire and the centre, is the equatorial interval of that wire.

If the star observed is very near the pole, the radius of curvature of its path will be so small, that it may not travel at right angles to all the vertical wires: a correction for curvature will then have to be applied to the wire intervals, *vide* Chauvenet's Astronomy, Volume 2, Section 131, page 147, and page 111 of this Handbook.

One division of the eye-piece micrometer.

13. The value of a division of the eye-piece micrometer, though it never practically varies, had best be determined at the commencement of every field season.

To enable the limb to be read by the microscopes, when the telescope is horizontal, the sectors should be unscrewed and revolved on their axis through 90°: their central diameter will then be parallel to the telescope instead of perpendicular, as it is in ordinary work. Now intersect the cross of a collimator with the horizontal wire of the telescope: clamp the telescope, and take the readings of the four microscopes and of the eye-piece micrometer. Move the telescope in altitude some 5 or 10 minutes by means of the tangent-screw, and again read the four microscopes. Now by working the micrometer intersect the collimator cross again with the horizontal wire and take the new reading of the micrometer-head. The angle in seconds of arc that the telescope was moved can be ascertained by comparing the two mean readings of the limb by the four microscopes, whilst the difference between the two readings of the micrometer gives it in micrometric divisions. The value of the angle is now therefore known both in seconds of arc and in micrometric divisions, and by equating the two measures, the relative proportion of the units can be found. Before the value of a division of the micrometer can be satisfactorily accepted as final, the above process should be gone through several times, as many different parts of the thread of the micrometer screw and as many different graduations of the limb being brought into play as possible.

The values of the level scales.

14. In all there are three levels employed with the zenith sector: two, mounted on the cradle, are to shew the inclination of the vertical axis in the meridian and should, if accurately made and mounted, always give identical results. The third is a striding level, and is used to find the dislevelment of the transit axis. In addition to these three it is advisable, as levels are easily broken, to subject two or three spare ones to the tests given hereafter, and to determine the values of their scales before leaving for the field.

The values of the divisions of the level scales are found in practice to vary both with temperature and age, and should consequently be determined annually. The most accurate and at the same time most simple method of determination is by means of the bubble tester, an instrument that can be had temporarily on loan either from the Mathematical Instrument Office, Calcutta, or the Dehra Office.

If no bubble tester can be obtained, the values of the divisions can easily be determined as follows:—The levels should be attached to the telescope with their axes parallel to the line of collimation: the telescope should be brought to a horizontal position, a collimator cross intersected with the horizontal wire, and the readings of both ends of each bubble and of the micrometer recorded. The telescope should now be moved in altitude by a slight tap, and the readings of the levels again taken: the collimator cross should then be intersected by means of the micrometer with the horizontal wire, and the new reading of the micrometer recorded. The number of divisions that each bubble has moved is equal to the difference between the two readings of the micrometer, and the value of a division of each level scale is therefore obtainable in terms of micrometric divisions, and thence in seconds of arc. The number of divisions that a bubble moves, when the telescope is tapped, is

$$\frac{N_1 + S_1 - N_2 - S_2}{2},$$

where

N_1 = first reading of northern extremity,

N_2 = second " " "

S_1 = first " southern "

S_2 = second " " "

$\frac{N_1 + S_1}{2}$ being the position of the centre of the bubble before it is disturbed, and $\frac{N_2 + S_2}{2}$ the position of the centre after the disturbance when it has again come to rest.

There is another method of determining the values of the level scales, which, owing perhaps to some difficulty in attaching the levels to the telescope as directed above, has been occasionally followed. The levels are mounted in their proper places on the instrument, and the readings of their bubbles taken, the position of the sectors at the moment is found from the four microscopes. The verticality of the vertical axis is now altered by means of the thumb-screws at its foot: the change in its inclination is then found in divisions from the levels, and in seconds of arc from the microscopes, and the value of one division deduced. This method is cumbersome and inaccurate, and is not recommended.

Testing a new level.

15. The zenith sector has been made with such delicacy that it is very difficult to obtain levels at all worthy of being employed with it, and no *new* level should ever be brought into use without being thoroughly *tested* first. In testing the fitness of a level the following points should be examined by means of the bubble tester:—

(i) If the inclination of the level be altered either *gradually* or *suddenly* and then brought back to what it was originally, the readings of the bubble before and after the disturbance should be identical.

(ii) A bubble should rapidly come to rest, after motion however violent has been imparted to it.

(iii) The uniformity of the curvature of the tube should be such, that no two individual divisions of its scale should differ by $0''\cdot 1$.

A great number of levels will be found, whose position at rest depends to a certain extent on the direction of motion last imparted to the bubble: they apparently overshoot their mark and have not the power of returning. They should never be employed with the zenith sector.

Many also will be found which after disturbance take a long time such as five minutes to settle, and there are a few to be met with that will not come to rest under twenty minutes. The accuracy of work with the zenith sector depends on the ability of the observer to measure and read several quantities *without* change of conditions, and unless a bubble settles in half a minute the level should be discarded. This point should be carefully decided on, as in many levels the bubble will go on moving a long time after it has apparently come to rest: its motion is so slow as to be invisible, but will be found to be real if its position be noted at the end of each minute. The values of the division scales of new and untried levels should be determined both by long sweeps extending from one end of the scale to the other, and by short ones of two or three divisions. Short sweeps up and down the whole length of the scale in both directions are most essential, as the value of every individual division should be determined.

It is useless to adopt the *mean* value of a division of the whole scale, unless the values of all the individual divisions agree with it very closely, as the ordinary amount of dislevelment to be corrected for in the zenith sector is under two divisions. In a really good level* a change of inclination of 1″ should make the bubble move lineally through $\frac{1}{20}$th of an inch *at least* and the value of no individual division† should differ from the mean by more than 0″·05.

In all levels the extreme 10 or 15 divisions at the two ends will be found to fail, although the centre portion may give excellent results: in determining therefore the value of the divisions, the two limits of reliable curvature on either side of the centre should be found and noted: in subsequent work no dislevelment should be accepted, if either end of the bubble has overstepped these limits.

The values of the divisions should be determined to 0″·001 at different temperatures and on several different days: at times a slow and gradual motion should be imparted, at others a sudden and turbulent one. The zero also of the micrometer of the bubble tester should be frequently changed by placing coins under the point of the screw, or the same portion of the thread of the screw will always be in play.

Precautions necessary in mounting the vertical axis levels on the cradle of the sector.

16. When the divisions of the level scales have been accurately determined, the levels that shew the inclination of the vertical axis should be placed in their working position on the instrument, where they should be allowed to remain on the march. In placing them on the instrument, the following points should be attended to:—

(i) A strip of the interior surface of the glass tube ¼ inch broad is ground to something like curvature from end to end: no

* If a change of inclination of 1″·00 makes the bubble move through $\frac{1}{20}$th of an inch, the curvature of the tube has been struck with a radius of 580 yards.

† The value of an individual division must not be taken from a single determination.

attempt is made to give curvature to the remaining portion of the interior surface, which is therefore no more suited for use as a level than an ordinary bottle. In only a single line of the ground strip running from end to end do the best makers pretend that the curvature is uniform: it is consequently essential that in mounting the level this particular line should be placed uppermost and in the same *vertical* plane as the axis of the level.

On most good levels there are two small opaque dots, one at each end: when the level is in its correct position, the longitudinal diameter of its bubble should coincide with the line joining the two dots, a condition that can be tested by running the bubble up to the two dots in succession.

In some levels instead of opaque dots, there are two broad arrows pointing inwards towards one another, about ¼ of an inch apart, near the middle of the tube, the bubble should then be made to float exactly between them. In other levels the tube has graduations cut on the glass along its whole length, in which case the centre of the bubble must be made to run down the line joining the centre of the graduation-cuts. The absence of opaque dots generally denotes an inferior level.

(ii) The glass tube of the level should be rigidly attached to the brass caps by wedging pieces of cork between the two, a very difficult operation. The pieces of cork that are fixed in the brass caps must first be scooped out on one side so as to fit the twisted extremities of the glass tube: one piece must then be placed in the centre of a brass cap, and one end of the level inserted in it: the glass cylinder must then be brought down round the level, until it takes up its proper position in the brass cap. Lastly, the second brass cap must be fitted on to the other end of the cylinder, the piece of cork at its centre receiving the free extremity of the level. If the pieces of cork employed are very thin, they will not support the level at all: if they are very thick, the risk is run of compressing the tube too forcibly and smashing it: if they are sufficiently thick to support the tube without breaking it, there is still a fear that the different portions of the level may not be quite rigidly connected, and that the tube may have slight longitudinal motion open to it in its brass case. The interposition of the glass cylinder prevents the tube being felt with the hand, and the rigidity of the connection cannot therefore be tested by touch. If the tube has play in its case, no level reading is reliable: in regular work the rigid attachment of the two is subjected to a severe test by the constant rapid rotation of the instrument in azimuth, and it has therefore to be carried out with great thoroughness.

The level should occasionally be taken to pieces and the connection of its parts examined, as in hot weather the pieces of cork are apt to shrink, and to lose their elasticity.

(iii) The brass case of the level must be rigidly attached to the Ys: this is done by means of springs, which are screwed down to the Ys and exert pressure on the pivots of the case. If this attachment be not rigid, the level will change its position in its Ys during the revolution of the instrument in azimuth.

(iv) The two levels when mounted on the cradle are parallel to one another, but are situated at unequal distances from the vertical axis, on which the instrument rotates in azimuth. As the rapidity with which they move varies with the square of their distance from the axis, a more violent motion is imparted during rotation to the distant one than to the near. The *more* sensitive level of the two should therefore be always mounted in the place of least disturbance.

Chronometer, barometer and thermometers.

17. Before leaving for the field the sidereal chronometer to be used during the latitude observations should be tested for uniformity of rate, and the index errors of the barometer and thermometers should be determined against the departmental standard. For actual work one mercurial barometer and two thermometers are required, but spare ones whose index errors are known should also be taken. The determination of the index errors must not be postponed till after the field season, as the instruments may get broken during work.

When the necessary preliminary operations that have been described have been carried out, the party should take the field under the observer and commence regular observations for latitude.

18. On arrival at the latitude station set up a small theodolite on the pillar and measure the angle between the meridian mark and the trigonometrical station: if it prove equal to 90° and if the meridian mark is afterwards found from observations with the zenith sector itself to be truly on the meridian of the latitude station, the latter has been correctly placed on the prime vertical of the trigonometrical station.

On arrival at a latitude station.

Examine the *material*, the bond and the isolation of the pillar: fill in the 3-inch space that has been left round the foundation with the sand collected for the purpose by the assistant. Fix the stone-cap to the pillar with cement, taking care that it is in its correct position* with regard to the meridian and that its upper surface is approximately level. The correct position for the stone-cap can be found by stretching a piece of string across the pillar in the direction of the meridian line, and by revolving the cap on the pillar until the line (that has been cut across its upper surface for the purpose of the adjustment) coincides with the string.

Pitch the observatory tent over the pillar, leaving the side towards the sun open until the new-laid cement is dry. Suspend the two thermometers to tent-ropes outside the observatory, one on the north side, the other on the south about 4 feet from the ground and out of the range of possible contact with other bodies. Hang the barometer inside the observatory to the frame-work of the tent with its cistern about 18 inches from the ground.

Make two pencil marks on the collimator pillar 9·6† inches on either side of the meridian line cut on the brick-work by the assistant: set up the collimators on the pillar, plumb the centres of their object glasses accurately over the pencil marks and place their optical axes by eye parallel to the meridian line. If the chronometer has stopped, or its travelling rate is unreliable, observe altitudes of the sun for time with a small theodolite.

* Holes have been bored for the foot-bolts of the sector in the stone-cap and the latter has therefore to be so placed in the horizontal plane that the holes will be vertically under the feet of the instrument when adjusted to the meridian.

† The line of collimation of the telescope is 9·6 inches to the east of the vertical axis with "telescope east" and the same amount west with "telescope west."

When the cement attaching the stone-cap to the pillar is dry, set up the zenith sector.

How to put up the sector.

19. Place the cast-iron pillar on the stone-cap with the azimuthal stud to the north, and see that all the three feet are resting firmly on it: place the hemispherical steel button under the base-plate screw, and test the rotation of the instrument by revolving the pillar several times on it. Bring the pillar to rest with the azimuthal stud to the south, and again see that the three feet are resting firmly on the stone-cap in this new position. It is advisable to commence work at a station with the azimuthal stud south, as the original adjustment in azimuth is facilitated by the observer being able to work the stud screw and intersect a star with the telescope simultaneously.

Loosen the base-plate screw: bring the screw that works in the azimuthal stud, and the four levelling thumb screws to the centres of their threads. Put in and *loosely* screw up the three foot-bolts that hold the pillar to the stone: if they all three go in easily without touching the sides of the holes in the stone, the instrument is roughly adjusted to the meridian. Find the dislevelment of the cast-iron pillar by means of an ordinary spirit-level, and make the pillar approximately level by placing tin washers of various thickness under the feet.

Thoroughly clean the cup that receives the point of the vertical axis, and the three hemispheres that are fixed in the neck of the pillar. Fill the cup with oil, and oil the hemispheres. (The hemispheres can be revolved by means of brass-heads, situated on the outside of the pillar near the top: these heads should be so screwed on, that the diameters cut across their surfaces are parallel to the grooves of the hemispheres: when the hemispheres are exerting pressure on the vertical axis, the grooves should be vertical, but as they are out of sight, their position can only be told by means of the diameters on the heads). Having screwed on the heads correctly, turn the hemispheres till their grooves are vertical.

Clean the vertical axis and oil it near its junction with the table: insert the axis into the cast-iron pillar, taking care that the flying stop is properly situated between the two fixed stops. Let down the dust-cap over the point of the axis: test the rotation of the axis in the cast-iron pillar by giving it a semi-revolution in azimuth, making the flying stop come into contact with the two fixed stops successively: glance at the brass heads, that turn the hemispheres, and see by the position of the diametrical cuts, whether the verticality of the grooves was disturbed by the insertion of the axis.

Revolve the whole pillar, which has been as yet only loosely attached to the stone-caps with bolts, in azimuth, until the centre line through the two fixed Ys of the table passes 3·2 inches to the east or west* of the meridian mark: by this means the adjustment of the instrument to the meridian can be somewhat improved. Place a spirit-level on the table and re-level the pillar approximately by means of washers under the feet. Tighten the three foot-bolts.

Put on the cradle; in raising the cradle *be careful not to lift by the microscopes,* which are hollow; when the cradle is in position, secure it *at once* to the table with the large bolt. See that the back studs of the cradle rest on their bearings: if one is resting and the other not, the box containing the spindle and the two bearing shafts must be opened, the endless screw must be put out of gear and the lower shaft raised to the same level as the higher by hand: to do this the cradle has to be removed. The dislevelment of the vertical axis is now shewn by the two vertical-axis levels: by means of the four thumb-screws make the axis perfectly vertical both in the meridian and the prime vertical: clamp the base-plate screw.

Put on the cylindrical brass counterpoise at the back of the table, and place ready to hand the two other counterpoises. Raise the telescope and sectors out of the box: *do not touch* the sectors in lifting, and remember too that the two wooden-tipped handles of the sectors are hollow and weak. So place the telescope that the horizontal axis points upwards and the brass cap, which shuts the window opposite the reflector in the telescope, rests on the truncated pyramid of the lid of the box. Remove the four capstan-headed screws situated round the telescope, and unclamp the sector-handles.

Consider whether 360° or 180° is to be under the index microscope; this is purely a matter of choice for the observer. Whilst two men hold the telescope steady, turn the sectors through 90°: on the direction, in which they are rotated depends, whether 180° or 360° comes under

* East or west, according to which of the fixed stops is in contact with the flying stop.

the index. When the sectors are in position, screw in again the four capstan-headed screws.

Screw on the clamping arm, taking care that it points towards the eye-end of the telescope. Tie up the spring of the clamping arm tight with string, and loosen the clamping cam and the tangent-screw of the arm attached to the cradle. Examine the friction rollers, situated at the junction of the telescope and the horizontal axis. Clean the horizontal axis; oil its collars and its steel bearings in the front wall and at the back of the cradle. Affix the wooden guides and conical cap to the horizontal axis; and clamp the handles of the sectors at right angles to the telescope. The telescope and sectors are now ready to be lifted.

Before lifting, see how the telescope should be moved so as to make the two clamping arms on the sectors and cradle face each other. So arrange the revolving iron guides that the two large relieving wheels will be *under* the telescope, when the latter is in position. Make three men raise the telescope, sectors and axis vertically, and when clear of the box turn the axis horizontal, keeping the eye-end micrometer *up*. Place the axis so far into its hole in the cradle, that the iron guides are just free, but on no account allow the axis to *rest* on the cradle. Direct the iron guides through their holes in the front wall of the cradle, making certain that the relieving wheels are *under* the telescope. Run the axis home through the cradle, and see that the two clamping arms meet and fit into each other. Two men must still support the telescope, or the whole instrument will over-balance.

Take off the wooden guides and conical cap; fix on the mushroom counterpoise: connect the iron guides with the brass yoke, and into the latter screw the black counterpoise. Screw on the dew-cap: put on the bridge-piece with screws towards the counterpoises: loosen the screws of the bridge-piece, to enable its springs to act. If the bridge-piece does not go on easily, turn the telescope slowly in altitude, while putting it on: if it does not go on then, the horizontal axis is not properly home. Screw on the oblique eye-piece and the brass handle for rotating the instrument in azimuth.

Until the bridge-piece is fixed, two men should support the telescope. When the bridge-piece is in position, the observer may direct the men to let go their hold.

20. When the sector has been put up, and been left for three or four hours to settle, the following adjustments should be made in the order given :—

To adjust the sector for work.

(1) Adjustment of the lamps.

(2) The telescope micrometer.

(3) The four microscopes.

(4) Elimination of the zero of the sectors.

(5) Verticality of the vertical axis in the meridian.

(6) Horizontality of the horizontal axis.

(7) Adjustment of the sector in azimuth.

21. Two lamps are provided for the general illumination of the instrument, each of which has one central and four surrounding lenses. They are set up east and west of the instrument on firm adjustable supports, and afford a perfect and constant illumination if properly placed. The centre lens of one lamp illuminates the reflector at the window and thence the interior of the telescope, while *at the same time* the four surrounding lenses of the other lamp illuminate the portions of the limb under the microscopes, the light of the latter being focused by the lenses attached to the microscopes: when the instrument is revolved in azimuth, the lamps simply interchange duties.

Adjustment of the lamps.

Perfect illumination though difficult to attain is all important; and though hours may be spent in adjusting the lamps the trouble is amply repaid afterwards.

The first step necessary is to bring the reflecting plate of the telescope to its mean position by means of the illuminating rod: if this is not done, the illumination of the wires may be obtained with the reflector in one or the other of its extreme positions, and the observer will not thus have the power of regulating the amount of light in the telescope at will.

Open the lamps and see that the heart of the flame in each is at the point where the principal axes of the five lenses meet: the flame may be too high, too low, or too far away from the lenses. It can be lowered by turning down the wick and raised by placing washers underneath the vessel containing the oil: if, when advanced as far as it will go, it is too distant from the lenses, either the vessel or the outer case has become distorted by wear or heat, and they must be made to fit each other by bending and filing.

Set up the wooden supports due east and west of the pillar, and place the lamps on them: the flames should be five feet from the vertical axis and the same height as the horizontal axis. Turn the central lens of each lamp towards the instrument, and cover up the eight surrounding lenses: then with the light from the central lenses only, obtain good illumination of the field in both telescopic positions: this can only be done by revolving the lamps slightly in azimuth backwards and forwards, by placing wedges under the back and front edges of the lamp and thus altering its inclination to the vertical, and by moving the whole lamp laterally parallel to itself.

When good illumination of the field has been obtained, the line joining the heart of the flame and the centre of the lens passes through the centre of the telescopic reflector; the distance of the lamp from the telescope may, however, still be wrong, and if it is so, the surrounding lenses will fail to answer their purpose.

When the results by the central lenses are satisfactory, uncover the surrounding lenses, and try whether the portions of the limb under the microscopes can be illuminated by means of the lenses *on* the microscopes and without touching the lamps. If the illumination of the limb cannot be obtained by means of the microscope lenses, try the effect of moving the lamp towards the instrument or away from it. This movement will probably disturb the adjustment of the central lens, which will have to be made again.

When both central lenses are illuminating the field and the surrounding lenses of one lamp also have been got to give good results, those of the other lamp have to be adjusted: the difficulties of doing this are now enhanced through the lenses of the microscopes having been placed to suit the former lamp and being therefore no longer available for the adjustment. As the lenses of the microscopes in one fixed position have to illuminate the limb with both lamps, they should never be placed in any abnormal position: it will often be seen that by inclining these lenses at extraordinary angles, illumination of the limb can be attained, but under these circumstances the position of the lamp is really at fault, and on revolving the instrument the lenses will be found to act no longer. When the lamps have been once adjusted, mark on the ground the exact position of the supports, and on the supports the exact position of the lamps, so as to have no difficulty in replacing them at any time.

Adjustment of the telescope micrometer.

22. With the oblique eye-piece get the *horizontal** wire into focus: examine Polaris for parallax, and whilst it is in the field, make use of it to test the horizontality of the intersecting wire. Revolve the micrometer-head on its axis, until it reads 0·0 division, when the horizontal wire is exactly midway between two teeth.

Adjustment of the four microscopes.

23. Loosen all the screws that fasten the microscopes to the arms of the cradle, and see that the microscopes can be revolved on their axes quite freely and moved longitudinally without any strain being set up. Get good focus of the wires by moving the eye-pieces in their cells, and then get distinct vision of the limb by moving the whole microscope either towards it or away from it: see that there is no parallax. By revolving the microscopes, make the wires accurately parallel to the longest divisions of the limb. Good illumination of the limb is all important: to have one side of a division light and the other dark is fatal.

By means of the small screw at the end of the micrometer box, bring the zero of the comb of A microscope as near the centre of the field as possible, and then with the tangent-screw of the sectors make a division of the limb exactly bisect that zero. Now intersect the division with the wires by turning the micrometer screw, when the head of the micrometer should read 0·0 division; if it does not do so correct it by revolving the graduated head on its axis. When A microscope has been thus adjusted, intersect the corresponding divisions of the limb with B, C, D microscopes, and by revolving their graduated heads on their axes, make them all in the position of intersection read 10·0 division. To guard against possible mistakes in the number of the minutes, the B, C, D microscopes should always read 10 divisions

* Focus is not the same for the horizontal and vertical wires.

more than A, but their combs need not be touched, as they are never referred to. The graduated heads of microscopes are held in position by friction only, and can easily be rotated, if they are first drawn a slight distance out from the micrometer box: it will, however, be found that this process often disturbs the intersection of the limb, which has consequently to be made again.

It is a peculiar feature of the zenith sector microscopes, that they cannot be adjusted for "run." The divisions of their heads therefore cannot be made to equal seconds of arc, when referred to the graduations of the limb. The mean value of the error of the "run" is about 0″·264 too small in five minutes, so that 299·736 micrometer divisions are equal to 300 seconds of arc: at different stations the alteration of the focussing adjustment may decrease the error of "run" to 0″·100 or increase it to 0″·400; variations of temperature are believed too to cause changes in its value.

The "run" of the microscopes has to be determined some 15 or 16 times a night, (but never twice over the same 5′ space) as follows:— Two consecutive divisions of the limb are intersected successively with each microscope, and the eight readings recorded: the mean of the four "low" readings is then deducted from the mean of the four "high" readings, the resulting difference being the value of the run correction. If the divisions of the micrometer heads were equal to seconds of arc, the mean of the "low" readings would equal that of the "high."

Owing to differences in the make of the four microscopes, an exceptional amount of care is required to avoid confusing in one or the other of them the "high" reading with the "low": the "high" reading can only be distinguished from the "low" by observing the direction in which the graduations of the head are moving: if the micrometer is being so turned, that its reading is constantly *increasing*, (that is to say the graduations of the head are passing the index in the order 0, 10, 20, 30, 40, 50 and 60), the wires *are approaching* the "high" division of the limb: if on the other hand the revolution of the micrometer is causing its readings to decrease and to pass the index in the order 60, 50, 40 . . . . . ., the wires are approaching the "low" division of the limb. It is a matter of indifference whether the "low" or "high" reading is taken first.

In taking a "run" over any 5′ space, say from 358° 10′ to 358° 15′, one is apt at first to imagine that the reading of the 15′ division, which is the *higher* division on the limb, must be the "high" reading: this is quite wrong, as the terms "high" and "low" when used in reference to the run have nothing whatever to do with the limb. One is liable too, if in a hurry, to assume that the "high" reading must be approached, if the micrometer head is turned with the "screwing-up" motion, this is also wrong, as in two of the microscopes, when the "screwing-up" motion is employed, the wires are receding from the "high" reading and approaching the "low." A confusion of the "high" and "low" readings can only be avoided by a strict observance of the following rule:—*Never fail, when a run is being taken, to watch whether the reading of each micrometer head is increasing or decreasing.*

An observer when reading the four microscopes must take care *what* graduation of the limb he intersects, or he will not afterwards be able to discover from the record the sign to be given to the run correction. Suppose the A microscope reads 176° 32′ 30″ approximately: the zero of the microscope will be situated nearly midway between the 30′ and 35′ graduations of the limb: if the 30′ graduation be intersected, the micrometer reading will be seen to increase as the screw is turned, and the "run" correction will be consequently + ; if the 35′ graduation be taken, the micrometer reading will decrease as the screw is turned, and the correction will be −. The two consecutive graduations will not read the same, but will differ by the amount of the "run" correction for a 5′ space. If the value of the "run" correction were 0″·264, the micrometer on intersecting the 30′ and 35′ graduations would read respectively 29·87 and 30·13 divisions; the correction for 2′ 30″ is half that for 5′ and is therefore ·13: in the first case the correction would be +, and the reading 29·87 + ·13 = 30″·00: in the second case the correction would be −, and the reading 30·13 − ·13 = 30″·00. If the observer will adhere to the three following rules, no error can creep into the sign of a run correction:—

(i) When the zero of A microscope is *conspicuously nearer* to one division of the limb than to any other, intersect that division.

(ii) If there is the slightest doubt as to which division of the limb is the nearer, intersect either, and tell the recorder to note which: the recorder will then write in the margin "referred to 30′" or "referred to 35′" as the case may be, meaning that the 30′ or 35′ division has been intersected.

(iii) Invariably intersect that division of the limb with the B, C, D microscopes, which corresponds with the one intersected with the A microscope.

The sign of the run correction is +, if the division intersected is less than the reading of A microscope, and − if greater. When the A microscope reads any thing between 0′ and 2′, the 0′ division being conspicuously nearer will be intersected, and the correction will be + : if it reads between 5′ and 7′, the 5′ will be intersected and the correction be + again: if it reads between 3′ and 5′ or between 8′ and 10′, the 5′ and 10′ being the nearer will be intersected and the correction be −. If it reads between 2′ and 3′ or between 7′ and 8′, the division intersected should be noted.

An officer, who is to be entrusted with the conduct of latitude operations, need not be told that on every possible occasion, when a division of the limb is being intersected with the wires of a microscope, the final motion given to the micrometer head must be in the "screwing-up" direction; if a division is being approached for the purpose of intersection with an "unscrewing" motion, whether it is to give an ordinary reading or a "high" or "low" one for run, the wires must be made to clearly pass the division and then brought back to it for intersection. In the zenith sector more discrimination is required than in other instruments to recognize instinctively, which is the "screwing-up" motion and which not, because two of the microscopes A and D possess a left-handed screw, and the other two B and C a right-handed one.

One more caution is necessary to anyone working with these microscopes, and that is to take care that no mistake arises in *reading* off the graduations from the heads, after the intersections have been made: the graduations in two microscopes increase from left to right, and in the other two from right to left: it may happen and often does, that one number only, say 40, can be seen by the observer, those on either side, *viz.*, 30 and 50 being invisible, unless he changes his position for the purpose of noting them. There is generally no time to spare, and the observer simply sees the number 40.

Suppose the index is pointing to the second division to the *left* of 40: in two microscopes this would mean that the reading was 42, and in the other two 38.

It will be seen from what has been said above, that the manipulation of the microscopes, without hesitation or flurry, requires considerable practice, and practice alone will enable a new observer to master their idiosyncracies: the following maxims however may at first be found useful:—

(i) In all the four microscopes the "screwing-up" motion is *inwards*; consequently when a division is to be intersected the final motion given to *all* the micrometer heads must be *towards* the mushroom counterpoise.

(ii) A's and D's micrometers read from right to left, B's and C's from left to right.

(iii) A and B read *downwards* on the limb, C and D *upwards*; that is to say, if a 30′ division of the limb is visible in all the microscopes, the division apparently* below it in A and B will be the 35′ division, and in C and D the 25′.

(iv) In taking runs the "high" reading with A and B is the apparent* *upper* division on the limb, with C and D the apparent *lower*.

In reading the four microscopes it will be found advisable to take them in the order A, B, D, C, for whilst intersecting with C microscope, one generally breathes on the eye-piece of D which prevents it being used for some minutes: if D is read before C, no such difficulty arises.

Elimination of the zero error of the sectors.

24. Test the adjustment of the setting-circle to read zenith distances by observing to some terrestrial object on both faces: if there is any facial difference, eliminate it by altering the bubble. When the setting-circle is in good adjustment, set it to read accurately 0° 0′: move the sectors in altitude until the index microscope reads 180° or 360°, and the A microscope 0′, and clamp. If now the bubble of the level of the setting-circle is at the centre of its run, the sectors have been placed in their correct position with regard to the telescope, and no zero error exists: if the bubble is not at the centre of its run, it must be brought there by changing the relative

* The word "apparent" is used to denote the direction of the limb graduations as they appear through the inverting microscopes.

positions of the sectors and telescope. To do this loosen the four large capstan-headed screws attaching the telescope to the sectors, and tap the telescope without moving the sectors, until the bubble comes to the centre. It will be found very difficult to move the telescope without moving the sectors too: after every tap therefore the reading of A microscope should be taken, and brought back to 0′ if disturbed. Continue tapping the telescope and re-adjusting the reading of A microscope, until at one and the same time the setting-circle bubble is at the centre of its run, the setting-circle vernier is pointing to 0° 0′, the index microscope to 180° or 360°, and the A microscope is reading 0′: when these four conditions all hold simultaneously, screw in and tighten the four large screws.* To guard against mistakes, test the zero error by observing a star on both faces. A zero error of 60″ or under may be accepted: a larger might entail several stars being lost. When the zero error of the sectors has been eliminated, examine the microscope adjustments, which will probably have been slightly disturbed.

Verticality of the vertical axis in the meridian.

25. Examine the two vertical-axis levels, and see that they are properly mounted on the cradle in accordance with the instructions given before. Take the readings of one of the levels in the instrumental position telescope east: reverse the instrument and take them again with telescope west. If the vertical axis is vertical in the meridian, the reading of the north end in the first position will be the same as that of the south end in the second. If the reading of the north end in one position differs by more than two divisions from the reading of the south end in the other position, the axis is not sufficiently vertical: its verticality can be improved by unclamping the base-plate screw with its spanner, and then moving the lower end of the axis in the meridian by means of the north and south thumb-screws.

This adjustment will often get disturbed and have to be re-made during actual work, when there is but little time to spare for the operation: in these cases always *remember to clamp the base-plate screw*, after the verticality of the axis has been secured: in the hurry of the moment *it is likely to be forgotten*, and if the base-plate screw is not clamped, the inclination of the axis will be always varying.

* These four screws should be put in loosely, and gradually screwed home together, one or two turns being given to each in rotation. In dealing with delicate instruments this rule is universal. If the first screw be screwed home before the second is inserted, and then the second made tight before the insertion of the third, and so on, torsion may possibly result.

26. The readings of the transit axis level depend on:—(i) The position of zero of the level* itself. (ii) The inclination of the vertical axis in the prime vertical. (iii) The inclination of the horizontal axis to the vertical axis. The first step necessary when levelling the horizontal axis is to bring the zero of the level to the centre of its run: if this is not done, the bubble, which is a very long one, will generally be found to settle too much to one side to be read, and on reversal too much to the other, rendering it impossible to decide what the dislevelment really is.

Horizontality of the horizontal axis.

It is *useless* to attempt to level the horizontal axis, until the vertical axis has been made vertical in the prime vertical. Suppose in the position telescope east, the vertical axis has an inclination θ to the east: if the horizontal axis is now made level by lowering the bearing shafts in the back of the cradle, the two axes will be inclined to each other at an angle of $(90^\circ \pm \theta)$. The result of thus making errors compensate instead of eliminating them will be, that on changing the instrumental position to telescope west, the horizontal axis instead of remaining level will assume an inclination of 2θ.

To make the vertical axis vertical in the prime vertical, first make it so in the meridian, and then note what the *right* end of one of the vertical-axis levels reads, when the telescope is in the meridian: rotate the instrument through 90° in azimuth, and then by means of the east and west thumb-screws, the base-plate screw being unclamped, make the right end of the same level read as it did before. The two fixed stops limit the motion of the telescope in azimuth to 180°, and it cannot therefore be brought both to the north and south of the instrument: the levels consequently can only be read in one position in the prime vertical and cannot be reversed end for end, as is practically done when the instrumental position is changed. When the reading of the end of a level remains the same throughout the rotation of the telescope in azimuth from its eastern position to its western, the vertical axis is vertical both in the meridian and the prime vertical, and the base-plate screw may be clamped.

The striding level can now be applied to the horizontal axis, and the dislevelment of the latter corrected by means of the capstan-headed

* The zero of the level is the reading of the centre of the bubble in the position of no dislevelment, *i.e.*, when the level is applied to a truly horizontal surface.

spindle, which raises and lowers the two bearing shafts: the spindle should not be turned, until the large screw, that attaches the cradle to the table, has been loosened. On the completion of the adjustment this screw should be tightened at once, or the safety of the instrument may be endangered.

From what has been said above, it will be seen that the dislevelment of the transit axis, arising as it does from two independent sources, will not be the same in the two instrumental positions, and two distinct values must always be deduced for it. In determining their amounts apply the level to the horizontal axis in the position telescope east, and take its readings: reverse the level end for end and take its readings again. Now change the instrument to telescope west and read the level a third time: reverse it end for end and read it a fourth time.

Adjustment of the sector in azimuth.

27. The "flying stop," affixed underneath the table, arrests the rotation of the instrument in azimuth in both directions by coming in contact with one or the other of the fixed stops. If the instrument be adjusted to the meridian when the flying stop is in contact with one of the fixed stops, there may still be a large deviation error when the instrument is rotated, and the flying stop brought into contact with the other fixed stop. The two directions of the line of collimation in the two telescopic positions are not necessarily parallel, but have to be made so. An adjustment to the meridian is therefore necessary in *both* telescopic positions.

One of the faces of the flying stop is a vertical plane, and the other carries a horizontal screw. In one* telescopic position the plane face is in contact with a fixed stop and in the other the screw face. When adjusting the instrument in azimuth, it is always necessary *to begin* with that telescopic position, in which the *plane face* of the flying stop is in contact with a fixed stop: the reason is, that when the plane face is in contact, the only method of correcting the deviation error is to rotate the whole instrument by means of the azimuthal stud screw, whilst when the screw face is in contact, the deviation of the telescope can be altered by making the screw protrude more or less from the stop: the adjustment therefore in the former case would disturb that

* Whether the eastern or western depends on the position of the azimuthal stud.

in the latter, if it had been performed first. If the telescope be first placed in the meridian with the plane face in contact, the horizontal screw of the flying stop enables the range of the instrument between the two fixed stops to be adjusted to exactly 180°, and the elimination of deviation error in the second telescopic position does not disturb the adjustment in the first.

If the directions for putting up the sector have been complied with, it has already been approximately adjusted in azimuth by means of the meridian line that has been cut on the collimator pillar. The error of the chronometer too by this time should have been discovered to within a minute or two.

Find the exact chronometer error by timing the transit of a Nautical Almanac star, selecting one as near the zenith as possible and setting for it with the setting-circle. Now compute out the chronometer times of transit of a circumpolar star over all the wires in both instrumental positions: recollect that in one telescopic position it will transit the wires in the order I to VII and in the other from VII to I. The time of transit of wire IV should be the same in both positions: the time of transit of wire I with telescope east will, as stated before, be approximately the same as that of wire VII with telescope west, but not, however, sufficiently near to allow of the same time being adopted for both events.

Bring the plane face of the "flying" stop into contact with one of the fixed stops: about half an hour before the time of transit of the circumpolar star, set the telescope to the correct zenith distance. If the circumpolar is not in the field, slowly *sweep* with the telescope to the right and left: as the fixed stop arrests the rotation of the instrument in azimuth, the observer should take care to so arrange the deviation error, that the flying stop is able to travel a little *past* the prime vertical, and is not arrested before it; unless this precaution is attended to, the observer will be only able to sweep with the telescope on one side of the meridian.

In sweeping with the telescope, be careful that it is set to the exact zenith distance, corrected for refraction: a great number of circumpolars are often met with in sweeping, and one is very liable to observe a wrong one. In deciding whether it is the correct star in the telescope, three tests can be applied, *viz.*, zenith distance, rate of motion, and magnitude. The rate with which a star passes from one wire to another is an excellent test: the equatorial intervals of the wires are known, and it is easy to compute the time, that a circumpolar of given declination takes to pass. In November and May Polaris gives ample time to get the meridian in both telescopic positions, and this is a star that no one can mistake.

When the correct circumpolar is in the field, bring it amongst the wires by sweeping the telescope: it is not necessary at this period to time it over any particular wire: the times of transit over all the wires are known, and the observer can judge from them fairly accurately, whereabouts the circumpolar should be at any given moment. When the circumpolar is approximately in its correct position amongst the wires at a given moment, the telescope is in the meridian, but as the flying and fixed stops are not now in contact, the adjustment in azimuth will be lost immediately the telescope is moved. Of the two stops it is the "flying" one that is correctly placed in azimuth, and the fixed one has to be brought up into contact with it. To do this, unscrew the nuts of the six steel bolts, that connect the upper portion of the cast-iron pillar to its base-plate, and rotate the pillar by means of the azimuthal stud-screw. The telescope and vertical axis must be held steady with one hand, whilst the azimuthal stud-screw is being turned, or they will revolve with the cast-iron pillar, and the whole essence of the adjustment is to rotate the latter independently of the former. The pillar should be revolved by means of the screw, until the "fixed" stop has been brought round into contact with the "flying" one.

The slotting of the six bolt holes will only permit the upper portion of the pillar being revolved on the base-plate through about 2°, so that if the azimuthal stud-screw is brougnt to the centre of its run, when the instrument is first put up, 1° is the extreme deviation error, that can be corrected by means of it: if too 1° even were corrected for, the bolts would be hard down against one side of their slots, the screw would be at the very end of its run, and any small deviation error that might arise in the future could only be eliminated by rotating the whole instrument on the stone-cap. As it is very inadvisable to commence work with no room on the screw for further azimuthal adjustments, 30′ may be considered the maximum allowable angle in both directions through which the iron-pillar should be rotated when *first* adjusting the instrument. If the meridian has not been laid down accurately, or if the zenith sector has been carelessly put up, the observer may find on placing a circumpolar amongst the wires, that the deviation error exceeds 30′: if this is the case, (and he must judge whether it is or not by the distance between the fixed and flying stops, when the telescope is in the meridian) he will have to loosen the three foot-bolts, that connect the instrument to the stone cap, and revolve the whole sector by means of a crowbar placed under the base-plate.

When the circumpolar is approximately in its correct position amongst the wires, and the fixed stop has been brought by means of the azimuthal stud-screw into contact with the plane face of the flying stop, take another N. A. Star, and if the chronometer error now differs from its earlier value, correct all the computed times of transit of the circumpolar over the several wires in accordance. Re-level the vertical and horizontal axes. The adjustment in azimuth must now be perfected by making the circumpolar star transit some particular wire at the computed time. One minute before the time, the recorder begins to count seconds, and the observer brings the star on to the wire by means of the stud-screw: as the star moves off the wire, the observer constantly brings it back, and keeps it there, until the recorder reaches the computed time of transit. When a northern circumpolar is being taken, and the azimuthal stud is north, the observer cannot turn the screw himself; this makes the adjustment more difficult, as a third person, the tindal, is introduced: the observer, recorder and tindal should, however, with a little practice work well together and perform the adjustment without difficulty. Be most careful, when the stud screw is being turned at this final phase of the adjustment, to see that the two stops *are* in contact; *they are very apt not to be.*

Do not eliminate deviation error by moving the fixed stop in its channel round the neck of the pillar.

When the circumpolar transits a wire at the right moment, the meridian has been found and the nuts of the six steel bolts may be tightly screwed down. In screwing the nuts down to the bolts, the azimuth adjustment will be slightly and inevitably disturbed, but as it is always by the same amount and in the same direction, an experienced observer soon learns how to allow for it by placing the circumpolar at the computed time of transit not exactly on the wire.

When the cast-iron pillar is rotated by means of the stud-screw, the vertical and horizontal axes will often become disleveled: unless however both are nearly level, no accurate adjustment to the meridian is possible, and consequently it is desirable to frequently take the readings of the levels throughout the process. Not only does the azimuthal adjustment disturb the levels, but the act of levelling alters the deviation of the telescope.

When the instrument has been accurately adjusted in azimuth with the plane face of the flying stop in contact with a fixed stop, change the telescopic position and again observe the same circumpolar star. Bring the screw head of the flying stop into contact with the fixed stop, and decide whether the deviation error in the new position is to the east or west, and whether the stop-screw must be made to protrude more or less in order to bring the telescope to the meridian. As the time of transit over a wire draws near, make the recorder count, and gradually bring the star nearer to the wire by turning the screw of the stop. When the time of transit is reached, the screw should have been made to so protrude from the stop, that when it is in contact with the fixed stop, the circumpolar is transiting the right wire. When the stop-screw protrudes the correct amount, clamp it in its position by the second screw situated on the outer side of the stop. If this precaution to clamp is omitted, the constant concussion, that takes place in regular work between the flying and fixed stops, gradually drives the screw into the flying stop and increases the deviation. The sector has been now adjusted to the meridian in *both* telescopic positions and is ready for work.

Selection of a meridian mark.

28. When the sector has for the first time been accurately adjusted in azimuth, bring the telescope horizontal, and move the two collimators, until their crosses are cut by the centre wire, one in the position telescope east, the other in the position telescope west: these collimators, which are found in practice to steadily retain one position, will give the direction of the meridian in both telescopic positions on any occasion, that a circumpolar star owing to clouds or other causes cannot be observed. If collimators have not been taken into the field, point the telescope, when adjusted in azimuth, to the horizon, and select some terrestrial meridian mark to serve for future reference. As a prominent natural object is not likely to exist exactly on the meridian, the observer will find it advisable to draw on paper, what he sees through the telescope, marking the position of the centre wire on his drawing: if he omits to do this, he may fail to recognise the view again.

29. Every night *before* work the vertical and transit axes must be accurately levelled, the microscopes examined for focus and parallax, the illumination of the limb tested, and the deviation of the telescope in both its positions from the meridian corrected by reference to the meridian mark or collimators. Before commencing work too, *always* see that the black counterpoise hangs freely and exerts its whole force: its suspension bar is apt to jam against the table.

Adjustments to be made every night.

30. The limb of the sectors extends through such a small arc, that a greater zenith distance than $17\frac{1}{2}°$ cannot be read: all stars, whose zenith distances are to be observed, must therefore be situated near the zenith.

How to observe the zenith distance of a star.

It is a fundamental principle in working with the zenith sector that no observation is to be accepted as complete, unless the star has been intersected in both telescopic positions: when a star has been intersected in one position only, the observation *must be rejected.* Intersection of the same star in both positions eliminates the zero error of the sectors: it is wholly inadmissible to take a single intersection of a star and correct it for zero error, the amount of the correction being determined from a double intersection of another star. The zero error of the sectors is not a constant quantity, but varies with the dislevelment of the vertical axis.

The procedure for observing a star for latitude is as follows:—

(i) Set the telescope to the correct zenith distance of the star by means of the Index microscope of the sectors; complete the setting to greater accuracy with the A microscope, and clamp the sectors.

(ii) Make the setting-circle read the same zenith distance, but the reverse way; that is, set the vernier to the proper reading *below* the zero of the setting-circle limb. The bubble will then be at the centre of its run when the telescope is inclined at the same angle to the vertical as it is in its present position, but on the opposite side of the zenith.

(iii) Bring the flying stop into contact with one of the fixed stops.

(iv) $2\frac{1}{2}$ minutes before the star is due, read the four microscopes, and then the two levels.

(v) At one minute before transit the recorder gives warning and the observer seats himself.

(vi) When the star is exactly *midway* between the first two vertical wires, intersect it with the horizontal wire and take the reading of the micrometer.

(vii) Rise up, unclamp the sectors, pass the telescope through the zenith, bring the bubble of the setting-circle to the centre of its run by tapping the telescope to the correct altitude, clamp the sectors, and rapidly rotate the instrument 180° in azimuth, bringing the flying stop up into contact with the second fixed stop *without jerk or jar*.

(viii) Again sit down and bring the star close to the horizontal wire by moving the telescope with the tangent screw of the sectors.

(ix) *Unclamp* and *re-clamp* the sectors.

(x) Intersect the star again with the horizontal wire, and take the second reading of the micrometer.

(xi) Read the two levels, and *lastly* the four microscopes. The observation is then completed.

Precautions to be taken when observing.

31. During an observation, take care that the flying stop *is* in contact with a fixed stop: it is apt not to be. Beware too in the hurry of the moment of reading the levels, *before* the bubbles are at rest.

Between the first reading of the microscopes and the first intersection of the star, no tangent-screw nor any other part of the instrument may be touched. The second intersection may be approximately made with the tangent-screw of the sectors, but when completed with the micrometer, the tangent-screw must not be touched again, till the final reading of the microscopes has been taken. The reason of the second intersection being first made approximately with the tangent-screw is, that the two intersections are then taken at equal distances from the zero of the telescope comb, and as the two readings of a *complete* observation are supplements of one another, error in the value of the telescope micrometer is cancelled.

Colonel Herschel, who knew the zenith sector well, laid great stress on the importance of invariably unclamping and re-clamping the sectors, after their tangent-screw had been used; "this step," he wrote, "should "become an acquired habit and not be left to memory: the observer's "hand after quitting the tangent-screw should at once move by instinct "to the milled heads of the clamping cam."

The equatorial interval between wires I and II is about 14 seconds of time: between each of the pairs II and III, III and IV, IV and V, V and VI it is about 7 seconds: between VI and VII it is again about 14 seconds. The time that elapses between the two intersections of one star is consequently 42 seconds, the first intersection being made 21 seconds *before* the centre wire, and the second 21 seconds *after*. For an observer who knows his work, 42 seconds is ample time to do all that has to be done.

It may often happen, owing to accidents, mistakes or clouds, that the first intersection or the second or even both are not made exactly midway between the two first or two last wires, and consequently not at 21 seconds from the meridian. In these *exceptional* cases the observer must cause the recorder to note on the record the distance in time from the centre wire that the intersection was made, so that in the computations the correction for "intersection off the centre" may be applied for the actual amount and not for the usual 21 seconds. Knowing the intervals between the inner consecutive wires to be 7 seconds each, and the two outer intervals to be 14 seconds, the observer can estimate fairly accurately at what distance from any particular wire he has made the intersection. Suppose, for instance, he makes the intersection, not as he ought midway between wires VI and VII, but just before the star transits wire VII: he says to the recorder "intersection made at VI + 13^s." Suppose again that he fails to intersect the star until it has passed wire VII by a distance slightly less than half one of the inner wire intervals, he informs the recorder "intersection at VII + 3^s." In the first of these hypothetical cases the correction for "intersection off the centre" would be for 27 seconds instead of 21, and in the second for 31 seconds.

When stars have been observed once, their correct readings on the sector should be taken out, and these latter used when setting on future occasions: it is desirable to *set* as accurately as possible, so as to keep the readings of the telescope micrometer within 30 or 40 divisions of its zero.

To reverse the instrument from "stud north" to "stud south," or *vice versâ*.

32. Suppose the instrument to be in position and in perfect adjustment with its azimuthal stud to the north. If there is no prominent meridian mark easily recognisable through the telescope, bring the latter down horizontal, and make a careful drawing of the distant terrestrial view as seen through it, marking the position of the centre wire on the drawing. If collimators are being used, and their crosses are not exactly on the centre wire of the telescope, when pointing in the meridian, do not trouble to move the collimators, but make a drawing of the wires of the telescope in both positions and mark on each the exact situation of the collimator crosses. These drawings are necessary to enable the meridian to be re-found after reversal, as during that process all adjustments are upset: it is safer to trust to a drawing than to memory.

Take out the three foot-bolts: place a crowbar under the foot *nearest* the telescope, and raise that side of the instrument: insert a wooden wedge under the foot, so as to raise the latter ¾ of an inch from the stone, and remove the crowbar. Insert the hemispherical steel button under the brass-nut in the centre of the base-plate. Remove the wedge. Rotate the instrument on the button through 180°. Again lift with the crowbar and insert the wedge. Remove the button, and then the wedge, lowering the instrument very gently with the crowbar on to the stone.

Bring the plane face of the flying stop into contact with a fixed stop: level the instrument fairly accurately by placing washers under the feet, and then shift it approximately into the meridian with the crowbar. Screw up the foot-bolts tight, and level again with the *four* thumb-screws. Point the telescope to the distant horizon or collimator, unscrew the nuts of the six steel-bolts and rotate the instrument in azimuth by means of the stud-screw, until its centre wire has taken up the same position in regard to external objects, as it occupies in the drawing made before reversal, when deviation error will have been eliminated. Screw down the nuts of the six bolts: re-level with the *four* thumb-screws: re-level the horizontal axis by raising or lowering the bearing shafts.

The steel button is of such a height, that when the zenith sector is balanced on it, the feet of the latter are raised about $\frac{1}{20}$th of an inch off the stone-cap. Care should be taken to place the button well in the centre of the brass nut, for if placed near the edge, the instrument might be rotated off it, and badly jarred.

To dismantle the sector.

33. To dismantle the sector unscrew the eye-piece and screw on the eye-end cover. Remove the dew-cap and screw on the telescope cap. Tie up the spring of the clamping arm with string. Loosen the clamping cam and tangent-screw of the arm attached to the cradle. Turn the telescope horizontal and have it supported by two men. Remove the bridge-piece. Tie on the wooden guides marked (1). Have the telescope-box brought near: see in which direction the telescope has to point when in the box, and arrange the latter accordingly: place the lid with its truncated pyramid on the box but without screwing it down. Remove the black counterpoise, and tie on the wooden guides marked (2). Remove the mushroom counterpoise and affix the conical wooden cap.

Three men must now pull at the telescope, whilst a fourth supports the outer end of the horizontal axis by means of the wooden cone: when the telescope has been pulled out some 4 or 5 inches, the horizontal axis must be supported between the cradle and sectors. The telescope and horizontal axis may now be entirely removed from the cradle, no one being allowed to touch the sectors.

Rest the telescope on the truncated pyramid of the box, and remove its clamping arm; wipe all oil from the horizontal axis; take out the four large screws that attach the sectors to the telescope, and rotate the former until they are parallel to the latter: replace the four screws. Turn the wooden-tipped handles of the sectors, so that they may lie in one of the diagonals of the box, when the telescope is packed. Place the telescope, sectors, and horizontal axis in the box, lowering it with every care.

Before packing the remaining parts of the instrument, wipe the oil off and thoroughly clean the cradle-bearings, vertical axis, hemispheres, and base-plate cup.

The stone-cap of the pillar must be taken with the instrument to the next station.

Rules by which the selection of stars for latitude observations ought to be governed.

34. The geodetic latitude of every station is known to the observer, and as it seldom differs by more than a few seconds from the astronomical latitude, the distance at transit of any star from the zenith can always be calculated with sufficient accuracy to allow of its being found with the telescope.

To observe 40 zenith stars for latitude is a hard night's work: to observe 30 is easy. The number of stars, included in a programme of work for *one* night on latitude operations, should therefore be about 36.

In selecting these 36 stars the following conditions should be fulfilled:—

(i) All stars must be taken from the latest Greenwich Catalogue.

(ii) No star is to be considered sufficiently trustworthy for observation unless its north polar distance is shewn in that catalogue as determined by at least six observations.

(iii) No star is to be included that has not a proper motion in north polar distance assigned to it in the Greenwich Catalogue.

(iv) Double stars and stars of the 1st and 2nd magnitude should be avoided.

(v) No star should be included that has a greater zenith distance than 13°.

(vi) The number of north stars must be the same as the number of south.

(vii) The mean zenith distance of all the north stars must not differ from the mean zenith distance of all the south by more than half a degree.

(viii) Stars that are 8° from the zenith or more should be paired as nearly as possible.

(ix) The minimum difference in right ascension between two consecutive stars should be six minutes.

(x) Two to four Nautical Almanac stars equally distributed north and south of the zenith should be included to enable the chronometer error to be determined.

(xi) Two to four stars within 1° of the zenith should be included to enable the collimation error in azimuth to be determined*.

35. The programme of work for one night is made up as follows :—

Programme of work for one night.

(i) The zenith distance of about 36 zenith stars must be measured. The stars being selected in accordance with the above rules, and each observed in both telescopic positions.

(ii) The time of transit of a circumpolar star, whose right ascension has been well-determined, must be taken over two wires in each telescopic position: from this is deduced the deviation error of the instrument.

(iii) The transit-axis level must be read before and after work in both telescopic positions, being reversed end for end in each position.

(iv) The two thermometers outside the tent should be read every 15 or 20 minutes.

(v) The barometer should be read every hour, the mercury in the cistern being lowered and raised again to the zero pointer each time.

(vi) The microscope run must be determined 15 times, the same 5′ space being never utilised twice; the high reading should be always recorded *above* the low, whether read first or not.

(vii) It is of importance, that the temperature of the interior of the observatory tent should not differ from that of the

* These eleven rules for governing the selection of the stars were laid down after much consideration by Colonel Heaviside, and have been closely adhered to by subsequent observers. Any departure from them will be surely followed by discordance in the results.

outside air by more than 1° or 2° Fahrenheit: the thermometer attached to the barometer should therefore be *occasionally* glanced at, but its readings need not be recorded.

(viii) The error of the chronometer and the collimation error of the telescope in azimuth have to be found nightly, but no special observations are needed for their determination, if in the selection of the stars attention has been paid to rules x and xi given in paragraph 34.

The observer is *again* reminded here of the necessity of noting on the record, *which* graduation of the sector limb he intersects with his microscopes, (*vide* para. 23) and *how far* from the centre wire, he intersects his stars with the telescope micrometer, (*vide* para. 31).

Programme of work for one station.

36. It must be borne in mind that the object of these observations is the determination of astronomical latitudes and not merely of zenith distances, and that therefore the accuracy of the results is affected by errors in north polar distance as well as by those of observation. The mean of *two* zenith distance observations with the sector is, as a rule, a more reliable measure than the north polar distance of a star as computed from the Greenwich Catalogue, a fact, that speaks very highly of the sector as an astronomical instrument.

Supposing an observer to devote four nights to one station and to take 40 observations per night, it has been found that the probable error of his final value of latitude will be

± 0″·063, if he observes the same 40 stars every night.

± 0″·049, if he observes 40 stars the first two nights and 40 different ones the last two nights.

± 0″·039, if he observes 40 different stars every night.

In each of these three cases the number of observations taken on the whole and the total amount of work carried out will be the same. The result attained by observing 40 fresh stars on each of the four nights will evidently be very much better than if 80 stars are observed twice each. It is moreover easy of proof that but slight advantage accrues at all from taking a third zenith distance *per* star, and that ***a fourth such observation is absolutely waste of time.***

It has been found that the zenith distance of a star when observed E to W is not the same as when observed W to E;* the difference between the two values is the same for every star, but varies with different observers. The existence of such a difference is probably due to the observer's personality, and necessitates every star being observed twice, once from E to W, and once from W to E: though these two observations may differ by $0''\cdot5$, their mean deserves the same weight as if they had been identical, personal error being in all probability entirely cancelled in it.

In Northern India as many suitable stars as can possibly be needed at any latitude station may be obtained; but south of the 18th parallel of latitude, it is difficult to get more than 35 to 40 well-placed south stars from the Greenwich Catalogue. An observer cannot of course work all night, but by commencing on one night shortly after sunset, on another at 12 midnight, and on a third at three o'clock in morning, he will be able to observe almost all the suitable stars, that transit the meridian, near the zenith, between sunset and sunrise.

Taking all these facts into consideration, the following may be laid down as the proper programme of work for a latitude station:—

(i) From 70 to 100 stars should be observed, 100 if possible.

(ii) Each star should be observed once E to W and once W to E. In order that this should be done without confusion, commence a programme of stars on its first night by taking the first star E to W, the second W to E, and so on alternately: on the second night take the first star W to E, the second E to W, and so on. The only reason for describing such an obvious method is to remind the observer, that when he misses a star on either night, he must change the telescopic position before taking the next, for unless he does so, every star after the miss, that was observed E to W the first night, will be observed E to W the second.

(iii) Four or six nights must be devoted to each station, an even number being advisable: four would suffice, if there was a paucity of stars and no misses occurred; but six are better,

* "Observed W to E" means that the first intersection of the star was made with the telescope west, and the second intersection with the telescope east.

as they allow of stars missed once being observed in their second direction E to W or W to E.

(iv) When half the observations have been secured, the instrument should be reversed, *i.e.*, if the azimuthal stud has been originally placed north, it should now be brought round to south.

(v) If only 70 stars are to be observed, they should be divided into two programmes of 35 each: during the first night the first programme should be worked through, during the second the second programme, and during the third those stars of both programmes, that have been missed, should be observed: on the morning of the fourth day the instrument should be reversed, and in the evening the first programme again worked through, those stars taken E to W on the first and third days being now taken W to E and *vice versâ*: on the fifth night the second programme should be again taken up, the direction E to W and W to E being changed for each star from what it was on the second night: on the last night, those stars of both programmes, that had been missed during the fourth and fifth nights, should be observed.

(vi) If 100 stars are to be taken they should be divided into three programmes: each programme should be worked through once with the azimuthal stud north, and once with it south: those stars observed E to W on the first occasion of each must be taken W to E on the second and *vice versâ*: As three programmes have to be got through in six nights, no spare nights are available for picking up misses, but by judicious interchanges of stars between the three a star missed from one programme can often be observed again in one of the other two.

(vii) The number of observations per station need not exceed 200, and should not be less than 140.*

The errors of the chronometer, collimation, inclination, and deviation during any night's work ought to be computed out before the next night's observations are commenced; the methods of computing them

* It will be found as a rule that all the observations taken on any one particular night will be burdened with some small constant error, running throughout; the more nights therefore that the observations are extended over, the better will be the final result.

have been therefore included under the heading "duties in the field" and not "duties in the recess," and are now described.

Chronometer error.

37. The chronometer error is only required for computing the deviation of the instrument from the meridian. It is found as follows:—Suppose the instrument is so placed for the first intersection that wire VII is the wire first reached by the star: wire VII will then be the last wire crossed by the star after the second intersection, the telescopic position having been changed between the two. When the observer seated at the telescope sees the star come into the field, he makes the recorder count seconds from the chronometer: as the star crosses wire VII, he notes the nearest half-second of time. Half-way between VII and VI he observes the zenith distance by intersecting the star with the horizontal wire and reading the micrometer-head. He then changes the telescopic position of the instrument and intersects the star again midway between VI and VII. The recorder again counts seconds, and the observer notes the half-second of time, that the star again transits wire VII: after this he reads off the micrometer-head, which should not have been touched since the last intersection. The mean of the two recorded times of transit gives the chronometer time, at which the star crossed the centre wire of the telescope. The difference between this mean and the right ascension of the star is the chronometer error: if the right ascension is the larger, the chronometer is slow.

The right ascension of a star, that is to be used for finding the chronometer error, should be accurately known: it is therefore advisable to always select one from the Nautical Almanac. If a star that is suitable for latitude observations is selected from the Nautical Almanac, it can be observed for zenith distance and time simultaneously as described above. If no Nautical Almanac star is included in the programme of latitude stars, special observations must be taken for the purpose of determining the chronometer error.

In timing the transit of a star over wire VII in both telescopic positions, error of collimation cancels out. Error arising from inclination of the transit axis does *not* cancel owing to the inclination being different in the two telescopic positions*; it should, however, be kept so

* *Vide* description of method of levelling the transit-axis, (para. 26).

small as to exercise no appreciable effect on a time of transit. This fact of non-cancelment should be borne in mind, for if the inclination were large, and the same end of the axis happened to be the higher in both telescopic positions, a correction would have to be applied to the two chronometer times of transit. To cancel the effect of deviation error on the times of transit, the chronometer error should be determined nightly from the observation of both a north and south star, or better still of two north and two south stars.

Inclination of the transit-axis.

38. A knowledge of the inclination of the transit-axis is necessary (i) for determining during the field season both the collimation and deviation errors of the instrument, and (ii) for computing during the recess the distances from the meridian, at which stars were intersected and the resulting corrections to be applied on that account.

To determine the inclination of the transit-axis, the striding level is read both with "telescope east" and "telescope west" and reversed in each position.

There are consequently eight readings, *viz.*,

With telescope east	Cross-level east	East-end of bubble	$= E_1$,
		West-end ,,	$= W_1$,
	,, west	East-end ,,	$= E_2$,
		West-end ,,	$= W_2$,
With telescope west	Cross-level east	East-end of bubble	$= E_3$,
		West-end ,,	$= W_3$,
	,, west	East-end ,,	$= E_4$,
		West-end ,,	$= W_4$.

If n = value of one division of the level-scale in seconds of arc, the dislevelment will be

$$\frac{(W_1 + W_2) - (E_1 + E_2)}{4} \times n \text{ with telescope east,}$$

and

$$\frac{(W_3 + W_4) - (E_3 + E_4)}{4} \times n \text{ with telescope west,}$$

in each case a positive sign denoting that the western end of the axis was the higher. This dislevelment has to be calculated every morning and reduced if large.

The *correction** for inclination to the time of transit of a star would be

$$+ \frac{(W_1 + W_2 - E_1 - E_2) \times n \cos \zeta \sec \delta}{4 \times 15}$$

with telescope east, and

$$+ \frac{(W_3 + W_4 - E_3 - E_4) \times n \cos \zeta \sec \delta}{4 \times 15}$$

with telescope west, where ζ and δ denote the zenith distance and declination of the star.

Collimation error in azimuth.

39. The collimation error has to be taken account of (i) in determining during the field season the deviation of the instrument from the meridian, and (ii) in computing during the recess the distances from the meridian, at which stars were intersected and the resulting corrections to be applied on that account.

The collimation error is found by timing the transit of a star, whose zenith distance is less than 1°, over the same wire in both telescopic positions: an ordinary latitude star should be used for the determination, as special observations for the purpose are then saved. The method of both timing the transit and intersecting the star in the two telescopic positions, that has already been described under the heading "Chronometer error," should be followed in finding the collimation error also. The only difference between the two is, that in the case of the chronometer error, the star may be at any zenith distance, but must have a well determined right ascension; whereas in the case of the collimation error, it must be close to the zenith, but need not be a Nautical Almanac star. Collimation stars have to be selected as close to

* If the western end is higher these expressions are positive, and as a star transits too soon with the west end high, a positive correction is wanted.

the zenith as possible, in order to avoid the observed times of transit being affected by the deviation error of the instrument. It is necessary to observe every night *at least* two collimation stars: one of these should be taken E to W, the other W to E.

In one telescopic position wire I is the first wire crossed and wire VII the last: in the other telescopic position wire VII is the first wire crossed and wire I the last. Suppose in the position telescope east wire I was the first wire crossed by a zenith star, and that the time of transit (corrected for the inclination of the transit-axis level) was $2^h\ 0^m\ 56^s\cdot 0$: suppose now in the position telescopic west, after the second intersection of the star has been made, the corrected time of transit over wire I, which has become the last wire, is again noted and is $2^h\ 2^m\ 2^s\cdot 0$. The elapsed time between the two transits is 66·0 seconds.

The equatorial interval of wire I is known: suppose it equal to $28^s\cdot 55$. The time, that the star would take to travel from wire I to wire IV, is equal to $28\cdot 55 \times \sec.\ \delta = 31\cdot 0$ seconds suppose, where δ = declination of star. Now, if there were no collimation error in the telescope, the centre wire would be transited by a star at the same time in both telescopic positions, and the interval that would elapse between its two times of transit over wire I would be $2 \times 31\cdot 0 = 62\cdot 0$ seconds. The collimation *error* in seconds of arc is

$$\left(\frac{62\cdot 0 - 66\cdot 0}{2}\right) \times 15 \cos\delta = -\ 27''\cdot 3.$$

In the above case the *actual* interval is *too long*, and therefore the system of wires is crossed *too soon* and is situated too far east with telescope east, and crossed too late and situated too far west with telescope west.

If the collimation star had been observed from W to E, the *first* time of transit would have been $2^h\ 1^m\ 0^s$, and the *second* $2^h\ 1^m\ 58^s$: the elapsed interval would then have been 58 seconds or 4 seconds *too short*. The collimation error in seconds of arc would then be

$$\left(\frac{62\cdot 0 - 58\cdot 0}{2}\right) \times 15 \cos\delta = +\ 27''\cdot 3.$$

The two values of the collimation error are the same but their signs different: this is right enough, for if a star takes too long a time when observed E to W, it will take too short a time when observed W to E.

The following rule will prevent confusion of sign in correcting transits for collimation :—

Let c″ represent the collimation constant for the correction of times of transits of stars, or in other words the quantity that has been called above the collimation error, but with a reverse sign: then $c'' = \frac{x-y}{2} \times 15 \cos \delta$, where x = observed interval, y = correct computed interval, both in seconds of time: if c″ is positive, the wires are too far east for that position of the telescope, *in which the first of the two intersections are made*, and the correction to a transit taken in that position is positive. For example in the first instance given above, $c'' = +27''\cdot 3$ when the star was taken E to W: the correction for collimation to the time of transit of a circumpolar, of declination 85°, which is being observed for deviation, would be $+\frac{27\cdot 3}{15} \sec 85°$, when the transit occurred with telescope east, and $-\frac{27\cdot 3}{15} \sec 85°$, when it occurred with telescope west. Transits observed in that position, in which the *first* intersection was made, *when collimation was determined*, take the sign of their correction as given by the formula for c″: transits observed in the other position take the opposite sign from c″.

40. The deviation of the telescope from the meridian has to be determined nightly: it has to be taken account of in the recess in computing the distances from the meridian, at which the stars were intersected. It is necessary to compute out the deviation error every morning, and to re-adjust the instrument, if it is found large.

Deviation error.

The deviation error is determined as follows:—The times of transit of a circumpolar star are noted at night over one or two wires in both telescopic positions. These times are "reduced to the centre" and the times of transit over wire IV in the two positions thus found. Corrections to the transits of wire IV have then to be applied, (i) for chronometer error, (ii) collimation, and (iii) inclination of the transit-axis. The corrections for chronometer error will have the same sign, when applied to the transits of the circumpolar in the two positions: the corrections for collimation error will have opposite signs: the corrections for inclination may have the same or opposite signs, as the errors in the two positions are semi-independent of one another. Let T = the corrected time of transit, δ = declination of the circumpolar, and ζ = its zenith distance; then $15\,(T - R.\,A.)\cos\delta\,\mathrm{cosec}\,\zeta$ = deviation of the instrument.

If T is greater than R. A., and the circumpolar has been observed at its $\frac{\text{upper}}{\text{lower}}$ culmination, the deviation error is $\frac{\text{west}}{\text{east}}$ of north.

Every night, after the time of transit of a circumpolar has been taken, the telescope should be brought down and the position of the collimator cross noted. The distance of the cross from the meridian can be computed, when the deviation of the centre wire is known. Then on any night when the pole is overclouded and no circumpolar observed, the distance between the centre wire and the collimator cross should be estimated accurately: if from the calculations of the preceding and succeeding nights, the collimator cross is found in the same position on both, it is but reasonable to assume that it has not moved in the interim. On a cloudy night therefore the deviation can be found from the observation of the collimator cross, whose distance from the meridian is known. If collimators are not being used, the distance of the centre wire from the meridian mark should be estimated every evening at sunset: if no circumpolar has been observed, this distance should be again estimated at sunrise: if no serious difference is found, the instrument may be considered to have remained unmoved in the interim, and the deviation error may be deduced from the position of the meridian mark. It should be borne in mind, however, that the collimators and the meridian mark are really intended solely for purposes of adjustment and not for determining deviation error, and that they should be used for the latter in cases of emergency only: there is always a chance that a collimator may have moved and returned to its original position between two observations made to it. The adjustment of the instrument in azimuth, and the determination of the deviation error are two distinct operations and must not be confused with one another.

CHAPTER III. DUTIES IN RECESS.

Under the head "duties in recess" are included the reduction of the observations and computations of the final results.

Description of stations.

1. The computations should always be prefaced by a description of each latitude station. The dimensions of the pillar should be given, and its situation with reference to the trigonometrical station described. The distances and directions of the neighbouring villages should be noted, so that no difficulty may afterwards be experienced in finding the site: if the station be situated in a dense jungle or on a precipitous hill difficult of access, the best road leading to it should be mentioned.

The observer should at the time of observation estimate the direction, in which he would expect a deflection of the plumb-line, and put on record the reasons for his opinion. With the assistance of maps he can tell the distances and masses of the neighbouring mountain ranges and hills; from the direction of the rivers he can ascertain the general slope of the ground in the vicinity; and from the Geological Survey records he can find the character of the soil of the surrounding country.

The computations may be divided into three principal parts:—
(1) Zenith distance of each star. (2) North polar distance of each star.
(3) Resulting latitude.

(1). *Zenith distance of a star.*

Before proceeding to describe the computations it may be as well to note, that each star observed is supposed to have been intersected both in the position telescope east and in the position telescope west.

The microscope readings, and "run" correction.

2. The first step in the computations is to take out the means of the four microscope readings, and to then correct these means for "run." A mean of the four readings has been designated c, and the "run" correction c'.

The amount of the "run" has first to be determined from the observations taken for the purpose, of which there ought to be from 70 to 90 per station. Suppose from all these we find that 299·736 micrometric divisions* are equal to a 5′ division on the limb: then 299·736 divisions are equal to 300 seconds of arc, and the correction to the reading, when the micrometer has been run over a whole 5′ space, is 0″·264. If the micrometer has been only run over a 1′ space on the limb, the correction to its reading will be $\frac{0''\cdot 264}{5} = 0''\cdot 053$, and the correction to be applied to *one* division of the micrometer to obtain its value in seconds of arc will be 0″·00088.

The above shews how the *amount* of the "run" correction is found: its *sign* is determined by the three following rules, always supposing that the *run is too small*, (*i.e.*, that one micrometric division is larger than a second of arc) :—

(i) In the case of an observation, where a run has been taken, and both the "high" and "low" readings are recorded, the "high" reading only should be used in deducing the zenith distance: the sign of the run correction in that case is + ve.

(ii) When a note has been made in the margin of the record thus "referred to *n*′", meaning that the *n*′ division of the limb has been intersected, the sign of the run correction is + ve, if the "reference minute" is less than the reading of A microscope. If the "reference minute" is greater than the reading of A microscope the sign is — ve.

(iii) In all other cases, when the reading of A microscope falls between 0′ 0″ and 2′ 30″ or between 5′ 0″ and 7′ 30″, the sign is + ve., and when between 2′ 30″ and 5′ 0″ or between 7′ 30″ and 10′ 0″, the sign is — ve.

There is one troublesome, but happily very rare exception to the third rule: it is when the reading of A microscope falls so closely *below* a division of the limb, that the mean of the four microscopes is *above* that division: the sign is then obviously + and not —.

Instrumental zenith distance, and zero error.

3. By the "instrumental" zenith is to be understood the intersection of the vertical axis of rotation with the celestial sphere. If the telescope is directed to this point, it will continue to point to it, when the instrument is

* A micrometric division here means the mean value of one division of each of the four microscopes.

turned about its axis, and the reading on the circle will remain the same. This reading has been designated the zero error ($= \omega$), and is considered positive, when it makes positive arcs too great.

We will start with the assumption that latitude = declination + zenith distance *always*, so that north declinations and south zenith distances are positive, and south declinations and north zenith distances are negative. Facing the limb the circle reading of the zenith sector decreases, as the object-glass rises on the right hand or sinks on the left: consequently when the telescope is east of the pillar, circle readings are positive, and when west they are negative, as regards the sign of zenith distance. The graduation is continuous from 0° to 360°, and the zero may be placed at either the zenith or nadir.

One division of the telescope-micrometer corresponds very closely to $\frac{3}{7}$ths of a second of arc; it is in fact exactly equal to $\frac{3}{7} \times 1''\cdot0007$. The reading of the telescope-micrometer at any intersection reduced to seconds of arc is designated by the symbol M. Suppose $N = c + c' + M$, N_E denoting that the readings c and M were taken for the intersection in the position "telescope east", and N_W for the intersection in the position "telescope west." Let Z = the distance of a star on the meridian from the "instrumental zenith."

Then $N_E = \omega + 180° + Z$, and $N_W = \omega + 180° - Z$, if the zero is at the nadir; and $N_E = \omega + Z$, and $N_W = \omega + 360° - Z$, if the zero is at the zenith. In both cases $Z = \frac{1}{2}(N_E - N_W)$, and $\omega = \frac{1}{2}(N_E + N_W) - 180°$.

The zero error does not enter in any way into the computations: it should be taken out by the recorder at the time of observation merely as a check against gross errors. It may also be *occasionally* employed as a criterion in rejecting observations, but great caution must be exercised in putting it to this use, as it is not even theoretically a constant quantity, but varies with the verticality of the vertical axis in the meridian.

Correction for inclination of the vertical axis.

4. The instrumental zenith distance has now to be corrected for the inclination of the vertical axis. This inclination is measured by two independent levels, and the results by both should agree closely, the mean of the two being adopted as the correction.

If the north end of the levels is too high, a south zenith distance is too small, and a north zenith distance is too large: having regard therefore to the signs of a zenith distance, the correction in both cases should be + ve. The proper sign will therefore be always attained in every case by deducting the south readings from the north. Suppose s = south reading of bubble, and n = north reading, and let $S_E = \overline{n-s}$, in the position "telescope east," and $S_W = \overline{n-s}$ in the position "telescope west". Then the correction for inclination will be $(S_W + S_E) \times \frac{x}{4}$, where x'' = one division of the level-scale. (In the printed forms for reduction S_W and S_E are put equal to $s - n$, and the correction made $-(S_W + S_E) \times \frac{x}{4}$: the result is the same).

The zero error of each level should be taken out for each star, for though it does not enter into the computations, it serves as a check against gross mistakes in reading the bubbles. In accordance with the system of signs adopted above the zero error is equal to $\frac{S_W - S_E}{4}$.

5. The instrumental zenith distance, when corrected for the inclination of the vertical axis, may be designated the *observed* zenith distance. Two corrections have to be applied to this observed zenith distance, *viz.*, (1) For refraction, (2) For intersection off the centre.

Observed zenith distance.

6. In computing the corrections for refraction the first step necessary is to find the readings of the barometer and thermometers at the time of observation of each star. A list of the stars observed on any one night is prepared, and their right ascensions to the nearest minute written against them. The corrected* reading of the barometer and the mean corrected* reading of the two thermometers are entered opposite those stars, which were observed immediately before the pressure and temperature were noted: barometric and thermometric readings are then interpolated between those actually taken during the night; the variation in both is considered proportional to the elapsed time, and this latter is derived from the right ascensions.

Correction for refraction.

* *i.e.*, corrected for index-error.

The actual correction for refraction is computed for each star by means of Bessel's three (Greenwich) tables. The argument with which to enter the first table is the zenith distance of the star, that for the second the barometric pressure, and that for the third the thermometer reading: a logarithmic number is obtained from each, the sum of the three being the logarithm of the "refraction."

The correction for refraction always *increases* a zenith distance, and is therefore—having regard to the signs of north and south zenith distance—*negative* for north stars and *positive* for south.

Intersection off the centre.

7. The observed zenith distance has to receive a correction on account of the extra-meridional position of the star at the time of intersection, due, in the first place to the observations being purposely and of necessity* made off the centre, and in the second, to imperfect adjustment of the instrument. At no step in the reduction of the results is an observer so likely to fall into error, as when computing this correction, a portion of the work that must fall to his personal share.

There are four causes operating, which prevent a star being intersected *on* the meridian: they are (i) Intentional intersection off the centre. (ii) Collimation error. (iii) Inclination of the transit-axis. (iv) Deviation error.

The mistake of supposing that the effect of collimation error is cancelled, if an intersection be made in both telescopic positions, can only arise from oversight; the result of collimation error is to cause the zenith distance to be measured on a *small* circle parallel to the meridian instead of on the meridian; the small circle is in one telescopic position east, and in the other west, of the meridian, but at the same distance from it in both.

There is another and more serious mistake which is very likely to occur in computing the correction for "intersection off the centre": it is to assume that the difference between a meridional and an extra-meridional zenith distance of any star depends only on the distance from the meridian, at which the star is intersected. Having made this assumption, the observer *combines* the four causes, that lead to

* In order to give time for the instrument to be rotated and for each star to be intersected in both telescopic positions.

extra-meridional intersection and computes the total hour-angle of the star at the instant of its observation: he then substitutes the seconds of the hour-angle in some formula that is applicable to either deviation or collimation only, and determines a wholly erroneous correction to the zenith distance.

It should be borne in mind that the difference between a meridional and an extra-meridional zenith distance depends not only on the distance from the meridian, at which the intersection is made, but also on the displacement of the zenith, and the position of the particular circle of the sphere, on which the measurement is made.

In the case of intentional intersection off the centre, as also in that of collimation error, the zenith is displaced laterally in the prime vertical, and the measurement is made on a small circle parallel to the meridian: in the case of deviation error the measurement is made on a great circle cutting the meridian in the vertical line, and the zenith is not displaced at all: in the case of dislevelment of the transit-axis the zenith is displaced laterally in the prime vertical, and the measurement is made on a great circle cutting the meridian in the north-and-south diameter of the horizon. In no case will the correction to north and south zenith distances be the same, as the path of a north star is convex and that of a south concave to the zenith.

From what has been said above, it will be seen that intentional intersection off the centre and collimation error *must* be treated together: the correction to a zenith distance (ζ) depending on these two causes is expressed by the following formula:—

$$d\zeta_{(k+c)} = \frac{15^2}{2}(k+c)^2 \tan\delta \sin 1'',$$

in which $(k + c)$ must be entered in seconds of time. This correction is the same in amount but opposite in sign for north and south zenith distances: it decreases the former as observed, and increases the latter. As north zenith distances have been considered negative and south zenith distances positive, the general formula* for the correction involving both cases may be written

$$d\zeta_{(k+c)} = + \frac{15^2}{2}(k+c)^2 \tan\delta \sin 1''.$$

* Chauvenet's Astronomy. Fourth Edition. Volume II, para. 229. τ = hour-angle = $(k + c) \sec\delta$.

From this formula it will be seen, that it is not only advisable but imperative to treat k and c together: if treated separately the formula would be

$$d\zeta_k + d\zeta_c = + \frac{15^2}{2} (k^2 + c^2) \tan\delta \sin 1'',$$

and this is too small by

$$15^2 \times k \times c \times \tan\delta \sin 1''.$$

Suppose for instance that the first intersection was properly made at 21 seconds from the centre wire, and that the second was late and not made till 33 seconds from the centre: suppose also that owing to collimation error the centre wire is 30″ too far east with "telescope west," which has the effect of making both intersections nearer the meridian. Then the mean value of k will be $+ 27^s$, and that of c will be $- 2^s$, and the value of $d\zeta_{(k+c)}$ will be

$$+ \frac{15^2}{2} \times 25^2 \times \tan\delta \sin 1''.$$

Now let the effect on zenith distance of an inclination in the transit axis be considered. If the transit-axis be inclined to the horizon at an angle b'', the zenith distance of a north star as observed will be *too great* by

$$\frac{b^2}{2} \cdot \frac{\cos(\Delta + \zeta) \cos\zeta \sin 1''}{\sin\Delta}, \text{ (Mr. Eccles's formula)},$$

where ζ = north zenith distance, and Δ = north polar distance of star.

As north zenith distances are negative, the correction will be

$$d\zeta_b = + \frac{b^2}{2} \cdot \frac{\cos(\Delta + \zeta) \cos\zeta}{\sin\Delta} \sin 1''.$$

When using this formula, the numerical value of ζ must be entered regardless of sign. Thus suppose $b = 40''\cdot 2$, $\Delta = 65°$, and ζ = a north zenith distance of 10°, the correction will be

$$d\zeta_b = + \frac{(40\cdot 2)^2}{2} \cdot \frac{\cos(65° + 10°) \cos 10°}{\sin 65°} \sin 1''.$$

It might perhaps have been more consistent, to have considered a north zenith distance $= -\zeta$, and to have so arranged the formula, that in the above example -10° would have had to be substituted for ζ: but this was tried, and was found to lead to much confusion; the formula has consequently been so written above, that in dealing with north stars the *numerical* value of the zenith distance has to be substituted for ζ *without* changing any of the signs as given.

In the case of south stars the *true meridional* zenith distance is larger than the *observed* zenith distance: the latter has therefore to be increased, and as south zenith distances are positive, the correction will be positive: the correction is

$$d\zeta_b = + \frac{b^2}{2} \cdot \frac{\cos(\Delta - \zeta)\cos\zeta \sin 1''}{\sin\Delta}.$$

b'' is the *mean* inclination of the transit-axis in the two telescopic positions: it is a matter of indifference which end of the axis is high, and only *amount* must be considered. Thus suppose with telescope east the inclination is 20″ east, and with telescope west 14″ west, the mean inclination is $\frac{20 + 14}{2} = 17''$, and not $\frac{20 - 14}{2}$. If with telescope east the inclination is 20″ east and with telescope west 14″ east, the mean inclination is 17″ also.

Now let the effect on zenith distance of a deviation in azimuth be considered. Suppose a'' to be the mean deviation of the telescope from the meridian in its two positions: the zenith distance of a north star as observed will be too great by

$$\frac{a^2}{2}\sin 1'' \times \frac{\sin(\Delta + \zeta)\sin\zeta}{\sin\Delta}, \text{ (Mr. Eccles's formula).}$$

As north zenith distances are negative the correction will be

$$d\zeta_a = + \frac{a^2}{2}\sin 1'' \times \frac{\sin(\Delta + \zeta)\sin\zeta}{\sin\Delta}.$$

When using this formula, the *numerical* value of ζ must be entered regardless of sign, as explained before: thus if $a'' = 112''\cdot 1$, $\Delta = 65^\circ$, and $\zeta =$ a north zenith distance of 10°, the correction will be

$$d\zeta_a = + \frac{(112\cdot 1)^2}{2}\sin 1'' \times \frac{\sin(65^\circ + 10^\circ)\sin 10^\circ}{\sin 65^\circ}.$$

In the case of south stars the *observed* zenith distance will also be too great: the latter has therefore to be decreased, and as south zenith distances are positive, the correction will be negative: the correction is

$$d\zeta_a = -\frac{a^2}{2}\sin 1'' \times \frac{\sin(\Delta - \zeta)\sin\zeta}{\sin\Delta}.$$

In calculating a'' only the *amount* of the deviation should be considered and no account should be taken of its sign. Thus suppose with telescope east the deviation is 60″ west, and with telescope west 38″ east, the value of a'' will be $\frac{60+38}{2} = 49''$ and not $\frac{60-38}{2}$.

Summing up the above conclusions, the three corrections to an observed zenith distance for intersection off the centre are:—

$$d\zeta_{(k+c)} = +\frac{15^2}{2}(k+c)^2\cot\Delta\sin 1''$$

for both north and south stars,

$$d\zeta_b = +\frac{b^2}{2}\cdot\frac{\cos(\Delta \pm \zeta)\cos\zeta\sin 1''}{\sin\Delta}$$

for both $\frac{\text{north}}{\text{south}}$ stars,

$$d\zeta_a = \pm\frac{a^2}{2}\cdot\frac{\sin(\Delta \pm \zeta)\sin\zeta\sin 1''}{\sin\Delta}$$

for $\frac{\text{north}}{\text{south}}$ stars.

The first formula is independent of zenith distance: in substituting a numerical value for ζ in the other two the sign should be retained as written whether north or south stars are being dealt with. The signs in front of the corrections in all three formulæ are given on the understanding, that north zenith distances are inherently negative.*

The *instrumental* zenith distance was first found: this was corrected for "inclination of the vertical axis" and the result designated the *observed* zenith distance: this latter has now received a correction on

* These formulæ are not applicable, if the zenith distance of a star is measured at its lower culmination.

account of refraction, and three corrections on account of the extra-meridional position of the star at the instant of intersection: the final result thus obtained is the *true meridional* zenith distance.

(2). *North polar distance of a star.*

Suppose the star No. 152 of the Greenwich Catalogue of 1872 to be observed in India for latitude on the 23rd December 1886: its apparent north polar distance at the moment of observation is required.

First correction.

8. The mean north polar distance on the 1st of January 1872 is given in the Catalogue as 75° 18′ 53″·91. The first correction applied is

$$\left\{p + \mu + \frac{sy}{200}\right\} y,$$

where y = number of years from 1872, p = annual precession in north polar distance = − 18″·703, μ = annual proper motion = 0″·00, and s = secular variation = + 0″·012. The amount of the correction is − 4′ 21″·67, and this applied to the above north polar distance gives 75° 14′ 32″·24 as the value of the mean north polar distance on January 1st, 1886.

Second correction.

9. The second correction is equal to Ee′ + Ff′ + Gg′ + Hh′ + L + l′ − 300, where E, F, G, H, L are numbers: the logarithms of E, F, G, H and the value of L are given in the Nautical Almanac; they change with the day of the year but are the same for all stars; e′, f′, g′, h′, l′ are functions of the star's place, the logarithms of e′, f′, g′, h′ and the value of l′ are given in the Star Catalogue and are independent of the date of observation. The second correction amounts in the present example to −16″·65.

Third correction.

10. The third correction consists simply of the product of the proper motion and the fraction of the year: in the present instance it is equal to $\mu \times \frac{357}{365}$ = 0″·00.

By applying the second and third corrections to the *mean* N. P. D. on January 1st, 1886, the *apparent* N. P. D. at Greenwich mean midnight on the date of observation is obtained and is given at 75° 14′ 15″·59.

11. Having computed the apparent N. P. D. at Greenwich mean midnight on December 23rd, the same process is again gone through for another date, (say December 31st,) and the result found to be $75^\circ\ 14'\ 16''\cdot 02$. In eight days the N. P. D. has increased $0''\cdot 43$, and the daily variation is therefore $+\ 0''\cdot 054$.

Daily variation in N. P. D.

12. By means of the daily variation, a fourth correction has now to be applied to reduce the apparent N. P. D. from Greenwich mean midnight to time of observation. The local mean time of the star's upper transit is the mean time at *sidereal* noon + the star's right ascension.* The local mean time of Greenwich mean midnight is 12^h + longitude (in time). Therefore the interval from Greenwich mean midnight to time of transit is expressed by

Fourth correction.

$$(\text{M.T. at Sidereal } 0^h + \text{R.A.}) - (12^h + \text{longitude}).$$

and this in the example quoted above is equal about to $-\cdot 4$ of a day, the negative sign denoting that the transit occurred *before* Greenwich mean midnight. As the daily variation is $+\ 0''\cdot 054$, the fourth correction is equal to $+\ \cdot 054 \times -\ \cdot 4 = -\ 0''\cdot 022$. The apparent N. P. D. at time of observation is therefore on December 23rd, $75^\circ\ 14'\ 15''\cdot 57$ and on December 31st, $75^\circ\ 14'\ 16''\cdot 00$. For any intermediate day it can be obtained by simple interpolation, thus for December 30th, it is $75^\circ\ 14'\ 15''\cdot 57 + \frac{(16''\cdot 00 - 15''\cdot 57)7}{8} = 75^\circ\ 14'\ 15''\cdot 57 + 0''\cdot 38 = 75^\circ\ 14'\ 15''\cdot 95$.

(3). *Resulting Latitude.*

As north polar distances and not declinations, are computed from the Catalogue it has been found advisable to take out the value of co-latitude by each observation instead of the latitude. Having regard to the adopted signs for zenith distance given above

$$\text{Co-latitude} = \text{north polar distance} - \text{zenith distance}.$$

The co-latitude should be taken out for each star: the mean co-latitude by north stars only and its probable error should then be computed:

* The star's R.A. should theoretically be converted into its equivalent in mean time, but practically this is an unnecessary refinement, as also is the correction of mean time at sidereal noon for longitude.

and the mean co-latitude by south stars and its probable error: results by north and south stars must be retained quite distinct. The probable error of a co-latitude by north or south stars is

$$\cdot 6745 \sqrt{\frac{[e^2]}{n(n-1)}},$$

where n = number of north or south stars, and $[e^2]$ = sum of squares of differences of all individual co-latitudes from the mean. The final value of co-latitude for a station is the mean of the two results by north and south stars respectively. If e_n = probable error of co-latitude by north stars only and e_s = probable error by south stars only, the probable error of the final mean value of co-latitude by all stars is

$$\sqrt{\left(\frac{e^2_n + e^2_s}{4}\right)}.$$

Results by north and south stars will be often found to differ, occasionally considerably. If the difference is constant for stars of all zenith distances, it must be due to a personal peculiarity of the observer, which makes him intersect every star too high or too low: in this case the difference however large is immaterial, and does not affect the accuracy of the final mean value of co-latitude. If the difference *increases* with the zenith distance of the stars observed, it is due to some instrumental defect, probably flexure of the telescopic tube: in this case the difference is a very serious matter, the final resulting co-latitude cannot be accepted as trustworthy, and the instrument should be discarded from use.

In order to keep a watch on instrumental and personal peculiarities, the co-latitude of stations should be computed from observations taken only when the azimuthal stud was north, and compared with its value calculated from observations taken with the stud south.

If the two levels do not indicate the same inclination of the vertical axis, the co-latitude of stations should be taken out, 1stly, employing the inclination for every star as shewn by one level only, 2ndly, employing the inclination per star as shewn by the other level only; if the probable error of final co-latitude, calculated when the readings of but one level were utilised, is less than the probable error of co-latitude obtained by utilising both levels, it is evident that the second level should be discarded.

PART VI.

Tidal Observations.

CHAPTER I. INTRODUCTORY REMARKS.

Object of tidal observations.

1. The immediate object of tidal observations is to determine the heights of the water, above some known datum mark at a station, for every instant of time during a more or less extended period. For this purpose, a self-registering tide-gauge is employed which exhibits the height of the tide in a graphical form by means of a pencil, driven by the rising and falling water with the help of suitable mechanical contrivances, marking a sheet of paper rolled round a drum driven by clock-work. The period during which the gauges are allowed to work is five years for minor stations, as this is considered sufficient to give a fair representation of the tidal oscillations at any place, and permanently at other stations or at least as long as the general tidal operations last, and certainly not less than nineteen years, 18·6 years being the period of a certain tide which is expected to give valuable information in regard to the rigidity of the earth. It is of the utmost importance that as few interruptions as possible may occur in the observations, and when they do occur that they may be of short duration; otherwise the method of interpolation employed in filling up the breaks, fails and a more complicated and less satisfactory one has to be adopted.

The results obtained are both of scientific and practical utility. With the scientific side of the subject we have little to do at present; but it may not be out of place to mention that a better knowledge of the laws of the tides is expected to lead to an evaluation of the mass of the moon, to more definite information regarding the rigidity of the earth, to an approximation to the depth of the sea from the observed velocities of the tide waves, and to information regarding the retardation of the earth's velocity due to tidal friction.

Practically, the correct determination of the tidal heights at any station, enables zeros of level to be fixed for the purposes of survey, and affords data for the calculation of the rise and fall of the tides at a future period and thus subserves the purposes of navigation.

Strength of a tidal party.

2. To carry out the work properly the strength of the Tidal Party (excluding the Levelling Party) should be as follows:—

Two officers of the Senior Branch;
Two officers of the Junior Branch;
Two Native mechanics with a knowledge of clock-work;
Twelve sub-surveyors and computers;
One writer;
Twelve daftris, peons and khalasies.

CHAPTER II. OBSERVATORY ARRANGEMENTS.

Selection of a site for a tide-gauge.

1. The choice of a site for the erection of a tide-gauge depends so much on local circumstances that a careful reconnaissance of the fore-shore is a necessary preliminary to the selection of the best of the generally limited number of suitable positions.

The gauge should be placed so as to obtain a fair representation of the tidal oscillations of the surrounding area, and to secure this it is necessary (1) that the sea should have direct communication with the gauge, and not approach it through tortuous channels; (2) that the spot chosen should be sheltered from heavy weather, and (3) that there should be deep water at low-tide close to it. For example, a good position would be the end of a pier or jetty, or the wall of a dock. It must, however, be pointed out that a position in a cove or in a minor bay at the head of a large bay, though it may apparently answer the above requirements, is not a good one for a tidal observatory, as experience shows that, at stations where the range is small (as in the south of India) the tidal curves in such a spot often present a zig-zag appearance all along the rise and fall. The irregularities are certainly not caused by rough or lumpy water; because it has frequently been noticed that they were being registered inside the float cylinder, at times when the surface of the water outside was perfectly smooth and no swell or ripple was apparent to the eye. There seems to be a slow throbbing or pulsatory action going on in such localities, during both rise and fall, which the eye does not readily detect; for instance, during a rising tide the recording pencil will remain stationary, sometimes for nearly five minutes, and then gradually fall for two or three minutes to an extent representing 2 or 3 inches in actual fall of tide, then again remain stationary for a few minutes, and afterwards move up on the rise. This will be repeated at intervals during the entire rising of the tide, and the same thing will recur in reversed order during the fall of the tide. In tidal rivers no such peculiarities have as yet been met with.

When a station has been selected near deep water, a vertical cylinder is fixed in the water in such a way as to admit it only through holes small enough to annul wave-motion and large enough to cause no sensible retardation of its rise and fall in the cylinder.

When an observatory has to be established at a place where such an adjunct as a pier, a jetty, or a dock-wall is not available, and cannot be specially constructed in deep water except at a prohibitory cost, it must be erected on the shore, the cylinder sunk in the ground, or in a masonry well and connected with the sea by piping, so as to maintain the level of the water to correspond at all times with that of the sea.

As an index to the method to be employed in determining the effect of a pipe connection, the subjoined example may be found useful.

Calculations to show the relative level of the water inside a cylinder of 22 inches diameter and that of the sea, the connection being made with a 2-inch pipe, 300 feet long, which has two bends of 90° each.

From Beardmore's Hydraulics, Table 8.

The discharge for a 2-inch pipe $= \dfrac{26\cdot 69}{\sqrt{\dfrac{l}{h}}}$ cubic feet per minute,

where

l = the length of the pipe,

h = the head of water.

Now, assuming that the water in the sea has risen 1 inch higher than in the well, there is a head of 1 inch, and

$$\sqrt{\frac{l}{h}} = \sqrt{\frac{300 \times 12}{1}} = 60.$$

Therefore, the discharge = 768 cubic inches per minute, supposing that there are no bends in the pipe.

Now the quantity of water required to raise the level in the cylinder 1 inch is πr^2 or $11^2 \times \frac{22}{7}$ cubic inches nearly, *i.e.*, about 380 cubic inches.

Thus, a head of 1 inch would discharge the quantity required in something less than half a minute.

But when there are bends in the pipe, the head required to overcome them varies as the angles of the bends and the velocity of the water through the pipe.

The area of the connecting pipe in section $= \pi r^2 = \frac{22}{7}$ square inches nearly.

Therefore the length of pipe which would contain a volume of water enough to raise the water in the cylinder 1 inch

$$= \frac{380}{\frac{22}{7}} \text{ inches} = 10 \text{ feet nearly,}$$

and if this amount is to be discharged into the cylinder in half a minute, the velocity of the water must be 20 feet per minute, diminishing as the head of water decreases.

Now by Beardmore's Hydraulics, Table 9, for a mean velocity of 20 feet per minute, and two bends of 90° each, we require a head of ·0048 × 2 inches or $\frac{1}{100}$ inch very nearly, so that the bends need hardly be taken into account.

Now suppose that there is an 18 feet rise and fall of tide at springs, *i.e.*, a mean of about 3 feet per hour, then the water will rise on an average $\frac{3}{10}$ of an inch in ½ minute.

Combining the above arguments, it follows that if the sea-level was an inch higher than the water in the cylinder, the latter would be raised this height in ½ minute, by which time the sea would be $\frac{3}{10}$ of an inch higher.

But the tide rising increases the head, and the water would tend on that account to flow quicker: also the sea-level would only be raised 1 inch in $1\frac{2}{3}$ minutes; so that the differences of level under the conditions specified may be considered as inappreciable.

2. To facilitate the detailed description of the self-registering tide-gauge, it will be best to begin with a general account of it.

Description of instruments.

3. Mention has before been made of the 'cylinder' placed in communication with the sea, either directly or by a pipe connection, its use being to render the water still and undisturbed.

General outline.

Resting on the water in the cylinder is a 'float' which rises and falls with it, and to it is attached a copper 'band' which passes over a wheel called the 'stud-wheel'.

The rise and fall of the 'float' communicates motion to the 'stud-wheel' by means of the 'band,' and the 'stud-wheel' in turn, by means of a projecting axle on which is fastened a 'toothed-wheel,' communicates motion to another 'toothed-wheel'. On the same axis as the latter, and consequently moving with it, is another wheel round which a flexible 'chain' is passed, one end of which is attached to the wheel and the other to the 'pencil'. The 'chain' is kept taut by a 'counterpoise weight' to ensure the 'pencil' following the movements of the 'float'.

The 'pencil' moves longitudinally along a cylindrical 'drum' touching the surface with its point; the 'drum' revolves once in 24 hours by means of 'clock-work' at the opposite end to the float. The 'drum' is supported on a cast-iron 'bed-plate,' and the whole instrument on wooden trestles.

The instruments in use in India are of Newman's pattern, their distinguishing characteristic being very long drums whereby the curves are delineated on comparatively large scales, alterable at will. The instruments were originally made by the late Mr. P. Adie, but improvements were effected from time to time, and their construction is now entrusted to Messrs. A. Légé & Co., London.

Having thus disposed of the apparatus as a whole, a detailed account of each part will now be given.

The cylinder.

4. The size of the cylinder varies at different places, but it is generally 24 inches in internal diameter, and is usually made of thin iron plate, in sections of from 4 to 8 or even 10 feet in length, with angle-iron flanges at each end for bolting the lengths together: the bottom of the cylinder is ordinarily closed with an iron plate, while the top reaches to the floor of the observatory and sometimes to one or two feet above it.

In some places where durable wood can be obtained cheap, stout wooden boxes are substituted for the iron cylinder. They are 24 inches square in internal section: the bottom is closed with a wooden grating, which does not rest on the ground but is at a depth of some 4 or 5 feet below the lowest tides, the box being supported by the piles of the pier or wharf on which the observatory stands.

5. The easiest and best inlet for the water to the cylinder is through a number of holes $\frac{1}{4}$ to $\frac{1}{2}$ an inch in diameter, near the bottom of the cylinder, below the lowest level of spring-tides, but at the same time well clear of the ground on which the cylinder rests: if however the cylinder does not rest on the ground, the bottom of the cylinder is the best place for the holes.

Inlet for the water.

When the cylinder cannot be fixed in deep water, a pipe connection between the sea and the cylinder is necessary. The piping used is ordinary gas-pipe, in lengths of about 14 to 22 feet, with an internal diameter of 2 or 3 inches. In order to render the connections perfectly air-tight as well as to facilitate the joining together, the lengths of piping are fitted with cast-iron flanges made to screw on each end.

The piping is connected with the cylinder, at about 9 inches above the bottom, by a small bend and is then brought up vertically outside the cylinder to a height of one or two feet below the lowest high-water. At this point there is another bend fitted with a stop-cock, for a purpose explained hereafter, from which the pipe is taken straight down to the sea along the slope of the shore to the low-water line. Here lengths of flexible suction-pipe are joined on and taken out to deep water. This piping is a helix of copper wire coated with india-rubber and canvas, and 2 inches in diameter. It is made in lengths of 50 or 60 feet and fitted with couplings and means of connection. At the end of the outer length, a cylindrical copper rose of about 15 inches in length and 2 inches in diameter, and having some 150 holes of $\frac{3}{10}$ of an inch bored in it, is screwed on. The rose is supported above the ground on a tripod fixed in the ground in deep water. Sometimes a small basket is made to answer the purpose of the copper rose.

The rigid and flexible pipes are connected as follows:—To the end of the rigid pipe a brass connecting piece is fitted. It is T-shaped and to either of its two outer extremities a length of the flexible pipe may be fixed, the other being closed with a brass disc with a good washer. When the flexible pipe has to be removed for cleaning, the brass disc is unscrewed, a length of spare flexible piping with a rose at its end attached and taken out to deep water temporarily: the original pipe is then taken off and cleaned, the disc being screwed on for the time in its place; then, when finished, the pipe and the disc are replaced in their original positions.

In connections of this sort, there is a decided tendency for air to collect in the piping and this causes a retardation of the flow of water into and out of the cylinder. It is to remedy this defect that the stop-cock, alluded to above, is placed at the upper bend of the rigid pipe. As this stop-cock is below high-water mark it is clear that, if it be opened at the time of any high-water, the air will be expelled and the levels of the water inside and outside the cylinder become identical. It is occasionally found necessary to keep the stop-cock open for 4 or 5 hours while the water is above the height of the cock.

The float band and stud-wheel.

6. The float is a cylindrical copper vessel 1 foot in diameter and 9 inches deep, and is of such a density that it will just sink if unsupported. The band is a copper ribbon, about 1 inch wide, perforated with holes about 2½ inches apart.

The stud-wheel is of brass, about 9½ inches in diameter, with a rim an inch wide: it has studs of the same diameter as the holes in the band, placed in the rim at intervals also of about 2½ inches, so that when the band is passed round the wheel the studs exactly fit into the holes, thus ensuring the revolution of the wheel as the float rises and falls. The band is attached at one end to the float, by means of two milled-headed screws passing through holes at the end of the band into a disc revolving underneath small rollers fixed on the top of the float: the arrangement forms a swivel and prevents the band being twisted. The band is cut to such a length that it passes over the stud-wheel and about 6 feet beyond when the float is in its lowest position in the cylinder. To the end of the band, as a counterpoise to the float, a weight is attached, and from its bottom a copper chain is suspended which theoretically should be equal in weight, length for length, to the copper band. The other end of the chain is attached to a hook below the float so as to form with the band a sort of endless chain passing over the stud-wheel and reaching to the bottom of the cylinder. This contrivance is introduced in order that the pull on the float shall be constant, otherwise a systematic error is introduced between rising and falling water. The counterpoise weight should be such as to give a decided preponderance, of say 3 or 4 lbs, on the float side; but when once adjusted it should not be altered, without noting the fact in the inspection book. When the whole system of float, band and counterpoise weight is hanging in position in the cylinder, there should be 3 inches space between the float and cylinder on the one side and the counterpoise weight and cylinder on the other. The stud-wheel is supported on two uprights fixed to the bed-plate: its axle is about 8 inches long and carries at its other end a toothed-wheel which is in gearing with another, this latter being fixed on the axle of the wheel over which the chain connected to the pencil passes: this axle is fastened to one of the uprights which support the drum.

The chain wheel.

7. The wheel round which the chain winds is about 5 inches in diameter and is grooved for its reception, so that the chain lies on the wheel and does not overlap itself.

The toothed-wheels.

8. The toothed-wheels are constructed in couples so as to enable the working scale of the tidal diagram to be varied at pleasure, from the natural or full scale to that of $\frac{1}{8}$th, according to the range of the tide. Six couples are supplied with each gauge, giving scales of $\frac{1}{1}$, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$ and $\frac{1}{8}$; and as the drum is 5 feet long any tide whose range does not exceed 30 feet can be safely registered. At Bhávnagar where the range is over 36 feet a $\frac{1}{10}$ scale has been employed. In practice the couple is selected which enables the tidal curves to be exhibited on the largest scale possible.

The bed-plate.

9. The bed-plate is of cast-iron about 7 feet long, 1 foot broad and $\frac{3}{4}$ of an inch thick, the upper surface being carefully planed. Underneath this plate and cast in the same piece with it, is a web 4 inches high which extends to within an inch of the edge of the upper plate both at the sides and at the ends, and has diagonals or stiffeners.

The friction-rollers.

10. At about 10 inches from each end of the bed-plate is a brass upright, and on each of these a pair of friction-rollers is fixed, the pins on which the rollers turn being screwed into the uprights some 5 inches above the bed-plate.

The drum.

11. The drum which is 5 feet 3 inches long and exactly 24 inches in circumference revolves between the uprights: it is composed of sheet-brass and is made as nearly as possible a true circular cylinder. Axles project from each end and rest on the friction-rollers: one axle is elongated and passing through an oblique slot in the upright carries a toothed-wheel which

gears with the driving clock. The position of the drum is horizontal and it carries the paper on which the tidal curve is registered.

Two grooves about $\frac{1}{16}$ of an inch deep are cut round the drum near each end and exactly 5 feet apart and a third is cut midway between the two first. The groove at the clock-end is generally adopted as the *zero-line* of the gauge. When the paper is fixed on the drum, the zero, middle and end lines can be indicated on it by rubbing over the grooves with a hard pencil. The paper is nearly 5 feet 3 inches long and extends well beyond the extreme grooves.

Arrangements for the pencil.

12. Parallel to the drum and fixed above it to the brass uprights are two bars of solid brass drawn to angle shape and between them moves a slide carrying the pencil-holder in such a position that the pencil is exactly over the centre line of the drum. The pencil-holder slide which is a double T-shape moves along the bars and is pushed towards one of them by means of a spring so that it has no lateral motion. The bars are prevented from buckling or having lateral motion by two arched stiffeners which are screwed on to the outer sides of the bars and allow the pencil-holder slide to pass through them.

The chain and counterpoise weight.

13. To each end of the upper plate of the pencil-slide, loops of wire are fixed: to the one of these nearest the float, a flexible chain is fastened and carried from thence round the chain-wheel, to which the end of it is made fast. The loop itself forms a swivel on a cylindrical capstan-headed screw which works into the upper plate of the slide and by means of which the pencil can be set exactly to the zero of the gauge for height. To the other loop, a silver wire or catgut-cord is tied which passes over a rimmed pulley at the top of the driving clock and has a weight of about 5 lbs attached to its other end. As the pencil-slide moves between the bars, the weight rises and falls and a sufficient space must be allowed for its drop.

The pencil-holder.

14. The pencil-holder is a small tube which screws into the slide and is adjusted so that it almost touches the paper on the drum. It is made to contain common leads which are pressed down on the paper from above by a weight of two or three ounces.

15. The driving clock is a 'regulator clock' with English lever escapement (gold hair-spring). The movements are boxed in by movable brass slides and the oil-cups are protected by bushes. The drum is driven by a toothed-wheel of the clock gearing with another on the axle of the drum. The arrangement for connecting and disconnecting the drum and the clock is as follows :—A clamping screw with a milled head is connected with an interior arbor so that when the screw is clamped the driving toothed-wheel of the clock is not movable on the arbor, but when the screw is released the wheel can turn freely and thus permit the drum to be placed in any position required. The drum can thus be set so that the pencil shows the correct time on the diagram and the clamping of the screw then brings the two toothed-wheels into connection with the rest of the wheel work of the clock.

The clock.

16. In order to prevent any back-lash which may exist between the gearing of the clock and the drum, a cord carrying a weight of about 5 lbs, is attached to and encircles a barrel on the axle of the drum and passes over a pulley on the bed-plate. This barrel carries a pawl which drops into a ratchet-wheel on the drum: it also carries a crown-wheel which gears with a bevel-pinion. This pinion turns freely in a socket fastened to the upright which supports the clock end of the drum. The outer end of the pinion is square so that the key which winds the clock fits it. In winding the pinion the barrel is also turned, thus winding up the weight which prevents the back-lash, while any backward motion is stopped by the pawl and ratchet on the drum.

The back-lash weight.

17. As the drum for the diagram may not be quite circular the position of the pencil, as the clock shows the exact hour at four different times of the day, is marked on the diagram which is afterwards re-divided in accordance with these marks.

Ellipticity of the drum.

18. The tide-gauge is set up on a wooden trestle the top of which is 5 feet long, 1 foot broad and 2 inches thick: the legs are splayed and firmly braced. The trestle is placed longitudinally in the observatory and touches the top of the float cylinder at one end.

Trestle.

19. No system of tidal observations can be considered complete which does not contain a continuous record of the atmospheric condition at the station. Consequently every tidal station (with the exception of Bombay where the information is supplied from Colaba) is furnished with a self-registering aneroid barometer to measure the momentary variations of pressure, and a standard mercurial barometer to check the aneroid from time to time and enable its index error to be determined; also with a self-registering anemometer indicating the velocity and direction of the wind at every moment, a maximum and a minimum thermometer and a rain-gauge.

The auxiliary instruments.

20. In the self-registering aneroid barometer there are seven vacuum-chambers or boxes coupled together; the top one is attached to a screw used for setting the metallic registering pointer and to the lowest is fixed a fork with hardened steel knife-bearings. On these bearings rests, by means of knife-edges, a lever which connects the balancing-spring with the vacuum-boxes, being pivoted on other knife-edges midway between the attaching points. The vacuum-boxes and balancing-spring are placed on a brass frame.

The self-registering aneroid.

The balancing-spring is a spiral one hooked at the bottom to the lower edge of the lever and at the top to a screw working in the upper part of the frame, by which its pull in connection with the lever is so adjusted that the reading of the instrument corresponds to that of the mercurial barometer under the particular atmospheric conditions existing at the time of its first adjustment. This adjustment, made by the maker, ought not to be altered unless the instrument has to be taken to pieces.

The amount of motion produced by the variation of the atmospheric pressure on the boxes is multiplied by the lever above mentioned and then again by a second lever which is supported on two uprights and counterpoised. These two levers are connected together by a steel rod pointed at both ends and pivoted in conical holes out of which it is prevented from slipping by means of forks.

Projecting from the clock is a third lever of the same length as the second and attached to it by a joint whose length is half the height of the recording barrel. The joint is movable and at its centre a metallic pointer is fixed for the purpose of marking on the prepared paper on the barrel; the pointer can be adjusted to press more or less heavily as required. The mode of marking is as follows:—Attached to the back of the third lever there is a sliding-piece which is pulled by the clock movement three times per hour and by this motion the joint is twisted and the pointer pressed against the paper.

To the brass frame on which the vacuum-boxes and balancing-spring are placed there is fixed a steel tube and on this the revolving drum pivots, being maintained in position by a nut screwing in the top of the tube. The tube is hollow to admit of a turnscrew being inserted to set the recording pointer to agree with the mercurial barometer. At the bottom of the revolving drum there is a toothed crown-wheel which gears with a pinion driven by the clock. The drum revolves in 8½ days. The recording barrel on which the specially prepared paper is fixed rests by its own weight on the revolving drum and has a knob on the top with a hole through it of the same size as the hollow in the steel tube. The several parts are fitted on a substantial brass plate, about 21 inches long and 6 inches wide, screwed on to a board an inch thick and the whole instrument is fixed in a neat case with a glass front, the top and front being made to open on hinges.

The self-registering anemometer.

21. The self-registering anemometer is fitted with Robinson's cups and steering-vanes and is about half the size of Beckley's standard anemometer. A special long protecting tube is fixed on the cross-bars carrying the cups to prevent dust or rain blowing into the bearings. The steering-vanes are made as large as possible with a sharper angle than usual and the screw working the wheel of the indicating shaft has a quicker thread so as to make the motion of the vane still more rapid.

The recording instrument has larger crown-wheels than usual so as to produce an easier gearing. The driving-barrel of the clock movement is drilled through its centre and a shaft passed through the hole having at one end a pinion gearing with the reading barrel and at the other, in front of the clock, a milled-headed screw. This arrangement allows the registering drum to be easily set, for by loosening the screw the shaft becomes free and after the drum is set to correct time, the clock connection is made by clamping the screw.

To prevent the back-lash of the recording barrel, the pinion on the centre spindle of the driving-barrel of the clock in gearing with the recording barrel is double; one part of this pinion is fixed on to the spindle and the other is loose but attached to the fixed part by means of a circular wire spring and this being pressed back when gearing the double pinion causes the tooth of the recording drum, which is in gear, to be clamped on both sides and thus prevents lost motion.

The clock has a good lever escapement with gold hair-spring and is driven by a weight; a dial showing minutes is fitted on to it.

The recording is done by the usual spiral metallic pencil marking on especially prepared paper.

Sometimes it is necessary to place the anemometer at a distance from the tidal observatory in order to obtain a site freely exposed to the wind from every direction.

The observatory.

22. A tidal observatory is constructed of wood and is usually so made as to be readily taken to pieces and put together again in order to be removed and re-erected.

It is about 12 feet by 9 feet in plan and 12 feet high in the middle up to the ridge, from which the roof slopes down to the sides which are 8 feet high.

The graduated staff.

23. Outside the observatory and attached to a pile of the pier or to the wharf wall, a graduated staff is fixed vertically, in such a position as to be easily read, so that a comparison of the level of the water outside and inside the cylinder may be readily made by simultaneous readings of the pencil on the diagram and of the water on the graduated staff.

Bench-marks.

24. Two or three bench-marks are laid down in the vicinity of every tidal observatory and connected with the bed-plate by first-class levelling described afterwards in Part VII. They are either cut in the dock-wall, or on the steps of some neighbouring building, or are cubical blocks of masonry, about 3½ feet each way, containing a large stone imbedded on the upper face. The stone is inscribed to show that it is a bench-mark of the Great

Trigonometrical Survey, and the year on which it was laid down is also given. In the centre of this stone, a square depression of 5-inch side and ¼ inch deep is cut and nicely smoothed, its size being just sufficient to allow the levelling-staff to turn freely in it.

To set up and start the self-registering tide-gauge.

25. The trestle is first put in position, longitudinally in the observatory and touching the float cylinder at one end. The tide-gauge is next set up on the trestle, in such a position that the centre of the float, the band and the counterpoise weight shall all be in a diametral plane of the float cylinder, and also so that the float and counterpoise weight shall each be about 3 inches from the sides of the cylinder. The trestle is then wedged up so that its top is nearly level, and its legs are firmly screwed to the floor. After this the bed-plate is levelled, longitudinally and transversely, by driving wedges in between the web and the top of the trestle; and the various parts of the instrument are tested, to see that they work freely and that there is enough drop, at extreme tides, for the counterpoise weight attached to the pencil traveller.

The wheels to govern the scale of the diagram are now placed in gear with the float-wheel, a trial diagram put on, and the instrument approximately adjusted so that at half-tide the pencil will be at the centre of the drum, the extreme rise and fall being roughly known beforehand. [Sometimes the zero of the gauge, hereafter defined, is made to correspond with some particular level which has been taken as the datum for local surveys and the instrument is adjusted accordingly. For example, when soundings are being taken in the vicinity and the times are noted, the gauge readings may be made to indicate the amounts to be subtracted from the soundings to find the distance of the surface of the water above the datum.]

Whatever be the approximate adjustments, careful measurements must next be taken to determine the distance of the water below the surface of the bed-plate when the pencil is on the zero of the gauge. In making these measurements, called 'zero measurements', a special apparatus is employed.

Zero measurements.

26. A flat strip of brass with a right-angled band is fixed by two or three countersunk screws to the top of a box-wood scale, divided into tenths, hundredths and

five-hundredths of a foot, so that when the flat piece of brass lies on the top of the bed-plate, the scale hangs vertically down; care is taken to have the under surface of the flat piece, which rests on the top of the bed-plate, corresponding exactly to the zero mark of the box-wood scale.

A small circular wooden disc, 3 inches in diameter and ½ an inch thick, bevelled at its edge from the bottom towards the top, is attached by means of a brass slip fixed on the top of the disc, to the end of a Chesterman's tape. The slip is made of two brass plates, about 2½ inches long and 1 inch broad, one fixed vertically nearly at the centre of the disc and the other attached to the first by four screws at its corners, so as to be removed at pleasure. The tape is held between the plates, and, when the screws are clamped, the disc is suspended by the tape; there is a line drawn across the tape, so as to show exactly how it is to be placed relatively to the top of the brass clip. The top of the disc is slightly loaded with lead, so that, when hanging from the tape, its under surface is quite horizontal. The distance from the bottom surface of the disc to the top of the clip is exactly 3 inches. The tape itself is now marked at every foot from 5 feet to 30 feet, by applying it to a standard levelling staff, the disc arrangement being held suspended all the time, so as to have the tape in tension under the same condition as it would be when in use.

The measurements are taken as follows:—The box-wood scale is suspended from the bed-plate above the float-cylinder, and the disc is lowered into the cylinder, care being taken to keep the tape close to the side of the scale; this is done by running off more tape than is required, passing it across the bed-plate and holding the tape on the top so that it can be paid out easily and in very minute increments of length. When the disc is seen to be close to the surface of the water, warning is given to the clerk who is standing by, ready to mark the exact position of the pencil on the barrel. The lowering is continued very carefully, and actual contact with the water is noticed by the disc causing a tremor on the surface; at the moment of contact a signal is made to the clerk who marks on the diagram the position of the centre of the lead pencil, and the distance in feet is read off the tape while the tenths, hundredths and thousandths are read off the scale. This distance is entered in the inspection book in one column: the measurement of the position of the pencil above the zero-cut on the drum is then carefully made, by using another scale, and entered in another column. The latter entry is multiplied by the denominator of the fraction indicating the working scale of the gauge and entered in a third column. The sum of the entries in the first and third columns gives a value of the distance of the water from the bed-plate with the pencil on the zero-line, or, in other words, the distance of the zero of the gauge from the bed-plate.

As a rule, twenty such measurements are made and the mean taken. The measurements have to be taken both during a rising and a falling tide (when it is well on the rise or fall) and the mean of the two sets is the adopted value of the zero below bed-plate: this eliminates the influence of the lost motion or back-lash between the two toothed-wheels connecting the stud-wheel and chain-wheel. If the value of the distance of the zero below bed-plate agrees with the true zero, previously fixed upon in regard to the datum, there is nothing more to be done; but if not, the chain must be lengthened or shortened by means of the milled-headed screw until the agreement is complete; the gauge may then be started and left to itself.

Duties of the clerk in charge.

27. For each observatory there is a clerk provided, who is generally one of the writers in the Port Office and receives a small increase of pay for the observatory duties; but in some cases a special man has to be engaged. Printed instructions are given to him concerning his work which should be carried out as follows:—

Hours of visiting the observatory.

28. The observatory should be visited each day at 7 and 10 A.M., and at 4 and 6 P.M., except on Sundays when two visits are considered sufficient; and also twice a month at some other hour to change the diagram.

The tide-gauge.

29. The tide-gauge clock must be wound up twice a week and the back-lash weight every evening. The positions of the pencil on the barrel should be marked by a circle of ink round the pencil, on each visit to the observatory, at the exact hour and the date of the month written alongside, the preceding day's curve should also be inked in with one of the coloured inks supplied. Simultaneous readings of the position of the pencil on the diagram and of certain white lines marked on the float-band should be taken once daily

and entered in the report, so as to make sure that the band has not been displaced. The heights of the tidal curve for each hour of the preceding 24 hours must be carefully measured off the diagram and entered in the report.

Tide-gauge clock.

30. The clock of the tide-gauge must be compared daily with the gun or time-ball, or with a watch previously taken to the telegraph office, where Madras time can be correctly obtained at 4 P.M., and local time deduced by applying a correction for longitude: the error of the clock should be noted in the report with a mark as to whether it has been corrected or allowed to stand. In setting the clock, supposing it is 1^m 40^s fast, the button on its face should be pulled out when the second hand is exactly at the 60 and released at the end of 1^m 40^s; but if it is slow, the minute hand should be turned forward 2^m and the clock stopped as before for 20^s. Any stoppage of the clock must be noted on the diagram and also in the report. The reading of the pencil on the diagram and the height of the water at the same moment on the graduated staff (if there is one) should be taken once a day and entered in the report. If there is an unmistakable difference of two-tenths of a foot when the water is calm, the cylinder should be flushed out and the communication holes examined. The diagram must have the zero, mid and end lines marked by rubbing a hard pencil over the grooves cut in the barrel at these points. A line about half an inch long should be rubbed at the hours of 10 A.M., 2 P.M. and midnight in a part of the paper not marked by any curve, if possible. The marking should be done on the day after the diagram is put on and again on the day it is taken off, and the date entered against each set of marks.

Changing diagram.

31. The diagram must be changed once a fortnight, generally every second Monday, when the tide has well turned so as to make sure of getting the highest and lowest tides. The change is made as follows:—the new diagram is numbered, dated, and has a narrow slip one inch long cut out in readiness for setting the zero-line: it is damped all over with clean water and paste applied to the overlap. The hour and date that the work was stopped is noted in ink on the old diagram and the pencil-holder is taken off. The diagram on the barrel is next cut carefully along the 12 o'clock line, the back-lash weight removed, the clips unscrewed and the diagram taken off, carefully rolled up and put

aside. The barrel and the clock are then disconnected and the new diagram put on the barrel, by first making the 12 o'clock line of the paper agree with that marked on the barrel and the zero of the diagram with that of the barrel; this can be done by means of the small slits that have been cut out of the diagram. The clips are now screwed down and the barrel turned round by hand till the outer edge of the diagram comes in contact with the 12 o'clock line and the height lines meet. The pencil-holder must then be fixed and the clock and barrel clamped, care being taken that the pencil is over that part of the diagram which corresponds, as nearly as possible in time, with the time of the clock. The back-lash weight is carefully and slowly put on and the hour of commencing work noted on the diagram. It only remains to regulate the position exactly for time. This is done by selecting any convenient hour, say noon, and at one minute before unclamping the clock-barrel connecting-screw and, when the second hand shows the complete hour exactly, bringing the centre of the pencil exactly over the hour-line of the diagram and clamping very firmly, otherwise the clock may fail to drive the barrel.

Stoppage of clock.

32. Any remarks regarding the stoppage of the clock, or in fact anything unusual, must be noted on the diagram and in the daily report. If the clock should stop, the weight must be removed from the pencil-cup and the pencil slightly raised; the clock-barrel connecting-screw must then be unclamped, the barrel, being held so that the back-lash weight does not run down, revolved by hand so as to bring the pencil over that part of the diagram which corresponds as nearly as possible to about 5 minutes in advance of the aneroid clock or reference watch time, and the pencil weight replaced: the clock must then be started and stopped again; when it shows the first exact hour after starting the pencil should be brought exactly over the hour-line, the clock and the barrel firmly clamped and the clock re-started when the exact hour is shown by the watch.

Displacement of band.

33. If by any chance the band should come off the stud-wheel, it should be replaced carefully by turning the wheel until the pencil is on the zero-line of the diagram and fitting the hole of the band marked with paint on the stud similarly marked; as the marks are made when the zero-line marked on the float-band is brought to agree precisely with the bed-plate, on the wheel being released, the pencil will assume its proper position.

34. Should the chain between the pencil-traveller and the float end break, the pencil-holder must be removed from the traveller, the counterpoise weight detached and the two pieces of the chain taken out and re-rivetted. The ends must then be attached to the traveller and the sheave as before, and the stud-wheel turned by hand till the 2·5 line painted on the band is on a level with the bed-plate. If the deviation of the pencil from the 2·5 line on the barrel is small, it can be set right by the adjusting screw attached to the pencil-holder; but if the deviation is more than the screw admits of, one of the change wheels will have to be taken off, turned one or two cogs and refixed, the final adjustment being made by the screw attached to the holder.

Breakage of chain or wire.

If a silver wire is in use instead of a chain, 6 feet 6 inches of new wire should be taken, one end inserted through the hole in the rim of the sheave and tied firmly with a stop-knot. The wire should then be wound round the groove of the sheave and the zero of the band held in position agreeing with the plane of the bed-plate. The pencil-holder should now be brought exactly over the zero-line of barrel and the other end of the wire securely fixed through the stud-loop on the traveller. The traveller should then be released, the float-wheel being still held so that the zero on the band is in position, in order to ascertain if the counterpoise weight will allow the pencil to be over the zero of the barrel; if not, it must be adjusted until it is so. In carrying out this, great care must be taken that the band does not kink.

35. The stop-cock should be opened at some time every day, at high water or as nearly so as convenient; but on no account must it be opened if the level of the water is nearly the same as the height of the stop-cock.

Stop-cock.

36. The aneroid and mercurial barometers and the thermometers attached thereto should be read daily at 7 and 10 A.M., and at 4 and 6 P.M.

Aneroid barometer.

Should the clock of the aneroid stop, the hand must be gently turned round till it points to the proper time as shown by the tide-gauge clock. The barrel of the diagram must then be turned until the pencil points to the proper time, but in doing this great care must be taken, otherwise the gold thread in Adie's pattern or the marker in Légé's may be broken. In the former, it should not be attempted if the clock hands are between 5 minutes to an hour and 10 minutes past the hour as the pencil marker suspended to the gold thread is, at these times, either pressing or close to the barrel; and in the latter, it may be best done at 5 minutes past a full hour.

The aneroid clock must be wound up every Monday morning and may be regulated by stopping for a few minutes if fast or pushing the minute hand forward if slow; but for one or two minutes' error it need not be altered.

The aneroid diagrams should be carefully numbered and changed every Monday morning, and the sheets should be carefully inked as they are taken off and put away.

Anemometer.

37. The anemometer clock must be wound up every morning by pulling the cord with the small weight and thus raising the heavy weight close to the bed-plate.

The anemometer diagram must be changed daily at 7 A.M. and the diagrams dated and numbered, the hour being recorded as put on at such a time and taken off at such a time. The number of miles of wind for the last 24 hours must be entered in the report, and is obtained by counting the number of velocity lines and multiplying by 10. The diagrams must be carefully inked in daily.

The instruments should be all oiled occasionally; and in the case of the anemometer, if the direction of the wind has been steadily from one point for many days without altering, as in the S.W. monsoon, the fans of the direction gear should be turned with the hand until the vane has made one or two complete revolutions.

Reports.

38. The daily reports must be made up in duplicate and one copy sent by post to the head office. Anything unusual must be marked on the diagram and noted on the back of the report, and if anything emergent is required to be done, the port officer must telegraph to the officer in charge of the tidal party.

Inspection of a tidal observatory. General remarks.

39. As a rule an inspection is made once a year, but sometimes oftener. Of course if any interruption has taken place, such as the removal of instruments by the port officer for safety on account of a cyclone, as has occurred more than once, or for the settlement of the observatory, thus necessitating a temporary suspension of the observations pending repairs, then an inspection should be made as soon as possible after the information has been received.

When it is intended to make an inspection, the first thing to be done is to test the accuracy of the one-foot graduations of the Chesterman's tape with which measurements for determination of working zero will be made, and which have been marked on the tape in the way described at page 195. The tape should not be used continuously for more than a couple of months without being retested and, if necessary, corrected.

The inspecting officer ought to carry in his inspection box, a copy of this handbook and of the current tide tables for Indian ports, and the necessary scales, measuring tape, and other instruments required at an inspection.

The inspecting officer, accompanied by a mechanic to dismantle, clean, repair if necessary, and refit the instruments, attends to the following points when making an inspection; and at the time of inspection writes in the observatory inspection book, under appropriate heads, a report, a copy of which is forwarded to the head office. The usual heads of the report are:—

General remarks.

Bench-marks.

Details of levelling.

Self-registering tide-gauge.

Details of determination of working zero.

Auxiliary instruments.

The general remarks should contain an account of the working of all the instruments since the last inspection, and should draw attention to the manner in which the observatory clerk performs his duties, and to anything else requiring special notice.

40. On arrival at the tidal observatory, it is necessary in the first place to ascertain if any settlement of the tide-gauge has taken place, by connecting the float end of its bed-plate by spirit levelling of precision with the bench-mark of reference, which in its turn should be similarly connected with the other bench-marks, and with the graduated staff, in order to test the accuracy of the zero of the latter. The report should give both the results and the details of the levelling, and should mention the condition in which each bench-mark and the staff is found; it should also mention whether the bed-plate is level both longitudinally and transversely.

Connection of bed-plate, bench-marks and staff.

41. Then, a set of measurements for the determination of working zero at a rising tide and another at a falling tide, each set to consist generally of not less than 20 measurements, should be taken; their mean will give a value of the distance of the working zero below the bed-plate. This eliminates the influence of the lost motion, or back-lash, between the toothed-wheels connecting the stud-wheel with the sheave for the wire of the pencil-slide (page 196); it also cancels the error arising from looseness of the pencil in the pencil-holder. If it should be necessary to continue these measurements, another pair of sets should be taken before proceeding with the inspection. These measurements for determination of the working zero should be made when the tide is well on the rise and fall, and not when it is almost high or low water. The method of taking them is explained at page 195 and, in entering them in the inspection report, it should be stated that they were made *before cleaning* the gauge.

Zero measurements and pencil and clock comparisons.

Concurrently with the measurements for the determination of the working zero, mentioned in the preceding paragraph, a comparison of the reading of the pencil on the drum with the reading of the bed-plate on the float-band, and with the reading of the graduated staff, should be made both at a rising and falling tide, and entered in the report.

Before cleaning the gauge, its clock should be compared with the telegraph or gun time by the inspecting officer, as a check on the previous recent comparisons entered by the observatory clerk in his daily reports.

42. The balance of the gauge ought also to be tested before beginning to clean the instrument. This may be done with sufficient accuracy by raising the float completely out of the water by gently turning the stud-wheel, and then taking the reading of a spring-balance hooked, for the purpose, to one of the holes of the band on the counterpoise side. The reading of the spring-balance will give the preponderance of the float. There ought to be a decided preponderance of, say, 3 or 4 lbs. on the float side, but the weight when once adjusted should require no alteration, as it would affect the value of the zero-line. Should the preponderance be found to have increased, it points to a probable flaw in the float sufficient to have admitted an influx of water.

Balance of gauge and setting of pencil.

After testing the balance of the gauge, and before cleaning it, the 2·5 painted line of reference on the band should be brought to the level of the bed-plate, when the pencil should be exactly on the 2·5 or mid-line of the drum (not diagram). If the pencil be found out of this position, the discrepancy should be measured and noted.

43. Then, the float and band should be raised into the observatory for examination and measurement, and the time noted. This is the first step in the dismantling of the gauge, preparatory to cleaning it. The total length of the band should be measured, also the distances from the painted 2·5 line and from the painted 0 line upon it to its junction with the float. During the inspection of the Bhávnagar tidal observatory in December 1887, it was found that the readings of the band and pencil agreed, but the measurements for determination of the working zero differed largely from what they ought to have been. The measurements and examination of the band disclosed that it had broken close to the float, and it was found that the observatory clerk had tried to conceal the breakage by attaching the float to the band at the place where the breakage had occurred. The float should be closely examined and, if any water is found in it, it should be repaired or renewed. In such a case it is interesting if the quantity of water which found its way into the float can be ascertained. The influx may have been sufficient to alter the balance of the instrument and raise the working zero. As an instance of this it was found that water had entered the float of the Dublat tide-gauge in November 1882, in sufficient quantity to raise the working zero 0·14 of a foot.

Dismantling gauge.

The dismantling of the gauge should now be completed, and it should be thoroughly cleaned and oiled where necessary; but the reference lines painted on the band and stud-wheel at the last inspection should not as yet be obliterated. It should be mentioned specially whether or not the driving clock required cleaning.

Refitting gauge.

44. After the several parts of the gauge are cleaned, they should be refitted carefully, the band being replaced so that when its painted 2·5 line of reference is level with the bed-plate, the pencil shall be at, or close to, the 2·5 line on the drum. The working of the band on the stud-wheel, while the pencil is being moved along the drum from its zero to the highest line of the diagram, should be tested; and it is best if this can be done at low water so as to reduce as much as possible the chance of the band kinking. The working should be smooth, and each hole of the band should fit over the studs freely. Any hole found too tight may be enlarged slightly with a file.

If the gauge is fitted, between the pencil and the float, with a silver wire, (which it is hoped will soon be superseded in every gauge by the new chain), and if the wire has been in use for a whole year, it should be renewed in the manner described at page 199, the best time for renewing it being about low water as there is then least danger of causing a kink in the band. The method described on page 199, indicates sufficiently well how a chain should be fitted, but a renewal of the latter is not likely to be often required.

The bed-plate should be made level both longitudinally and transversely if necessary, and if this operation be found, by spirit-levelling, to have altered the level of the bed-plate relatively to that of the bench-mark of reference, the alteration of level should be recorded.

Adjusting gauge.

45. The refitted gauge being now clean, level, and connected by levelling with the bench-mark of reference, is still unadjusted. To ascertain the amount of adjustment required, measurements for determination of the working zero at rising and falling tides should now be taken and recorded as having been made *after cleaning* the gauge. If a combination of the results of these measurements with the final level of the bed-plate make the distance below the bench-mark of reference of the

working zero, thus obtained, to differ from the distance of the true zero below the same bench-mark by a quantity appreciable on the scale of the diagram, the position of the pencil must be adjusted until the working zero coincides with the true zero. It is usual to take one more pair of sets of measurements for determination of the working zero (which should be registered as having been made *after cleaning* the gauge) as a final test of the perfect adjustment of the instrument.

The gauge being in adjustment, the reference lines painted on the band at last inspection should be compared with the pencil readings on the drum, and if the former are found out of position, they may now be obliterated and new lines substituted for them, special care being taken in the painting of the streaks, the *upper edges* of which mark where the readings of the bed-plate on the band correspond with the readings of the pencil on the engraved lines of the drum.

Auxiliary instruments.

46. The dismantling, cleaning, and refitting of the auxiliary instruments proceed hand in hand with the similar duties in connection with the self-registering tide-gauge. The auxiliary instruments are:—

Standard mercurial barometer.
Self-registering aneroid barometer.
Maximum and minimum thermometer.
Anemometer.
Rain-gauge.

Of these only the self-registering aneroid barometer and the anemometer require dismantling, cleaning, and refitting, and these operations are carried out as described in the following paragraphs.

Aneroid barometer.

47. The aneroid should be compared with the mercurial barometer, and its clock should be rated.

The position of the pencil-marker on the diagram should be made to agree with the reading shown on the dial, and adjusted, if necessary, by the screw; the diagrams should be examined to see if the marker is working freely; if they show a straight line, *i.e.*, no rises at 10 o'clock nor depressions at 4 o'clock, then the marker will be moving stiffly and requires cleaning.

Thermometer comparisons should be made between that on the aneroid and that attached to the mercurial barometer.

The clerk should be made to read the aneroid and mercurial barometers and the thermometer, and to set the maximum and minimum thermometers.

The diagrams should be examined, and the clerk told if the inking in has been properly done or not: the supply of blank diagrams should be noted, to see that there are plenty for future work.

Anemometer.

48. The direction of the vane with the wind, and the marking of the direction on the barrel should be tested. The upper part of the instrument should be oiled and the cups so marked that they cannot be put wrong.

The diagrams should be examined and the clock looked at, to see if new catgut or anything else is required. If the diagrams are faintly marked the bearing of the helices should be looked at—they should be quite free.

Miscellaneous duties before closing inspection.

49. Before ending his inspection, the inspecting officer should see the observatory clerk make an accurate comparison of the tide-gauge clock (which, like all the other clocks in the observatory, must keep *local time*) with the telegraphic or gun time, and enter the comparison in the report. The clerk must also show that, in addition to being able to rate the clock, he knows how to bring it to correct time when it is fast or slow, according to his printed instructions. A comparison of the reading of the pencil on the drum with that of the band at the bed-plate, and with the level of the water on the graduated staff, should be made and recorded after the tide-gauge has been put into adjustment. The inspecting officer should see that a conspicuous note is contained in the observatory report book, for the information not only of the observatory clerk but of the local official appointed to supervise him and superintend the working of the observatory, to the effect that whenever any interruption in the working of the tide-gauge takes place, owing *e.g.* to the stoppage of the driving clock, hourly readings should be taken on the diagram by day and night during the interruption. If for any reason these readings cannot be taken, then hourly readings of the graduated staff (the zero of which should agree with that of the

gauge) should be taken by day and night and entered in the daily reports. Should this amount of frequency be unattainable, then it is indispensable that readings at high and low water should be taken day and night and registered in the daily reports. If the cause of the interruption be of so serious a nature as to render necessary the removal of the instruments from the observatory, the promptest information should be sent to the officer in charge of the tidal operations to enable him to arrange for an inspection at the earliest possible date. The inspecting officer ends his inspection by taking a note of whatever diagrams, ink, books, pencils or other necessaries are required to be sent to the observatory.

CHAPTER III. COMPUTATIONS.

Preparation of tidal diagrams for reduction.

1. The tidal diagrams are examined and prepared for reduction in the head-quarter office in the following manner:—Vertical lines in red ink are drawn through each set of the points which have been marked by the clerk of the observatory, showing the position of the pencil at the exact hours of 7 A.M., 10 A.M., 4 P.M., and 6 P.M., and these lines are the bases for drawing vertical lines at the intermediate hours of the day. The daily reports are next examined to see if there are any clock errors amounting to 3 minutes or more, as compared with telegraphic time or gun signal; if there are, then (×'s) crosses in red ink are made *on* the tidal curves to show the exact position of each hour. The limit of 3 minutes error has been adopted because $\frac{1}{20}$ of an inch is the smallest distance which can be conveniently and accurately laid down in measuring along the curve, and $\frac{1}{20}$ inch = 3 minutes.

If the clock is fast, the cross is placed in *advance* of the hour-line; if slow, then *behind* the vertical time-line. Thus, suppose the clock 4 minutes *fast* the cross (×) is placed between the 2 and 3 P.M. lines at $\frac{1}{15}$ of an inch from the 2 P.M. line; if, however, the clock was *slow* by 4 minutes, then the cross (×) is put between the 1 and 2 P.M. lines at $\frac{1}{15}$ of an inch from the 2 P.M. line. As a rule, however, there is rarely any correction of this kind required, for when the clocks are properly attended to, errors of over 30 seconds are at once corrected by the clerks.

Interrupted curves or non-recorded curves caused by the stoppage of the clock, or other suspension of the tidal registration, are carefully filled in by drawing a curve in dotted lines exactly between the two contiguous instrumental curves.

The zero-lines to which all the measurements for height are referred are now laid down as indicated in the rules below, in which the terms 'true zero,' 'working zero,' 'accepted value of true zero,' and 'adopted level of bed-plate' have the following meanings:—

True zero.

2. The true zero is that which has been adopted in determining the datum-line for heights in the tide-tables. Its relative level with regard to the bench-mark of reference is fixed. As a rule, the zero corresponds to that originally adopted when the gauge was started, and its distance below the bed-plate was determined when the level of the bed-plate with regard to the bench-mark was fixed.

Working zero.

3. The working zero is the level of the water with reference to the bench-mark, corresponding to the pencil being on the zero-groove cut on the drum. In starting the instrument the working zero of course corresponds to the true zero, but from various causes the instrument may get out of adjustment, and its working zero may be altered. The position of the working zero on the diagram is always marked by the clerk rubbing over the groove cut in the drum with a hard pencil. In general at an inspection, the working zero is made to agree with the true zero by adjusting the instrument.

Accepted value of true zero.

4. The accepted value of the true zero is the distance of the true zero from the bed-plate, which was determined when the bed-plate was fixed as regards its relative level with the bench-mark of reference.

Adopted level of bed-plate.

5. The adopted level of the bed-plate means the level of the bed-plate with reference to the bench-mark, which has been adopted in determining the true zero; as a rule this will correspond to the level obtained when the observations commenced.

Rules for fixing true zero on diagrams.

6. The inspection book must first of all be examined to see if the bed-plate has altered in level relatively to the bench-mark. If there is any difference from

the adopted level exceeding ·02 of a foot, a correction will have to be applied on this account. The measurements for the determination of the *working zero* at the various inspections are next examined. If no alteration has been made in the adjustment of the gauge during the inspection, then the whole of the sets of measurements should be grouped, and the mean value would represent the distance of the working zero from the bed-plate on the day of the inspection.

If an adjustment has been made during an inspection, then those measurements for determination of zero before and after adjustment must be grouped separately, and the means respectively applied to the preceding and the following diagrams.

In treating the diagrams for any period between two inspections, the distance of the working zero from the bed-plate must be taken as the mean of the values obtained at the inspections.

The following are the cases which may occur and the ways of adjusting for them.

I. *Bed-plate settled below adopted level.*—The true zero will have to be placed above the working zero at a distance proportioned to the amount of the settlement in accordance with the scale of the diagram; hence the measurements from the true zero will be *less* than from the working zero.

II. *Bed-plate raised above adopted level.*—In this case the true zero will be placed below the working zero.

III. *Bed-plate unaltered and working zero at greater distance from bed-plate than accepted value for true zero.*—The true zero in this case will be placed above the working zero.

IV. *Bed-plate unaltered and working zero at less distance from bed-plate than accepted value for true zero.*—In this case the true zero will be placed below the working zero.

V. *Bed-plate settled and working zero at greater distance from bed-plate than true zero.*—In this case the true zero would be placed *above* the working zero at a distance equal *to the sum of the corrections* on account of each event.

VI. *Bed-plate settled and working zero at less distance from bed-plate than true zero.*—If the correction for settlement is the *greater of the two* the true zero will be placed *above* the working zero, and if the correction on account of the difference of zero-measurements was the greater then the true zero should be placed below the working zero. Obviously the amount in each case would be the difference of the two corrections.

VII. *Bed-plate raised and distance of working zero from bed-plate less than that accepted for true zero.*—In this case the true zero would be placed *below* the working zero at a distance equal to the sum of the two corrections.

VIII. *Bed-plate raised and distance of working zero from bed-plate greater, &c.*—The true zero should be placed *below* the working zero if the correction on account of the raising is the greater, and *above* if the latter correction is the greater. The distance between the two zeros is difference of the corrections.

N.B. If the determination of the true shows that the working zero comes within 0·005 of the true zero *on the diagram,* then no correction is considered necessary, and the working zero is used as the line of reference in measuring the heights from the diagram. What is meant by being within ·005 *on the diagram* is the actual difference between the true and working zeros reduced to scale.

Water getting into the float would have the effect of making the working zero *nearer* the bed-plate than the value formerly obtained, and this would have to be treated under IV, VI, or VII, according as the bed-plate had remained unaltered, had settled, or had been raised.

A kink in the band.—If this occurred, and zero-measurements were taken, it would have the effect of showing the zero so determined as being nearer the bed-plate than it would be if the kink were removed, and if the band righted itself in the course of working, the determination for zero at next inspection would be at a greater distance from the bed-plate than formerly.

Information to be recorded in the book entitled 'Determination of the True Zero on the Diagram' should be somewhat as follows:—

(1) Level of bed-plate with reference to B. M. unaltered, or settled by . . . . . . or raised by . . . . . .

No correction necessary, or correction under rule equal to . . .
has been applied to all diagrams from . . . . to . . . . . .

(2) Distance of working zero from bed-plate at inspection of . . . . 189 } feet } = value from . . . to . . . .
Distance of ditto ditto at inspection of . . . . 189 }

Correction on account of (1) or (2) or (1) and (2) = ... applied, and the true zero has been placed . . . . above/below the working zero from . . . . . to . . . . .

No other inspection having taken place, the value of the working zero at the inspection of . . . . . 189 , as given above, has been used in determining the true zero for the remainder of the diagrams, and for these diagrams the true zero has been placed . . . . . . above/below the working zero in accordance with rule No. . . . . .

Cases may occur which will have to be specially treated. All ordinary cases are here dealt with.

Intermediate lines, generally about 6 inches apart, are now laid down in red ink parallel to the true zero-line to facilitate the measurements. These are made with paper scales differently divided, according to the scale which may be adopted for the tidal diagram in each instance.

Harmonic analysis of tidal observations.

7. Before proceeding to an account of the method in which the observations are manipulated numerically, it will be advisable to give a brief sketch of some of the properties of harmonic curves, their connection with the tidal observations and the means of determining the various constants.

Harmonic curves.

8. Any curve of the form $y = a \cos (n x + b)$ is called a harmonic curve. The curve is periodic, that is to say, after a certain period it takes its original form: for if $x + \frac{2\pi}{n}$ is put for x, y again becomes $a \cos (n x + b)$.

If $\frac{2\pi}{n}$ is put equal to λ, then the quantity λ is called the 'wave-length' of the curve; for it is the distance along the axis of x between two successive equal and similarly placed ordinates. The constant a is called the 'amplitude' of the curve because its value is that of the greatest displacement. The angle $nx + b$ or $\frac{2\pi}{\lambda}x + b$ is called the 'phase' of the curve: the constant b is therefore known if the phase is given for any value of x.

Any two curves of equal wave-length may be combined into another of the same wave-length. For the equation

$$y = a_1 \cos\left(\frac{2\pi}{\lambda}x + b_1\right) + a_2 \cos\left(\frac{2\pi}{\lambda}x + b_2\right)$$

$$= (a_1 \cos b_1 + a_2 \cos b_2) \cos\frac{2\pi}{\lambda}x - (a_1 \sin b_1 + a_2 \sin b_2) \sin\frac{2\pi}{\lambda}x$$

$$= \text{A} \cos\left(\frac{2\pi}{\lambda}x + \text{B}\right),$$

where $$\text{A}^2 = a_1^2 + a_2^2 + 2\,a_1\,a_2 \cos(b_1 - b_2),$$

and $$\tan \text{B} = \frac{a_1 \sin b_1 + a_2 \sin b_2}{a_1 \cos b_1 + a_2 \cos b_2},$$

represents a harmonic curve of the same wave-length as the two components. Similarly for any number of curves of the same wave-length.

Two or more curves of different wave-lengths can however not be combined into a single harmonic curve; but if the wave-lengths are commensurable the resultant curve is periodic. For let

$$y = a_1 \cos\left(\frac{2\pi}{\lambda_1}x + b_1\right) + a_2 \cos\left(\frac{2\pi}{\lambda_2}x + b_2\right) + a_3 \cos\left(\frac{2\pi}{\lambda_3}x + b_3\right) + \text{\&c.},$$

and let λ be the least common multiple of λ_1, λ_2, λ_3, &c., so that their actual values are $\frac{\lambda}{m_1}, \frac{\lambda}{m_2}, \frac{\lambda}{m_3}$, &c., where m_1, m_2, m_3, &c., are integers;

then $$y = a_1 \cos\left(\frac{2\pi}{\lambda}m_1 x + b_1\right) + a_2 \cos\left(\frac{2\pi}{\lambda}m_2 x + b_2\right) + \text{\&c.},$$

and if $x + \lambda$ is put for x the value of y is unaltered, so that the resulting curve is a periodic curve of wave-length λ.

9. Such are a few of the properties of harmonic curves, and the next thing to be done is to point out the connection between harmonic curves and tidal observations.

Connection between harmonic curves and tidal observations.

In a *Report of a Committee for the Harmonic Analysis of Tidal Observations* for the British Association, 1883, Professor G. H. Darwin has deduced an expression for the height of the tide at any time: each term, which is of the harmonic form $R \cos (nt - \zeta)$, arises from some specific cause in the elaboration of the equilibrium theory of tides and is regarded as a separate tide due to this cause. Thus there are as many tides as there are terms in the series, and the height of each simple tide is equal to a constant, R, multiplied by the cosine of a certain angle $nt-\zeta$ called the 'argument', which is partly made up of a simple function of the time and partly dependent on the position of the sun or moon or both.

The maximum value of the cosine being unity, the constant, R, gives the greatest height above the mean of the particular tide, that is, the 'semi-range' or 'amplitude'.

The part of the argument which is a function of the time is of the form nt, so that n represents the rate at which the argument increases: it is called the 'speed' of the tide and is reckoned in mean solar hours. Also since the tide's maximum occurs when the remainder of the argument, *viz.*, ζ is equal to nt, it follows that $\frac{\zeta}{n}$ gives the time which must elapse from the beginning of the observations till the time of the first high water of the tide: ζ is therefore called the 'epoch'.

10. For the purpose of arithmetical calculation the form $R \cos(nt-\zeta)$, in which the tide is presented, is not convenient and it is therefore expanded into

Determination of constants R and ζ.

$$A \cos nt + B \sin nt,$$

so that
$$R^2 = A^2 + B^2 \quad \text{and} \quad \tan \zeta = \frac{B}{A};$$

and the immediate object of the numerical reductions is to find the A's and B's, from which the R's and ζ's are at once obtained by means of the above equations. It now remains to explain how the A's and B's are determined from the observations.

The expression deduced by Darwin gives for the height of the tide at any time,

$$h = A_0 + \Sigma (A \cos nt) + \Sigma (B \sin nt).$$

Now among these n's will be found n_1, $2n_1$, $3n_1$ &c.; n_2, $2n_2$, $3n_2$; and so on, and it would be practically impossible to determine the corresponding A's and B's in a direct manner. It has, however, been found possible, by a method of manipulation of the observed quantities which will be explained below, to separate the terms containing n_1 from all the others and then the problem presents no difficulty. As will be explained, it is reduced to the question of determining the constants from a series of equations of the form

$$h = A_0 + A_1 \cos nt + B_1 \sin nt + A_2 \cos 2nt + B_2 \sin 2nt + \&c.,$$

where t has any integral value from 0 to $\frac{2\pi}{n}$, so that nt goes through its variations from 0 to 2π.

Now it is clear, if r and s be any two integers, and the summation extends from $t = 0$ to $t = \frac{2\pi}{n}$, that

$$\Sigma \cos rnt = 0, \text{ and } \Sigma \sin rnt = 0,$$

since to each positive value, there is a corresponding negative value of the cosine or sine.

$$\text{Also } \Sigma \cos^2 rnt = \tfrac{1}{2} \Sigma (1 + \cos 2rnt) = \frac{\pi}{n},$$

and

$$\Sigma \sin^2 rnt = \tfrac{1}{2} \Sigma (1 - \cos 2rnt) = \frac{\pi}{n};$$

$$\Sigma \cos rnt \cos snt = \tfrac{1}{2} \Sigma \cos(r + s)nt + \tfrac{1}{2} \Sigma \cos(r - s)nt = 0,$$

and similarly

$\Sigma \cos rnt \sin snt$ and $\Sigma \sin rnt \sin snt$ are each equal to zero.

Consequently for determining the constants there are the following equations:—

$$\Sigma h = \frac{2\pi}{n} A_0, \qquad \text{or} \quad A_0 = \frac{n}{2\pi} \Sigma h;$$

$$\Sigma h \cos rnt = \frac{\pi}{n} A_r, \quad \text{or} \quad A_r = \frac{n}{\pi} \Sigma h \cos rnt;$$

$$\Sigma h \sin rnt = \frac{\pi}{n} B_r, \quad \text{or} \quad B_r = \frac{n}{\pi} \Sigma h \sin rnt.$$

The A's and B's being now determined, the R's and ζ's are calculated from the two formulæ given above.

Determination of constants H and κ.

11. But in order that a comparison of the records of different years may be made, it is necessary to exhibit the height of the tide in yet a different form; for when it is represented by $R \cos (nt - \zeta)$, it is clear that ζ may have any value from 0 to 360° and that the results of the analysis of successive years of observations will not be comparable with each other.

Such being the case, let it be supposed that the results of the analysis are presented in a number of terms of the form

$$f \, \mathrm{H} \cos (V + u - \kappa).$$

Here V is a linear function of the moon's and the sun's mean longitudes, the mean longitudes of the moon's and the sun's perigees, and the local mean solar time at the place of observation reduced to angle at 15° per hour. V therefore increases uniformly with the time and its rate of increase per mean solar hour is the n of the first method, or the 'speed' of the tide.

It is supposed that u stands for a certain function of the longitude of the node of the lunar orbit, at an epoch half a year later than 0^h of the first day. Strictly speaking, u should be taken as this same function of the longitude of the moon's node, varying as the node moves; but as the variation is but small in the course of a year, u may be treated as a constant and put equal to an average value for the year, which average value is taken as the true value of u at exactly mid-year.

Together $V + u$ constitute the whole 'argument'* according to the equilibrium theory of tides, with the sea covering the whole earth; and it therefore follows that $\frac{\kappa}{n}$ is the lagging of the tide which arises

* See Schedules [B, i], [B, ii], [B, iii] and C of Professor Darwin's Report.

from kinetic action, friction of the water, imperfect elasticity of the earth, and the distribution of the land.

It is also supposed that H is the mean value in British feet of the semi-range of the particular tide in question; and f is a numerical factor of augmentation or diminution due to the variability of the obliquity of the lunar orbit: the method of determining it is fully explained in, and its values tabulated at the end of, Professor Darwin's Report and also in the Auxiliary Tables appended.

It is obvious, then, that, if the tidal observations are consistent from year to year, H and κ should come out the same from each year's reductions: and it is only when the results are presented in such a form as this, that it will be possible to judge whether the harmonic analysis is giving satisfactory results.

The determination of H and κ from R and ζ is made as follows:— Clearly $H = \frac{R}{f}$ and is at once found; also $nt - \zeta$ is identical with $V + u - \kappa$, so that if V_0 be the value of V at 0^h of the first day, that is when $t = 0$, then,

$$-\zeta = V_0 + u - \kappa;$$

so that

$$\kappa = \zeta + V_0 + u.$$

Thus the rule for the determination of κ is:—add to the value of ζ the value of the 'argument'* at 0^h of the first day.

12. Hitherto the tides have been spoken of as if they each were treated in the same way, but this is not the case. For the purposes of calculation they are divided into 'short-period' and 'long-period' tides. The short-period tides are still further sub-divided into semi-diurnal and diurnal tides. The former have periods equal or nearly equal to 12 mean solar hours and the latter have periods equal or nearly equal to 24 mean solar hours. Besides these there are also some 'over-tides' and 'compound-tides'; their origin is explained in Professor Darwin's Report, and their periods approximate to some submultiple of 12 mean solar hours. The long-period tides have periods of about a fortnight, a month, half a year or a year as the case may be.

Short and long-period tides.

* See Schedules [B, i], [B, ii], [B, iii] and C of Professor Darwin's Report.

The method of determining the A's and B's described above is only applicable to the short-period tides: the means of determining them for the long-period tides will be described hereafter.

Description of numerical harmonic analysis for short-period tides.

13. The tide-gauge gives a graphical record of the height of the water above some known datum for every instant of time. The first operation performed on the tidal record is the measurement, in feet and decimals, of the height of the water above the true zero of the gauge (the height of which relatively to the datum is known) at every mean solar hour. The period chosen for analysis is about one year, and the first measurement corresponds to noon, but it has been found inconvenient hitherto to have the same initial noon at the several ports.

It would seem, at first sight, preferable to take the measurements at each mean lunar hour; but the whole of the actual process in use is based on measurements taken at the mean solar hours, and a change to lunar time would involve a great deal of fresh labour and expense.

If T be the period of any one of the diurnal tides, or the double period of any one of the semi-diurnal tides, it approximates more or less nearly to 24 mean solar hours, and if it be divided into 24 equal parts, each part may be spoken of as a T-hour, while for brevity mean solar time will be referred to as S-time.

Suppose, now, that there are two clocks, each marked with 360° or 24 hours, and that the hand of the first or S-clock goes round once in 24 S-hours and that of the second or T-clock goes round once in 24 T-hours; and suppose that the two clocks are started at 0° or 0^h at noon of the initial day. For the sake of distinctness, imagine that a T-hour is longer than an S-hour so that the T-clock goes slower than the S-clock.

The measurements of the tide-curve give the height of the water exactly at each S-hour; and it is required from these data to determine the height of the water at each T-hour. For this end, it is necessary to count T-time; but this must be done with reference to S-time and, moreover, the time must always be specified as an integral number of hours.

Beginning, then, with 0^h of the first day, it is necessary to count 0, 1, 2, &c., as the T-hand comes up to its hour marks. But as the

S-hand gains on the T-hand, there will come a time when the T-hand, being exactly at the p hour-mark, the S-hand is nearly as far as $p + \frac{1}{2}$. When, however, the T-hand has advanced to the $p + 1$ hour-mark, the S-hand will be a little beyond $p + 1 + \frac{1}{2}$; that is to say, a little less than half-an-hour before $p + 2$. Counting, then, the T-hours in S-time, it is necessary to jump from p to $p + 2$. The counting will go on continuously for a number of hours nearly equal to $2p$, and then another number will be dropped, and so on throughout the whole year. If the T-hand went faster than the S-hand, it is obvious that one number would be repeated at two successive hours instead of one being dropped. Each such process may be described as a 'change'.

Now if there is a sheet marked for entries of heights of water according to T-hours from results measured at S-hours, the S-measurements must be entered continuously up to p: then comes a 'change' and the dropping of one of the S-series, after which the entry goes on continuously until another 'change' when another is dropped, and so on.

Since a 'change' occurs at the time when a T-hour falls almost exactly between two S-hours, it will be more accurate to insert the two S-entries which fall on each side of the truth. If this be done the whole of the S-series of measurements is entered on the T-sheet. Similarly if it is the T-hand which goes faster than the S-hand, a gap may be left in the T-series instead of duplicating an entry. For the analysis of the T-tide there is, therefore, prepared a sheet arranged in rows and columns: each row corresponds to one T-day and the columns are marked 0^h, 1^h, ... 23^h; the 0^h's may be called T-noons. A dot is put in each space for entry, and where there is a 'change' two dots are put if there is to be a double entry, and a bar if there is to be no entry; black vertical lines mark the end of each S-day. These black lines will, of course, fall into slightly irregular diagonal lines across the page, being steeper and steeper the more nearly T-time approaches to S-time. They slope downwards from right to left if the T-hour is longer than the S-hour, and the other way in the opposite case. The 'changes' also run diagonally with a slope in the opposite direction to that of the black lines when the T-hour is longer than the S-hour, and in the same direction in the opposite case.

A sample is annexed of parts of pages drawn up for the entries of the M-series and J-series of tides, in the former of which T-time is mean lunar time.

M—SERIES

	0h	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h	12h	13h	14h	15h	16h	17h	18h	19h	20h	21h	22h	23h	S-hour
1	·	·	·	·	·	·	·	·	·	·	·	·	·	·	:	·	·	·	·	·	·	·	·	·	2d 0h
2	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	:	·	·	·	·	·	3 1
3	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	:	4 2
4	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	5 2
5	·	·	·	:	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	6 3
				&c.				&c.					&c.				&c.				&c.				
67	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	70 7
68	·	·	·	:	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	71 8
69	·	·	·	·	·	·	·	·	:	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	72 9
70	·	·	·	·	·	·	·	·	·	·	·	·	·	:	·	·	·	·	·	·	·	·	·	·	73 10
71	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	:	·	·	·	·	·	·	74 11
Sum	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	No. of days
No.	73	73	74	74	73	73	73	73	75	73	73	73	74	74	74	73	73	75	74	73	73	74	73	74	73d 12h

J—SERIES

	0h	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h	12h	13h	14h	15h	16h	17h	18h	19h	20h	21h	22h	23h	S-hour
1	·	·	·	·	·	·	·	·	·	·	·	·	·	—	·	·	·	·	·	·	·	·	·	·	1d 22h
2	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	—	·	·	·	·	·	·	·	2 21
3	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	—	·	·	·	·	3 20
4	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	—	·	·	4 19
5	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	5 19
				&c.				&c.					&c.				&c.				&c.				
72	·	·	·	·	·	·	·	·	·	·	·	·	·	—	·	·	·	·	·	·	·	·	·	·	70 6
73	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	—	·	·	·	·	·	·	·	71 5
74	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	—	·	·	·	·	·	72 4
75	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	—	·	·	73 3
76	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	74 3
Sum	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	No. of days
No.	73	74	73	73	74	73	73	74	73	73	73	74	73	72	74	74	71	74	73	73	73	72	74	73	73d 4h

Since the first day is numbered 1 and the first hour 0^h, it follows that to find the number of hour values entered in the form from 0^h of the first day, it is necessary to subtract 1 from the number of the day and to add 1 to the number of the hour.

For each class of tide there are five pages similar to the annexed examples, giving in all about 370 values for the height of the water at each of the 24 special hours: the number of values for each hour varies slightly according as more or less 'changes' fall into each column.

The numbers entered in each column are summed on each of the five pages; the five sets of results are now summed and the results divided each by the proper divisor for its column, thus giving a mean value for that column. In this way 24 numbers are found which give the mean height of the water at each of the 24 special hours.

Eliminating effects of other tides.

14. It is obvious that if this process were continued over a very long time, in the end, the tide under analysis would be extracted from among all the others; but as the process only extends over about a year, the elimination of the others is not quite complete. The elimination of the effects of the other tides may be improved by choosing the period for analysis not exactly equal to one year. For, suppose that the expression for the height of the water is

$$A_1 \cos n_1 t + B_1 \sin n_1 t + A_2 \cos n_2 t + B_2 \sin n_2 t,$$

where n_2 is nearly equal to n_1, and that it is required to eliminate the n_2-tide so as to be left only with the n_1-tide.

The expression may then be put equal to

$$\{ A_1 + A_2 \cos (n_1 - n_2) t - B_2 \sin (n_1 - n_2) t \} \cos n_1 t$$

$$+ \{ B_1 + A_2 \sin (n_1 - n_2) t + B_2 \cos (n_1 - n_2) t \} \sin n_1 t;$$

which shows that the tide may be regarded as oscillating with a speed n_1, but with slowly varying range. Now, in the column appertaining to any hour in the form, $n_1 t$ is a multiple of 15° if n_1 be a diurnal, and of 30°, if n_1 be a semi-diurnal tide. Consider the column headed 'p-hours'; then $n_1 t = 15^\circ p$ for diurnals and $30^\circ p$ for semi-diurnals.

Hence the sum of all the entries, of which suppose there are q, in the column numbered p-hours, is for diurnal tides,

$$\cos 15^\circ p\Big\{A_1 q + A_2\Big[\cos(n_1 - n_2)\frac{15p}{n_1} + \cos(n_1 - n_2)\Big(\frac{2\pi}{n_1} + \frac{15p}{n_1}\Big)$$

$$+ \cos(n_1 - n_2)\Big(2\frac{2\pi}{n_1} + \frac{15p}{n_1}\Big)\Big] + B_2[\&c.]\Big\} + \sin 15^\circ p\{\&c.\} \quad . \quad . \quad . \quad . \quad (a)$$

and for semi-diurnal tides, the arguments of all the circular functions in the expression (a) are to be doubled.

Now such a number of terms is to be chosen, that the series by which A_2 and B_2 are multiplied may vanish. This is exactly the case, if the series is exactly re-entrant and is nearly the case, if nearly re-entrant.

The condition is exactly satisfied, if

$$(n_1 - n_2)\,q\,\frac{2\pi}{n_1} = 2\pi r \quad \text{for diurnal tides,}$$

or

$$(n_1 - n_2)\,q\,\frac{4\pi}{n_1} = 2\pi r \quad \text{for semi-diurnal tides,}$$

where r is either a positive or negative integer.

That is to say, if

$$(n_1 - n_2)\,q = n_1 r \quad \text{for diurnal tides,}$$

or

$$(n_1 - n_2)\,q = \tfrac{1}{2}\,n_1 r \quad \text{for semi-diurnal tides.}$$

It is not worth while attempting to eliminate the effects of the semi-diurnal tides on the diurnal tides and *vice versâ*, because the periods could only differ by the fraction of a day and owing to the incommensurability of the speeds, it is impossible to avoid being wrong to that amount.

It is of course impossible to choose for each tide n_1, a period which shall minimise the effects of more than one of the tides of short-period n_2, in vitiating the values of the mean semi-ranges of the tide n_1; and accordingly the periods have been chosen so as to minimise the effect of the principal solar semi-diurnal tide S_2 on the principal lunar semi-diurnal tide M_2, and of the M_2 tide upon the other semi-diurnal tides: in the case of the diurnal tides, the periods are chosen to minimise the effects of either O or K_1.

Returning now to the general notation and considering the 24 mean values, each pertaining to the 24 T-hours, it may be supposed that all the tides, excepting the T-tide, are adequately eliminated, and, in fact, a computation of the necessary corrections for the absence of complete elimination, which is given in the Tidal Report of 1872, of the Tidal Committee of the B. A. under the presidency of Sir William Thomson, shows that this is the case.

Augmenting factors for correcting heights.

15. Now it is obvious that any one of the 24 values does not give the true height of the T-tide at that T-hour, but gives the average height of the water, as due to the T-tide, estimated over half a T-hour before and half a T-hour after that hour. A correction must therefore be determined on this account.

The required expression for the height of the tide at any T-hour is

$$h = A_1 \cos\theta + B_1 \sin\theta + \text{\&c., \&c.} + A_r \cos r\theta + B_r \sin r\theta + \text{\&c.}$$

But the results of analysis give instead of this the mean of all the h's between the limits $\theta + \frac{a}{2}$ and $\theta - \frac{a}{2}$.

That is $ha = \Sigma h$ between these limits

$= \text{\&c.} + \Sigma A_r \cos r\theta + \Sigma B_r \sin r\theta + \text{\&c.}$ between these limits,

or $$ha = \text{\&c.} + A_r \frac{2}{r} \sin\frac{ra}{2} \cos r\theta + B_r \frac{2}{r} \sin\frac{ra}{2} \sin r\theta + \text{\&c.}$$

whence $$h = \text{\&c.} + \frac{\sin\frac{ra}{2}}{\frac{ra}{2}} A_r \cos r\theta + \frac{\sin\frac{ra}{2}}{\frac{ra}{2}} B_r \sin r\theta + \text{\&c.}$$

Consequently the coefficients that express the oscillation which goes through its period r times in 24 T-hours, must be augmented by the factor $\frac{\frac{ra}{2}}{\sin\frac{ra}{2}}$ to give the true A_r and B_r.

Remembering that a is 15° and putting for r, 1, 2, 3, &c., in succession, the augmenting factors for the diurnal, semi-diurnal, ter-diurnal oscillations, &c., become

$$\frac{7{\cdot}5\,\pi}{180 \sin 7^\circ\, 30'};\quad \frac{15\,\pi}{180 \sin 15^\circ};\quad \frac{22{\cdot}5\,\pi}{180 \sin 22^\circ\, 30'};\ \text{\&c.}$$

Thus, the augmenting factors are:—

for	A_1, B_1	1·00286
	A_2, B_2	1·01152
	A_3, B_3	1·02617
	A_4, B_4	1·04720
	A_6, B_6	1·11072
	A_8, B_8	1·20920

In the reduction of the S-series of tides, the numbers treated are the actual heights of the water exactly at the S-hours, and therefore no augmenting factor is requisite.

Determination of A's and B's.

16. If now t denotes T-time expressed in hours and n is 15°, the height h, as expressed by the averaging process explained above, is given by the formula

$$h = A_0 + A_1 \cos nt + B_1 \sin nt + A_2 \cos 2nt + B_2 \sin 2nt + \&c.,$$

where t is 0, 1, 2 .... 23.

Then, if Σ is the sum of the series of 24 terms found by giving t its 24 values, as before shown,

$$A_0 = \tfrac{1}{24}\Sigma h; \quad A_1 = \tfrac{1}{12}\Sigma h \cos nt; \quad B_1 = \tfrac{1}{12}\Sigma h \sin nt;$$

$$A_2 = \tfrac{1}{12}\Sigma h \cos 2nt; \quad B_2 = \tfrac{1}{12}\Sigma h \sin 2nt; \quad \&c., \&c.$$

Also, since $n = 15^\circ$ and t is an integer, all the cosines and sines involved are equal to one of the following:— 0; $\pm\sin 15^\circ$; $\pm\sin 30^\circ$; $\pm\sin 45^\circ$; $\pm\sin 60^\circ$; $\pm\sin 75^\circ$; ± 1. These are denoted in the computation forms by 0, $\pm S_1$, $\pm S_2$, $\pm S_3$, $\pm S_4$, $\pm S_5$, ± 1.

This enables the forms to be arranged in the neat tabular form on pages 1, 2 and 3 of the Analysis of Short-Period Tides, a specimen of which is given in Form No. VI, where the 24 hourly values to be submitted to analysis are written continuously down columns I and II. The subsequent operations are sufficiently indicated by the headings to the columns, and it will be found on examination that the results are in reality the sums of the several series given above.

LIST OF

Initial	Speed in symbols and in m. s. hours		Name of tide
S_1	$\gamma-\eta$	15°	Principal Solar Diurnal
S_2	$2(\gamma-\eta)$	30	„ „ Semi-Diurnal
S_4	$4(\gamma-\eta)$	60	Principal Solar Series Over-Tides
S_6	$6(\gamma-\eta)$	90	
S_8	$8(\gamma-\eta)$	120	
P_1	$\gamma-2\eta$	14 ·9589314	Solar (Declinational) Diurnal
T_2	$2\gamma-3\eta$	29 ·9589314	Larger Solar Elliptic
M_1	$\gamma-\sigma-\varpi$ and $\gamma-\sigma+\varpi$	14 ·4920521	Principal Lunar Diurnal
M_2	$2(\gamma-\sigma)$	28 ·9841042	„ „ Semi-Diurnal
M_3	$3(\gamma-\sigma)$	43 ·4761563	Principal Lunar Series Over-Tides
M_4	$4(\gamma-\sigma)$	57 ·9682082	
M_6	$6(\gamma-\sigma)$	86 ·9523126	
M_8	$8(\gamma-\sigma)$	115 ·9364164	
K_1	γ	15 ·0410686	Luni-Solar Diurnal (Declinational)
K_2	2γ	30 ·0821372	„ „ Semi-Diurnal „
N_2	$2\gamma-3\sigma+\varpi$	28 ·4397296	Larger Lunar Elliptic
$2N_2$	$2\gamma-4\sigma+2\varpi$	27 ·8953548	Lunar Elliptic, 2nd order
L_2	$2\gamma-\sigma-\varpi$ and $2\gamma-\sigma+\varpi$	29 ·5284788	Smaller Lunar Elliptic
ν_2	$2\gamma-3\sigma-\varpi+2\eta$	28 ·5125830	Larger Lunar Evectional
O_1	$\gamma-2\sigma$	13 ·9430356	Lunar (Declinational) Diurnal
Q_1	$\gamma-3\sigma+\varpi$	13 ·3986609	Larger Lunar Elliptic Diurnal
J_1	$\gamma+\sigma-\varpi$	15 ·5854433	Lunar Elliptic Diurnal
$(MS)_4$	$4\gamma-2\sigma-2\eta$	58 ·9841042	Compound Tides
μ_2 or 2MS	$2\gamma-4\sigma+2\eta$	27 ·9682084	
$(2SM)_2$	$2\gamma+2\sigma-4\eta$	31 ·0158958	
$(M_2K_1)_3$	$3\gamma-2\sigma$	44 ·0251728	
$(2M_2K_1)_3$	$3\gamma-4\sigma$	42 ·9271398	
$(M_2N)_4$	$4\gamma-5\sigma+\varpi$	57 ·4238338	
MSf	$2\sigma-2\eta$	1 ·0158958	Luni-Solar Synodic Fortnightly
Mm	$\sigma-\varpi$	0 ·5443747	Lunar Monthly
Mf	2σ	1 ·0980330	„ Fortnightly
Sa	η	0 ·0410686	Solar Annual
Ssa	2η	0 ·0821372	„ Semi-Annual

TIDES.

Initial Argument $= V_0 + u$	Factor for reduction $= \frac{1}{f}$	Initial
Zero	Unity	S_1
"	"	S_2
"	"	S_4
"	"	S_6
"	"	S_8
$-h_0 + \frac{1}{2}\pi$	"	P_1
$-(h_0 - p_1)$	"	T_2
$(h_0 - \nu) - (s_0 - \xi) + Q - \frac{1}{2}\pi$ where $\tan Q = \frac{1}{2}\tan P$	Fac. O $\div \sqrt{\frac{5}{2} + \frac{3}{2}\cos 2P}$	M_1
$2(h_0 - \nu) - 2(s_0 - \xi)$	$\left(\frac{\cos\frac{1}{2}\omega\cos\frac{1}{2}i}{\cos\frac{1}{2}I}\right)^4$	M_2
$\frac{3}{2}$ Arg. M_2	(Fac. $M_2)^{\frac{3}{2}}$	M_3
2 Arg. M_2	(Fac. $M_2)^2$	M_4
3 Arg. M_2	(Fac. $M_2)^3$	M_6
4 Arg. M_2	(Fac. $M_2)^4$	M_8
$h_0 - \nu' - \frac{1}{2}\pi$ where $\tan\nu' = \frac{\sin\nu}{\cos\nu + \cdot464\,k}$	$\frac{1\cdot46407\,k}{\sqrt{1 + (\cdot464\,k)^2 + \cdot928\,k\cos\nu}}$ where $k = \frac{\sin 2\omega(1 - \frac{3}{2}\sin^2 i)}{\sin 2I}$	K_1
$2h_0 - 2\nu''$ where $\tan 2\nu'' = \frac{\sin 2\nu}{\cos 2\nu + \cdot464\,k}$	$\frac{1\cdot46407\,k}{\sqrt{1 + (\cdot464\,k)^2 + \cdot928\,k\cos 2\nu}}$ where $k = \frac{\sin^2\omega\,(1 - \frac{3}{2}\sin^2 i)}{\sin^2 I}$	K_2
Arg. $M_2 - (s_0 - p_0)$	Fac. M_2	N_2
Arg. $N_2 - (s_0 - p_0)$	Fac. M_2	$2N_2$
Arg. $M_2 + (s_0 - p_0) - R + \pi$ where $\tan R = \frac{\sin 2P}{\frac{1}{6}\cot^2\frac{1}{2}I - \cos 2P}$	Fac. $M_2 \div \sqrt{1 - 12\tan^2\frac{1}{2}I\cos 2P}$	L_2
Arg. $M_2 + (s_0 - p_0) + 2h_0 - 2s_0$	Fac. M_2	ν_2
$(h_0 - \nu) - 2(s_0 - \xi) + \frac{1}{2}\pi$	$\frac{\sin\omega\cos^2\frac{1}{2}\omega\cos^4\frac{1}{2}i}{\sin I\cos^2\frac{1}{2}I}$	O_1
Arg. $O_1 - (s_0 - p_0)$	Fac. O_1	Q_1
$(h_0 - \nu) + (s_0 - p_0) - \frac{1}{2}\pi$	$\frac{\sin 2\omega\,(1 - \frac{3}{2}\sin^2 i)}{\sin 2I}$	J_1
Arg. M_2	Fac. M_2	$(MS)_4$
Arg. M_4	Fac. M_4	μ_2 or 2MS
$2\pi -$ Arg. M_2	Fac. M_2	$(2SM)_2$
Arg. M_2 + Arg. K_1	Fac. $M_2 \times$ Fac. K_1	$(M_2K_1)_3$
Arg. M_4 − Arg. K_1	Fac. $M_4 \times$ Fac. K_1	$(2M_2K_1)_3$
Arg. M_2 + Arg. N_2	Fac. $M_2 \times$ Fac. N_2	$(M_2N)_4$
$2\pi -$ Arg. M_2	Fac. M_2	MSf
$s_0 - p_0$	$\frac{(1 - \frac{3}{2}\sin^2\omega)\,(1 - \frac{3}{2}\sin^2 i)}{(1 - \frac{3}{2}\sin^2 I)}$	Mm
$2(s_0 - \xi)$	$\frac{\sin^2\omega\cos^4\frac{1}{2}i}{\sin^2 I}$	Mf
h_0	Unity	Sa
$2h_0$	"	Ssa

17. The preceding table contains a list of the tides, at present subjected to analysis, together with their speeds, arguments and factors for reduction. The symbols in the table have the following meanings:—

List of tides.

γ = earth's angular velocity of rotation.
σ = mean motion of the moon.
η = „ „ „ sun.
ϖ = „ „ „ lunar perigee.
h = sun's mean longitude.
p = mean longitude of the moon's perigee.
ν = right ascension of the 'intersection' or descending node of the equator on the lunar orbit.
s = moon's mean longitude.
ξ = longitude 'in the moon's orbit' of the 'intersection'.
I = obliquity of the lunar orbit to the equator.
ω = „ „ ecliptic.
i = inclination of the moon's orbit to the ecliptic.
P = longitude of the moon's perigee at mid-year measured from the 'intersection'.

The letters with the zero subscript represent the values of the corresponding functions at 0^h of the 1st day of the year of observation.

18. For the purpose of determining the tides of long period, it is necessary to eliminate the oscillations of water-level arising from the tides of short period. As the quickest of the tides of long period has a period of many days, the height of the water at one instant for each day gives sufficient data. Thus there will, in a year's observations, be 365 heights to be submitted to harmonic analysis. In leap-years, the last day's observation must be dropped, because the treatment is adapted for analysing 365 values.

Long-period tides.

In finding the value of the height of the water for each day, the algebraical mean of 24 consecutive hourly values, beginning with the height at noon, is taken: the result will then apply to the middle instant of the period 0^h to 23^h, that is to say to $11^h\ 30^m$ at night.

19. The formation of a daily mean does not obliterate the tidal oscillations of short period, because none of the tides, excepting those of the principal solar series, have commensurable periods in mean solar time.

Clearance for short-period tides.

A correction, or 'clearance of the daily mean', should therefore be applied for all the important tides of short period, excepting for the solar tides.

Let $R\cos(nt-\zeta)$ be the expression for one of the tides of short period, as evaluated by the harmonic analysis for the same year: and let a be the value of $nt-\zeta$ at any noon. Then the 24 consecutive hourly heights of the water due to this tide, beginning with that noon, are:—

$$R\cos a\,;\quad R\cos(n+a)\,;\quad R\cos(2n+a)\,\ldots\,R\cos(23n+a)\,;$$

the sum of these is $R\,\dfrac{\sin 12n}{\sin\frac{1}{2}n}\cos(a+11\tfrac{1}{2}n)$, so that the 'clearance of the daily mean' is $-\tfrac{1}{24}R\,\dfrac{\sin 12n}{\sin\frac{1}{2}n}\cos(a+11\tfrac{1}{2}n)$, and is additive.

It has been found, practically, that only three tides of short period *viz.*, M_2, N, O, exercise any appreciable effect, so that clearances for them have to be applied. It was formerly the custom to compute the clearances for these three tides, for every day in the year, as above and to correct the daily means accordingly: but the procedure now is different, and a single correction, for each short-period tide, is applied to each of the final equations, instead of to each daily mean. The process will be explained more fully below.

Uncleared equations.

20. The mean of the 365 quantities is now taken to give the mean height of the water for the year; and it is evident that, even if the daily means are uncleared from the effects of the short-period tides, as is the case in practice, this yearly mean cannot be sensibly vitiated.

The yearly mean height is next subtracted from each of the 365 daily means, and 365 quantities, δh, are found giving the mean daily height of the water above the mean yearly height.

These quantities are to be the subject of harmonic analysis: and the tides chosen for evaluation are those which have been denoted above as Mm, Mf, MSf, Sa and Ssa.

Let

$$\begin{aligned} \delta h = \; & A\cos(\sigma-\varpi)t + B\sin(\sigma-\varpi)t \\ & + C\cos 2\sigma t + D\sin 2\sigma t \\ & + C'\cos 2(\sigma-\eta)t + D'\sin 2(\sigma-\eta)t \\ & + E\cos\eta t + F\sin\eta t \\ & + G\cos 2\eta t + H\sin 2\eta t, \end{aligned}$$

where t is time measured from the first $11^h\ 30^m$.

Then a little manipulation, for which the reader is referred to Professor Darwin's Report of 1883, gives the following equations:—

	Coefficient of A	Coefficient of B	Coefficient of C	Coefficient of D	Coefficient of C′	Coefficient of D′	Coefficient of E	Coefficient of F	Coefficient of G	Coefficient of H
$\Sigma\delta h \times \cos(\sigma-\varpi)t =$	+183·05	+ 2·14	+ 0·73	+ 4·29	+ 0·77	+ 5·04	+ 4·88	− 0·34	+ 4·96	− 0·69
„ $\times \sin(\sigma-\varpi)t =$	+ 2·14	+181·95	− 4·15	+ 1·02	− 4·90	+ 1·07	+ 3·80	+ 0·34	+ 3·88	+ 0·69
„ $\times \cos 2\sigma t =$	+ 0·73	− 4·15	+183·18	+ 0·88	+ 0·61	+ 0·92	− 1·50	− 0·10	− 1·51	− 0·19
„ $\times \sin 2\sigma t =$	+ 4·29	+ 1·02	+ 0·88	+181·82	+ 0·92	− 0·75	+ 3·05	− 0·08	+ 3·06	− 0·17
„ $\times \cos 2(\sigma-\eta)t =$	+ 0·77	− 4·90	+ 0·61	+ 0·92	+183·19	+ 0·97	− 1·68	− 0·11	− 1·70	− 0·23
„ $\times \sin 2(\sigma-\eta)t =$	+ 5·04	+ 1·07	+ 0·92	− 0·75	+ 0·97	+181·81	+ 3·25	− 0·10	+ 3·27	− 0·23
„ $\times \cos\eta t =$	+ 4·88	+ 3·80	− 1·50	+ 3·05	− 1·68	+ 3·25	+182·43	+ 0·00	− 0·14	+ 0·00
„ $\times \sin\eta t =$	− 0·34	+ 0·34	− 0·10	− 0·08	− 0·11	− 0·10	+ 0·00	+182·57	+ 0·00	+ 0·00
„ $\times \cos 2\eta t =$	+ 4·96	+ 3·88	− 1·51	+ 3·06	− 1·70	+ 3·27	− 0·14	+ 0·00	+182·43	+ 0·00
„ $\times \sin 2\eta t =$	− 0·69	+ 0·69	− 0·19	− 0·17	− 0·23	− 0·23	+ 0·00	+ 0·00	+ 0·00	+182·57

21. The left-hand sides of these equations must now be cleared from the effects of the three tides of short period. This is done in the following manner:—

Numerical determination of clearances.

It has been shown before that the 'clearance' is

$$-\tfrac{1}{24}\,R\,\frac{\sin 12n}{\sin\frac{1}{2}n}\cos(nt-\zeta+11\tfrac{1}{2}n).$$

The proper clearances therefore to be applied to the left-hand sides of the first and second equations will be

$$-\tfrac{1}{24}\,\Sigma\,R\,\frac{\sin 12n}{\sin\frac{1}{2}n}\cos(nt-\zeta+11\tfrac{1}{2}n)\cos(\sigma-\varpi)t,$$

and
$$-\tfrac{1}{24}\,\Sigma\,R\,\frac{\sin 12n}{\sin\frac{1}{2}n}\cos(nt-\zeta+11\tfrac{1}{2}n)\sin(\sigma-\varpi)t,$$

the summation extending over 365 days.

Writing $R\cos\zeta = A$ and $R\sin\zeta = B$, Professor Darwin in his report of 1883 has deduced these clearances in the forms

$$X_1 A + X_2 B \quad \text{and} \quad Y_1 A + Y_2 B,$$

where the X's apply to the left-hand sides containing a cosine and the Y's to those containing a sine, and the A's and B's are taken from the analysis of the corresponding short-period tides. Simple formulæ are also given for computing the X's and Y's; their values are shown in the annexed table, and are to be applied with the signs there given :—

			$\sigma - \varpi$	2σ	$2(\sigma - \eta)$	η	2η
M_2	$n = 2(\gamma - \sigma)$	X_1	− 0·05557	+ 0·00302	+ 5·7393	− 0·10410	− 0·10465
		X_2	− 0·17036	− 0·03773	− 2·9228	− 0·07525	− 0·07546
		Y_1	− 0·17075	+ 0·04170	− 2·8400	− 0·00176	− 0·00353
		Y_2	+ 0·04410	+ 0·01052	− 5·7271	+ 0·00476	+ 0·00958
N	$n = 2\gamma - 3\sigma + \varpi$	X_1	− 0·05884	+ 0·03680	+ 0·02938	− 0·01760	− 0·01760
		X_2	− 0·07758	− 0·22337	− 0·19384	+ 0·00254	+ 0·00254
		Y_1	− 0·02059	− 0·15245	− 0·12210	+ 0·00020	+ 0·00041
		Y_2	+ 0·11381	− 0·08544	− 0·08081	+ 0·00007	+ 0·00015
O	$n = \gamma - 2\sigma$	X_1	− 0·06485	+ 0·01673	+ 0·01582	− 0·19240	− 0·19340
		X_2	− 0·34765	− 0·07788	− 0·08158	− 0·18260	− 0·18311
		Y_1	− 0·34523	+ 0·08418	+ 0·08748	− 0·00460	− 0·00926
		Y_2	+ 0·04052	+ 0·03379	+ 0·03295	+ 0·00897	+ 0·01802

Solution of equations.

22. Having now obtained the 'cleared' values of the left-hand sides of the equations, the left-hand side of the first equation is divided by the coefficient of A in that equation, the left-hand side of the second equation by the coefficient of B in that equation and so on, the results being approximate values of A, B, C, &c. These are now substituted in the ten equations and the final values of A, B, C, &c. deduced. But the initial instant of time is the first $11^h\ 30^m$ in the year, instead of the first noon. Hence, if as before,

$$R^2 = A^2 + B^2 \quad \text{and} \quad \tan \zeta_1 = \frac{B}{A},$$

then, in order to reduce the results to the normal form in which noon of the first day is the initial instant of time, the increment of the corresponding argument for 11^h 30^m must be added to ζ_1 to get ζ. Having thus determined R and ζ, H and κ will be found as before by multiplying R by its proper factor of reduction and by adding to ζ the initial argument.

Interruption in record.

23. It may happen from time to time that the tide-gauge breaks down for a few days, from the stoppage of the clock, the choking of the tube, or some other accident and that other readings are not taken during the interruption. In this case there will be a hiatus in the values of δh. Now the whole process employed depends on the existence of 365 continuous values of δh. Unless, therefore, the year's observations are to be sacrificed, this hiatus must be filled. If not more than three or four days observations are wanting, it is best to plot out the values of δh graphically on each side of the hiatus and, filling in the gap with a curve drawn by hand, use the values of δh given by the conjectural curve. If the gap is somewhat longer several plans might be adopted, for example, if there is another station in the neighbourhood the values of δh for that station might be inserted; or, the values of δh for another part of the year, in which the moon's and sun's declinations are as nearly as possible the same as they were during the gap might be used and, as a matter of fact, these methods have been used. When the hiatus is of considerable length the preceding methods are inapplicable, and the plan employed is as follows:—The actual δh's are entered in their proper places; then in the ten final equations all the terms with small coefficients are neglected, and in the terms whose coefficients are approximately 182·5, a coefficient equal to 182·5 diminished by half the number of days of hiatus, is substituted; the computations are then carried out, as if there was no gap, until the values of R and ζ are obtained for each long-period tide. From these approximate values of R and ζ, the height of each tide for each day of the gap is computed from the formula $R\cos(nt-\zeta)$, where t is the number of days since the commencement of the year of observation and n the speed of the particular long-period tide per mean solar day. Thus five heights, above or below mean-water level, are obtained for each day of the gap. These five heights are added together and the sum is the missing δh for the particular day. The gap having been thus

filled in with computed δh's, the whole computation is repeated with the completed series.

Where a break extends over two or three months in the first half of a working year, the observations antecedent to the break in that half-year are rejected, and the date of the working year put forward to the date following the end of the break: but if the break occurs in the second half of the working year, that year is considered to end at the commencement of the break, and to begin 365 days before it. And the year following the break will begin at the end of the break.

Method of equivalent multipliers.

24. As the determination of each of the ten quantities $\Sigma\delta h \cos(\sigma-\varpi)t$, $\Sigma\delta h \sin(\sigma-\varpi)t$, &c., by multiplying each of the 365 δh's by its proper cosine or sine and adding the results together, would be extremely laborious, the method of equivalent multipliers has been devised by Professor Adams. The values of the respective cosines and sines are divided into eleven groups, according as they fall nearest to 1·0, ·9, ·8, ·7 . . . ·2, ·1, 0. Then, as all the values of δh are to the multiplied by some value of the cosine or sine, and that value must fall into one of these groups, all the values of δh which belong to one of these groups are collected together, summed and the sum multiplied by the corresponding multiplier. Since there are as many positive as negative values of the cosine or sine, the signs of half of the δh's must be changed: this is effected mechanically as follows:—In the spaces in the forms for the entry of the δh's, those δh's whose signs are to be unchanged are to be entered on the left side of the space if positive, and to the right if negative. Thus, in the column corresponding to each multiplier, there are two sub-columns: these are separately summed and the difference of these sums gives the total of the column for the δh's whose signs are to be unchanged. This process is carried out in the upper half of the form and the result is called a. Exactly the same course is adopted in the lower half of the form with the δh's whose signs have to be changed, and the result is denoted by b. The complete sum of the δh's is thus $a-b$, and the value of $a-b$ in each column is multiplied by the multiplier corresponding to that column, when the sum of the products will give the result required. A pair of forms, one for the cosine and the other for the sine series, is of course required for each long-period tide.

25. From the analysis of the observations at any port, one value of H (the mean semi-range) and one value of κ (which may be called the mean epoch) are obtained for each constituent for each year of observation analysed. For the larger tides M_2, S_2, K, &c., the values obtained are very accordant, but in the smaller tides there are considerable discrepancies from year to year. The means of the values of H and κ for each tide are accepted as the best results.

Preparation of data for prediction of tides.

Suppose, now, it is required to predict the tides at Karáchi for 1891; the values of the R and ζ are computed for 0^h January 1st, 1891 for Karáchi for each of the constituent tides represented on the tide-predicting machine. The value of R gives the proper throw of the corresponding crank and ζ the angle at which it has to be set in starting the tide-predicting machine belonging to the Indian Government at present located in the India Store Depôt at Lambeth.

The computations of the R's and ζ's are carried out in the way described in the account of the reductions of the observations, the only difference being that to find R from H and ζ from κ, the formulæ $R=fH$ and $\zeta=\kappa-(V_0+u)$ are used.

When all the R's and ζ's have been computed, the results are sent to Mr. E. Roberts of the Nautical Almanac Office for employment in the tide-predicter.

The depth, also, below mean sea-level of the datum-line of reference from which the heights are estimated is required, in order to enable the pen which draws the datum-line on the tide-diagram to be set. Until lately there was no scientific definition of the usual term "Low-water Ordinary Spring-Tides", but, after consultation between Professor Darwin, Major Baird and Captain Wharton, the British Hydrographer, it was determined to use for Indian charts a datum termed the "Indian Low-water Spring-Tides". This is the depth below mean sea-level of the sum of the mean lunar semi-diurnal, the mean solar semi-diurnal, the mean luni-solar diurnal and the mean lunar diurnal tide, or in the notation used in this handbook,

$$A_0-(\text{H of } M_2+\text{H of } S_2+\text{H of } K_1+\text{H of } O_1).$$

The machine is started with the proper setting of the cranks and the tide-curve for a whole year is automatically traced on a roll of

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PAGE 232 line 7 from bottom *for* "Indian Low-water Spring-Tides" *read* "Indian Spring Low-water Mark".

, the mean solar semi-diurnal, the mean luni-solar diurnal and the mean lunar diurnal tide, or in the notation used in this handbook,

$$A_0 - (H \text{ of } M_2 + H \text{ of } S_2 + H \text{ of } K_1 + H \text{ of } O_1).$$

The machine is started with the proper setting of the cranks and the tide-curve for a whole year is automatically traced on a roll of paper in about four hours. The time and height of each high and low water are measured, and the tabulated results form the tide-tables for the port.

This is the procedure followed in regard to open coast stations: the riverain stations are treated in a different way, the observations being reduced separately for each month in the year, and the semi-monthly curves obtained from the means of the corresponding months throughout the total period of the observations.

Forms.

26. In entering the heights of the water, read off the diagrams, in the computation forms, various precautions are taken to guard against error. The procedure adopted in regard to these forms will now be explained, in so far as is not self-evident from the forms themselves. The forms given below have for convenience been numbered consecutively from I to XI, but this is not the case in the actual forms.

Form I. S-Series.

27. After the diagrams have been prepared, the heights are successively measured and entered in the respective hour columns of this form, the first entry being that of 0^h of the 1st day. The first day is called 1 in the forms for short-period tides and 0 in those for long-period tides. The date for each day is generally written in pencil on the left-hand margin.

The measurements are made to hundredths of a foot, by means of a paper scale divided into tenths and hundredths in accordance with working scale of the instrument which registered the diagram under treatment. When the reader has called out the entry for each 23rd hour, the recorder checks the record by calling out 'end of the day' and then gives the date of the next day.

The reading and entry of the heights in the S-Series is done in duplicate, the original set being generally measured by a surveyor and the duplicate by a native computer. While the measurement for the duplicate is going on, a second clerk watches the entries in the original and if the reading differs by more than 0·01, 0·03 or 0·05 of a foot according as it is the natural, half or smaller scale, that height is at once remeasured and a correct value entered both in the original and the duplicate. The original and duplicate are now compared, and if there are any discrepancies larger than those above mentioned, they are noted, remeasured at the completion of the comparison, and correct values entered in both copies.

Form II, &c. M, &c., Series.

28. The heights are next copied from the original of the S-Series into the M-Series, from the M-Series into the O-Series, from that into the K-Series and so on, the last being now the 2MK-Series.

Where a double dot occurs in the forms, it indicates that two successive hourly values of the S-Series are to be entered, the first above the second: when a horizontal line takes the place of one of the usual dots, it means that no entry is to be made there, but that the next entry is to be in the next column to the right.

A black vertical line means that the solar day divides at the line and that the height immediately preceding corresponds to a 23rd solar hour. A double dot with a short black vertical line opposite the upper one, means that the entry made at the upper dot is the height corresponding to the 23rd hour of one solar day, and that at the lower dot to the 0 hour of the next solar day. By these marks, the copyist knows that he is at the end of a mean solar day.

At the right-hand side of all the forms, except the S-form, is a column headed S-hour giving the day and hour of the S-Series corresponding to the 23rd hour of the particular day of the series in question. This is a further check but is rarely used.

Comparison of entries.

29. When the 2MK-Series has been copied from the MK-Series, it is compared simultaneously with the original and duplicate of the S-Series, and if it agrees with the original the copying is perfect. The comparison with the duplicate guards against gross errors which may have escaped notice in the comparison of the original and duplicate.

Errors found in the 2MK-Series are searched for in the other series in the reverse order until one is reached in which the error does not occur, and corrections are made accordingly.

Addition of hourly heights.

30. The heights in each column are then added together, the units, tens and hundreds being separately summed, and the sums entered at the bottom of the page; some weeks afterwards they are verified by fresh computers.

Besides this, for the S-Series, the sum of the 24 hourly heights is taken for each day and entered in the column 'daily sum', which, divided by 24, gives the quantity in the column 'daily mean'.

Checks.

31. For the S-Series the total of the 'daily sums' should be equal to the total of the horizontal line at the bottom of the page in each of the five pages of the form.

For the other series, for example the M-Series, the total of the heights on page 1 of M should be the total of the heights on page 1 of S, less the sum of the last 12 hours on page 1 of S, since the last entry on page 1 of M corresponds to $74^d\ 11^h$ of S. In comparing the totals after the first page, account must be taken of the number of entries in excess or defect at the beginning as well as at the end of each page, as compared with the entries on the corresponding page of S.

Form V. Summations and means.

32. This requires no explanation; but care must be taken that the number of observations is correct: it should be the sum of the five quantities, one on each page, at the bottom of the page under the corresponding hour.

The remainder of the forms for short-period tides are self explanatory.

Form IX. Long-period tides.

33. The mean height of the water for each day is taken from the column 'daily means' in the S-Series, and the mean height of the water for the whole year, or A_0, is determined in the form for the S-Series which corresponds to Form VI. The latter mean is subtracted from each of the former quantities, giving a number of small positive and negative quantities δh, one for each day. These are entered in Form IX, of which there are two for each long-period tide, in the manner described in the foot-note.

Form XI.

In solving the first equation, the second line is obtained by introducing for B, C, D, &c., the first approximate values obtained in the preceding form. In solving the second equation, the second approximate value of A is introduced and the first of the other quantities, C, D, &c., and so on.

The rest of the computations is sufficiently evident from an inspection of the forms given below..

SHORT-PERIOD TIDES.

KARACHI, 1883–84.

Commencing o hours.

Astronomical time, 1st May, 1883.

Argument $(\gamma - \eta)$.

FORM I.—SERIES S.

Motion per mean Solar hour = 15°.

	h 0	h 1	h 2	h 3	h 4	h 5	h 6	h 7	h 8	h 9	h 10	h 11	h 12	h 13	h 14	h 15	h 16	h 17	h 18	h 19	h 20	h 21	h 22	h 23	Daily Sum	Daily Mean
1	5·22	6·35	7·60	8·62	9·42	9·86	9·76	9·00	8·16	7·30	6·30	5·67	5·44	5·76	6·46	7·18	7·92	8·56	8·80	8·36	7·41	6·52	5·62	4·91	176·20	7·342
2	4·81	5·51	6·56	7·81	8·98	9·82	10·36	10·24	9·34	8·10	6·90	5·77	4·75	4·36	4·97	5·86	6·95	8·15	9·07	9·42	9·09	8·14	6·90	5·76	177·62	7·401
3	5·00	4·83	5·50	6·72	8·10	9·28	10·18	10·74	10·48	9·26	7·81	6·20	4·59	3·36	3·32	4·05	5·50	6·91	8·50	9·64	10·14	9·76	8·48	7·08	175·43	7·310
4	5·79	4·96	4·71	5·37	6·77	8·24	9·51	10·44	10·92	10·27	8·76	6·96	5·05	3·23	2·07	2·34	3·45	5·28	7·06	8·84	10·24	10·84	10·20	8·72	170·02	7·084
5	7·24	5·96	4·90	4·48	5·33	6·84	8·51	9·84	10·74	11·04	10·20	8·36	6·13	4·04	2·21	1·32	1·75	3·29	5·48	7·48	9·40	10·82	11·40	10·59	167·25	6·969
70	10·02	10·67	10·74	9·90	8·52	7·06	5·83	5·03	5·05	5·72	6·77	7·87	8·80	9·20	8·78	7·80	6·77	5·74	5·02	4·80	5·01	5·66	6·77	8·07	175·60	7·317
71	9·23	10·04	10·50	10·14	9·00	7·60	6·43	5·30	4·72	5·00	5·84	6·87	7·68	8·18	8·50	8·27	7·32	6·35	5·83	5·64	5·48	5·40	6·24	6·48	172·04	7·168
72	6·83	7·27	7·66	8·04	8·24	8·31	6·90	5·86	5·06	4·77	5·08	5·77	6·28	6·63	6·96	7·26	7·41	7·14	6·71	6·47	6·20	5·87	6·10	6·66	159·48	6·645
73	7·27	7·60	7·87	8·13	8·30	8·41	7·50	6·41	5·53	4·94	4·81	4·98	5·52	6·33	6·93	7·33	7·61	7·58	7·44	7·10	6·71	6·38	6·34	6·54	163·56	6·815
74	6·96	7·68	8·45	8·92	8·96	8·46	7·88	6·87	5·92	5·00	4·44	4·32	4·64	5·11	5·69	6·52	7·19	7·74	7·87	7·76	7·38	7·02	6·87	6·77	164·42	6·851
Sum	646·01	647·27	641·04	624·89	604·65	591·02	584·87	587·09	591·62	592·54	587·31	570·00	535·33	489·49	442·73	408·68	393·70	398·91	430·37	472·53	519·28	564·88	604·18	634·41	13162·80	

Measured from Diagram by ________________ *and* ________________

Summed by ________________ *Compared by* ________________

Checked by ________________

Commencing o hours.
Astronomical time, 1st May, 1883.
Argument $(\gamma - \sigma)$.

SHORT-PERIOD TIDES.

[KARACHI, 1883-84.

FORM II.—SERIES M.

Motion per mean Solar hour = $14^\circ \cdot 4920521$.

	0ʰ	1ʰ	2ʰ	3ʰ	4ʰ	5ʰ	6ʰ	7ʰ	8ʰ	9ʰ	10ʰ	11ʰ	12ʰ	13ʰ	14ʰ	15ʰ	16ʰ	17ʰ	18ʰ	19ʰ	20ʰ	21ʰ	22ʰ	23ʰ	S. hour
1	5·22	6·35	7·60	8·62	9·42	9·86	9·76	9·00	8·16	7·30	6·30	5·67	5·44	5·76	6·46 7·18	7·92	8·56	8·80	8·36	7·41	6·52	5·62	4·91	4·81	2ᵈ 0ʰ
2	5·51	6·56	7·81	8·98	9·82	10·36	10·24	9·34	8·10	6·90	5·77	4·75	4·36	4·97	5·86	6·95	8·15	9·07	9·42 9·09	8·14	6·90	5·76	5·00	4·83	3 1
3	5·50	6·72	8·10	9·28	10·18	10·74	10·48	9·26	7·81	6·20	4·59	3·36	3·32	4·05	5·50	6·91	8·50	9·64	10·14	9·76	8·48	7·08	5·79	4·96 4·71	4 2
4	5·37	6·77	8·24	9·51	10·44	10·92	10·27	8·76	6·96	5·05	3·23	2·07	2·34	3·45	5·28	7·06	8·84	10·24	10·84	10·20	8·72	7·24	5·96	4·90	5 2
5	4·48	5·23	6·84	8·51 9·84	10·74	11·04	11·20	8·36	6·13	4·04	2·21	1·32	1·75	3·29	5·48	7·48	9·40	10·82	11·40	10·59	8·86	7·36	5·99	4·90	6 3
67	5·22	6·16	7·14	7·97	8·68	9·10	8·50	7·32	5·84	4·60	3·96	3·96	4·64	5·70	7·33	8·88	10·02	10·67	10·74	9·90	8·52	7·06	5·83	5·03	70 7
68	5·05	5·72	6·77	7·87 8·80	9·20	8·78	7·80	6·77	5·74	5·02	4·80	5·01	5·66	6·77	8·07	9·23	10·04	10·50	10·14	9·00	7·60	6·43	5·30	4·72	71 8
69	5·00	5·84	6·87	7·68	8·18	8·50	8·27	7·32	6·35 5·83	5·64	5·48	5·40	6·24	6·48	6·83	7·27	7·66	8·04	8·24	8·31	6·90	5·00	5·06	4·77	72 9
70	5·08	5·77	6·28	6·63	6·96	7·26	7·41	7·14	6·71	6·47	6·20	5·87	6·10	6·66 7·27	7·60	7·87	8·13	8·30	8·41	7·50	6·41	5·53	4·94	4·81	73 10
71	4·98	5·52	6·33	6·93	7·33	7·61	7·58	7·44	7·10	6·71	6·38	6·34	6·54	6·96	7·68	8·45	8·92	8·96 8·46	7·88	6·87	5·92	5·00	4·44	4·32	74 11
Sum	381·34	447·11	555·96	641·43	700·00	735·57	722·91	647·44	565·07	456·26	375·65	330·69	347·89	425·92	512·91	596·94	673·81	739·51	728·38	662·97	575·35	489·01	403·65	366·47	No. of Days.
No.	73	73	74	74	73	73	73	73	75	73	73	73	74	74	74	73	73	75	74	73	73	74	73	74	73ᵈ 12ʰ.

13802·24

*Copied by*________ *Summed by*________ *Checked by*________

SHORT-PERIOD TIDES.

[KARACHI, 1883-84.

Commencing o hours.

Astronomical time, 1st May, 1883.

Argument ($\gamma = 2\sigma$).

FORM III.—SERIES O.

Motion per mean Solar hour $= 13^\circ \cdot 9430356$.

	0^h	1^h	2^h	3^h	4^h	5^h	6^h	7^h	8^h	9^h	10^h	11^h	12^h	13^h	14^h	15^h	16^h	17^h	18^h	19^h	20^h	21^h	22^h	23^h	S. hour
1	5·22	6·35	7·60	8·62	9·42	9·86	9·76 / 9·00	8·16	7·30	6·30	5·67	5·44	5·76	6·46	7·18	7·92	8·56	8·80	8·36	7·41 / 6·52	5·62	4·91	4·81	5·51	2^d 1^h
2	6·56	7·81	8·98	9·82	10·36	10·24	9·34	8·10	6·90 / 5·77	4·75	4·36	4·97	5·86	6·95	8·15	9·07	9·42	9·09	8·14	6·90	5·76	5·00	4·83 / 5·50	6·72	3 3
3	8·10	9·28	10·18	10·74	10·48	9·26	7·81	6·20	4·59	3·36	3·32	4·05 / 5·50	6·91	8·50	9·64	10·14	9·76	8·48	7·08	5·79	4·96	4·71	5·37	6·77	4 4
4	8·24 / 9·51	10·44	10·92	10·27	8·76	6·96	5·05	3·23	2·07	2·34	3·45	5·28	7·06	8·84 / 10·24	10·84	10·20	8·72	7·24	5·96	4·90	4·48	5·23	6·84	8·51	5 6
5	9·84	10·74	11·04 / 10·20	8·36	6·13	4·04	2·21	1·32	1·75	3·29	5·48	7·48	9·40	10·82	11·40	10·59	8·86 / 7·36	5·99	4·90	4·56	5·48	7·26	8·96	10·28	6 8
65	4·64 / 5·70	7·33	8·88	10·02	10·67	10·74	9·90	8·52	7·06	5·83	5·03	5·05	5·72	6·77	7·87 / 8·80	9·20	8·78	7·80	6·77	5·74	5·02	4·80	5·01	5·66	70 21
66	6·77	8·07	9·23	10·04 / 10·50	10·14	9·00	7·60	6·43	5·30	4·72	5·00	5·84	6·87	7·68	8·18	8·50	8·27 / 7·32	6·35	5·83	5·64	5·48	5·40	6·24	6·48	71 23
67	6·83	7·27	7·66	8·04	8·24	8·31 / 6·90	5·86	5·06	4·77	5·08	5·77	6·28	6·63	6·96	7·26	7·41	7·14	6·71	6·47 / 6·20	5·87	6·10	6·66	7·27	7·60	73 1
68	7·87	8·13	8·30	8·41	7·50	6·41	5·53	4·94 / 4·81	4·98	5·52	6·33	6·93	7·33	7·61	7·58	7·44	7·10	6·71	6·38	6·34	6·54	6·96 / 7·68	8·45	8·92	74 3
69	8·96	8·46	7·88	6·87	5·92	5·00	4·44	4·32	4·64	5·11	5·69 / 6·52	7·19	7·74	7·87	7·76	7·38	7·02	6·87	6·77	6·73	7·20	7·88	8·37	8·73 / 8·74	75 5
Sum	536·93	520·49	523·78	509·52	509·01	493·46	508·06	509·57	535·11	532·66	566·22	569·10	587·27	588·44	606·51	594·24	583·08	572·95	571·37	565·98	562·28	560·03	551·53	552·86	No. of Days.
No.	75	73	75	74	74	74	74	75	75	74	74	75	73	74	75	74	74	73	75	74	75	74	75	74	74^d 6^h.

13210·45

Copied by ______ *Summed by* ______ *Checked by* ______

Commencing o hours.
Astronomical time, 1st May, 1883.
Argument $(\gamma + \sigma - \varpi)$.

SHORT-PERIOD TIDES.

FORM IV.—SERIES J.

[KARACHI, 1883-84.

Motion per mean Solar hour = $15^{\circ} \cdot 5854433$.

	0^h	1^h	2^h	3^h	4^h	5^h	6^h	7^h	8^h	9^h	10^h	11^h	12^h	13^h	14^h	15^h	16^h	17^h	18^h	19^h	20^h	21^h	22^h	23^h	S. hour.
1	5·22	6·35	7·60	8·62	9·42	9·86	9·76	9·00	8·16	7·30	6·30	5·67	5·44	—	5·76	6·46	7·18	7·92	8·56	8·80	8·36	7·41	6·52	5·62	1^d 22^h
2	4·91	4·81	5·51	6·56	7·81	8·98	9·82	10·36	10·24	9·34	8·10	6·90	5·77	4·75	4·36	4·97	—	5·86	6·95	8·15	9·07	9·42	9·09	8·14	2 21
3	6·90	5·76	5·00	4·83	5·50	6·72	8·10	9·28	10·18	10·74	10·48	9·26	7·81	6·20	4·59	3·36	3·32	4·05	5·50	—	6·91	8·50	9·64	10·14	3 20
4	9·76	8·48	7·08	5·79	4·96	4·71	5·37	6·77	8·24	9·51	10·44	10·92	10·27	8·76	6·96	5·05	3·23	2·07	2·34	3·45	5·28	—	7·06	8·84	4 19
5	10·24	10·84	10·20	8·72	7·24	5·96	4·90	4·48	5·23	6·84	8·51	9·84	10·74	11·04	10·20	8·36	6·13	4·04	2·21	1·32	1·75	3·29	5·48	7·48	5 19
72	5·22	6·16	7·14	7·97	8·68	9·10	8·50	7·32	5·84	4·60	3·96	3·96	4·64	—	5·70	7·33	8·88	10·02	10·67	10·74	9·90	8·52	7·06	5·83	70 6
73	5·03	5·05	5·72	6·77	7·87	8·80	9·20	8·78	7·80	6·77	5·74	5·02	4·80	5·01	5·66	6·77	—	8·07	9·23	10·04	10·50	10·14	9·00	7·60	71 5
74	6·43	5·30	4·72	5·00	5·84	6·87	7·68	8·18	8·50	8·27	7·32	6·35	5·83	5·64	5·48	5·40	6·24	6·48	—	6·83	7·27	7·66	8·04	8·24	72 4
75	8·31	6·90	5·86	5·06	4·77	5·08	5·77	6·28	6·63	6·96	7·26	7·41	7·14	6·71	6·47	6·20	5·87	6·10	6·66	7·27	7·60	—	7·87	8·13	73 3
76	8·30	8·41	7·50	6·41	5·53	4·94	4·81	4·98	5·52	6·33	6·93	7·33	7·61	7·58	7·44	7·10	6·71	6.38	6·34	6·54	6·96	7·68	8·45	8·92	74 3
Sum	530·35	540·31	525·07	529·69	543·21	541·22	538·65	552·76	540·83	548·94	551·26	556·58	551·56	542·43	559·33	557·96	535·36	553·85	542·24	544·27	537·84	529·88	541·24	535·56	No. of Days.
No.	73	74	73	73	74	73	73	74	73	73	73	74	73	72	74	74	71	74	73	73	73	72	74	73	73^d 4^h.

13030·39

Copied by ______ *Summed by* ______ *Checked by* ______

SHORT-PERIOD TIDES. [KARACHI, 1883-84.

FORM V.—SUMMATIONS AND MEANS OF SERIES M.

	0h	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h
Sum (p. 1)	381·34	447·11	555·96	641·43	700·00	735·57	722·91	647·44	565·07	456·26	375·65	330·69
„ (p. 2)	343·74	428·05	526·68	615·67	695·14	721·19	699·40	641·14	544·08	442·65	368·45	332·16
„ (p. 3)	345·81	419·59	505·63	596·08	681·28	718·74	709·65	634·70	545·90	459·52	385·92	345·39
„ (p. 4)	364·14	435·23	536·65	632·80	691·73	730·42	711·59	651·35	553·59	451·15	366·11	324·73
„ (p. 5)	353·49	423·23	527·79	625·39	690·07	724·24	714·22	631·25	539·98	433·03	358·82	318·28
Sum(pp. 1—5)	1788·52	2153·21	2652·71	3111·37	3458·22	3630·16	3557·77	3205·88	2748·62	2242·61	1854·95	1651·25
No. of Obs.	369	369	371	370	369	368	369	369	372	368	369	369
Means ...	4·847	5·835	7·150	8·409	9·372	9·865	9·642	8·688	7·389	6·094	5·027	4·475

12h	13h	14h	15h	16h	17h	18h	19h	20h	21h	22h	23h	
347·89	425·92	512·91	596·94	673·81	739·51	728·38	662·97	575·35	489·01	403·65	366·47	13082·24
345·45	412·26	519·02	599·31	680·90	707·83	697·21	630·58	532·87	434·23	363·64	330·45	12612·10
360·08	422·98	511·07	602·91	657·22	693·69	676·94	624·31	528·24	424·97	354·08	320·60	12525·30
351·46	417·95	512·74	605·23	695·54	736·54	733·18	660·95	570·73	482·95	394·27	345·06	12956·09
330·04	400·14	503·91	591·91	660·43	706·30	702·20	627·34	541·38	443·44	362·66	328·93	12538·47
1734·92	2079·25	2559·65	2996·30	3367·90	3583·87	3537·91	3206·15	2748·57	2274·60	1878·30	1691·51	63714·20
370	369	371	368	368	369	369	368	369	369	368	369	
4·689	5·635	6·899	8·142	9·152	9·712	9·588	8·712	7·449	6·164	5·104	4·584	

SHORT-PERIOD TIDES. [KARACHI, 1883-84.

FORM VI.—ANALYSIS OF SERIES M.

	I		II	III I+II	IV I−II	V Lower half of IV reversed	VI IV+V	M	VII M×VI	M	VIII M×VI	IX IV−V	M	X M×IX	M	XI M×IX	XII First half of III	XIII Second half of III	XIV XII+XIII	XV XII−XIII
0h	4.847	12h	4.689	9.536	+.158	....	+.158	0	.000	0	.000	+.158	1	+ .158	1	+.158	9.536	19.230	28.766	− 9.694
1	5.835	13	5.635	11.470	+.200	−.109	+.091	S_1	+.024	S_3	+.064	+.309	S_5	+ .298	S_3	+.218	11.470	17.400	28.870	− 5.930
2	7.150	14	6.899	14.049	+.251	−.077	+.174	S_2	+.087	1	+.174	+.328	S_4	+ .284	0	.000	14.049	14.838	28.887	− 0.789
3	8.409	15	8.142	16.551	+.267	−.070	+.197	S_3	+.139	S_3	+.139	+.337	S_3	+ .238	$-S_3$	−.238	16.551	12.258	28.809	+ 4.293
4	9.372	16	9.152	18.524	+.220	−.060	+.160	S_4	+.139	0	.000	+.280	S_2	+ .140	−1	−.280	18.524	10.131	28.655	+ 8.393
5	9.865	17	9.712	19.577	+.153	−.024	+.129	S_5	+.125	$-S_3$	−.091	+.177	S_1	+ .046	$-S_3$	−.125	19.577	9.059	28.636	+10.518
6	9.642	18	9.588	19.230	+.054	....	+.054	1	+.054	−1	−.054	+.054	0	.000	0	.000				
7	8.688	19	8.712	17.400	−.024															
8	7.389	20	7.449	14.838	−.060															
9	6.094	21	6.164	12.258	−.070															
10	5.027	22	5.104	10.131	−.077															
11	4.475	23	4.584	9.059	−.109															
								12	+.568	12	+.232		12	+1.164	12	−.267				
								$B_1=$	+.0473	$B_3=$	+.0193		$A_1=$	+.0970	$A_3=$	−.0223				

M	XVI M×XV	M	XVII M×XV	M	XVIII M×XV	M	XIX M×XV	XX First half of XIV	XXI Second half of XIV	XXII XX−XXI	M	XXIII M×XXII	M	XXIV M×XXII	XXV XX+XXI	M	XXVI M×XXV	M	XXVII M×XXV
0	.000	1	− 9.694	0	.000	1	−9.694	28.766	28.809	−0.043	0	.000	1	−0.043	57.575	0	.000	1	+57.575
S_2	− 2.965	S_4	− 5.136	1	− 5.930	0	.000	28.870	28.655	+0.215	S_4	+0.186	S_2	+0.108	57.525	S_4	+49.818	$-S_2$	−28.763
S_4	− 0.683	S_2	− 0.395	0	.000	−1	+0.789	28.887	28.636	+0.251	S_4	+0.217	$-S_2$	−0.126	57.523	$-S_4$	−49.817	$-S_2$	−28.761
1	+ 4.293	0	.000	−1	− 4.293	0	.000												
S_4	+ 7.269	$-S_2$	− 4.196	0	.000	1	+8.393				12	+0.403	12	−0.061		12	+ 0.001	12	+ 0.051
S_2	+ 5.259	$-S_4$	− 9.109	1	+10.518	0	.000												
12	+13.173	12	−28.530	12	+ 0.295	12	−0.512				$B_4=$	+0.0336	$A_4=$	−0.0051		$B_8=$	+0.0001	$A_8=$	+0.0043
$B_2=$	+1.0978	$A_2=$	−2.3775	$B_6=$	+0.0246	$A_6=$	−0.0427												

o = nat. sin 0° = .00000
S_1 = nat. sin 15° = .25882
S_2 = nat. sin 30° = .50000
S_3 = nat. sin 45° = .70711
S_4 = nat. sin 60° = .86603
S_5 = nat. sin 75° = .96593
1 = nat. sin 90° = 1.00000

Computed by ________ Checked by ________ Compared by ________

SHORT-PERIOD TIDES. FORM VII.—VALUES OF $V_0 + u$ FOR KARACHI, 1883–84.

1883, May 1st, 0^h. N. Lat. 24° 47′, Long. 66° 58′ E. ($=4^h{\cdot}4645$).

Average Long. Moon's Node for year beginning with 1st May, 1883, Midyear 0^h 31st October, 1883.

Auxiliary Tables N = 211°·884.

Extract from Auxiliary Tables.

$I = 19°{\cdot}267$, $\nu = -8°{\cdot}256$, $\xi = -7°{\cdot}695$.
For K Tides $\nu' = -5{\cdot}366$, $2\nu'' = -9{\cdot}906$.

h_0

Sid. Time at M. N. (*Naut. Alm.*, p. II.)	=	2h 36m 17s·48
	=	2 36·2913
	=	2·604855
		1·302428
	=	39°·073
E. Long. Cor.	=	−0·183
h_0	=	38·890
ν	=	−8·256
$h_0 - \nu$	=	47·146
$2h_0$	=	77·780

s_0

Moon's mean long. (*Naut. Alm.*)	=	332° 32′·4
	=	332·540
E. Long. Cor.	=	−2·448
s_0	=	330·092
ξ	=	−7·695
$s_0 - \xi$	=	337·787
$2s_0 - 2\xi$	=	315·574
$2s_0$	=	300·184

Average Long. Moon's Perigee for L Tide (round numbers).

(*from below*) p_0	=	16°·079
motion in $184^d\ 13\frac{1}{2}^h$	=	20·560
$P + \xi$	=	36·639
$-\xi$	=	7·695
P	=	44·334
$\cot \frac{1}{2}I$	= +	5·891
$\cot^2 \frac{1}{2}I$	= +	34·704
$\frac{1}{6}\cot^2 \frac{1}{2}I$	= +	5·784
$-\cos 2P$	= −	·023
$\frac{1}{6}\cot^2\frac{1}{2} I - \cos 2P$	= +	5·761

p_0

Hansen's Tables de la Lune, p. 300. π for 1883, Jan. 0.	=	2°·484
Motion in interval from Jan. 0, or 121 days.	=	13·480
Constant	=	0·136
Sum	=	16·100
E. Long. Cor.	=	−0·021
p_0	=	16·079
s_0	=	330·092
$s_0 - p_0$	=	314·013

Computation of R and Q (round numbers).

colog. $[\frac{1}{6}\cot^2 \frac{1}{2}I - \cos 2P]$	= +	9·23950
$\log \sin 2P$	= +	9·99988
$\log \tan . R$	= +	9·23938
for L Tide, R	=	9°·845
$\log \tan P$	= +	9·98990
$\log \frac{1}{2}$	=	9·69897
$\log . \tan Q$	= +	9·68887
for M_1 Tide, Q	=	26°·036

p_1

Extract from Auxiliary Table	$p_1 = 280°{\cdot}940$
	$h_0 = 38{\cdot}890$
	$h_0 - p_1 = 117{\cdot}950$

Motion per mean Solar hour.	East Long. Correction.
$\eta = 0°{\cdot}0410686$	$(-0°{\cdot}183)$
$\sigma = 0°{\cdot}5490165$	$(-2°{\cdot}448)$
$\varpi = 0°{\cdot}0046419$	$(-0°{\cdot}021)$

M SERIES.

	$h_0 - \nu$ =	47°·146
	$-(s_0 - \xi)$ =	22.213
§	M_1 =	69·359
(× 2)	for M_2 =	138·718
(× 3)	for M_3 =	208·077
(× 4)	for M_4 =	277·436
(× 6)	for M_6 =	56·154
(× 8)	for M_8 =	194·872
§ as above	M_1 =	69·359
(*see above*)	$+ Q$ =	26·036
	$-\frac{1}{3}\pi$ =	270·000
	for M_1 =	5·395

K_2 SERIES.

$2h_0$ =	77°·780
$-2\nu''$ =	9·906
for K_2 =	87·686

K_1 SERIES.

h_0 =	38°·890
$-\nu'$ =	5·366
$-\frac{1}{3}\pi$ =	270·000
for K_1 =	314·256

λ SERIES.

for N =	184°·705
$-2h_0$ =	282·220
$+2s_0$ =	300·184
$+\pi$ =	180·000
for λ =	227·109

ν SERIES.

for M_2 =	138°·718
$+(s_0 - p_0)$ =	314·013
$+2h_0$ =	77·780
$-2s_0$ =	59·816
for ν =	230·327

M_2N SERIES.

for M_2 =	138°·718
+ for N =	184·705
for M_2N =	323·423

$2M_2K_1$ SERIES.

for M_4 =	277°·436
− for K_1 =	45·744
for $2M_2K_1$ =	323·180

Mm SERIES.

$s_0 - p_0 = 314°{\cdot}013$

Mf SERIES.

$(2s_0 - 2\xi) = 315°{\cdot}574$

SHORT-PERIOD TIDES. [KARACHI, 1883-84.

FORM VIII.—EVALUATION OF SHORT-PERIOD TIDES. SERIES M.

Augmenting Factors.—For A_1, B_1...1·00286, A_2, B_2...1·01152, A_3, B_3...1·02617, A_4, B_4...1·04720, A_6, B_6...1·11072, A_8, B_8...1·20920.

	1	2	3	4	6	8
Log B =	+8·67486	+0·04052	+8·28556	+8·52634	+8·39094	+6·00000
Log A =	+8·98677	−0·37612	−8·34830	−7·70757	−8·63043	+7·63347
L. tan ζ =	+9·68809	−9·66440	−9·93726	−0·81877	−9·76051	+8·36653
ζ =	25°·995	155°·215	139°·124	98°·631	150°·053	1°·332
V_0+u =	5·395	138·718	208·077	277·436	56·154	194·872
κ =	31·390	293·933	347·201	16·067	206·207	196·204
$(B)^2$ =	·002237	1·205165	·000372	·001129	·000605	·000000
$(A)^2$ =	· 9409	5·652506	· 497	· 26	· 1823	· 18
$(R)^2$ =	·011646	6·857671	·000869	·001155	·002428	·000018
R =	·1079	2·6187	·0295	·0340	·0493	·0043
Aug^{tn} =	· 3	· 301	· 8	· 16	· 55	· 9
Aug^{td} R =	·1082	2·6488	·0303	·0356	·0548	·0052
1/f =	·7443	·9689	·9537	·9388	·9096	·8813
H =	·0805	2·5664	·0289	·0334	·0498	·0046

*Computed by*________ *Checked by*________ *Compared by*________

$\Sigma dh \sin (\sigma - \varpi)t$. FORM IX.—LONG-PERIOD TIDES. TIDE Mm. [KARACHI, 1883-84.

Multiplier	1·0		·9		·8		·7		·6		·5		·4		·3		·2		·1		0
No. of the day	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	
7 to 0	· ·	·27 ·20	·	·25	·	·22			·	·11			·12	·			·21	·			·15
8— 13	·	·09	·08	·	·19	·			·18	·			·18	·			·33	·			
34— 28	·	·01	·18	·	·23	·	·17	·			·16	·			·22	·			·25	·	
35— 41	·	·02	·13	·	·27	·	·44	·			·52	·			·54	·			·67	·	—
			&c.												&c.						
311—316	·16	·	·	·06	·	·04			·	·07			·	·09			·	·22			
337—331	·	·20	·	·32	·	·34	·	·38			·	·21			·	·29			·	·32	—
338—344	·	·16	·	·07	·	·05	·	·08			·	·18			·	·19			·	·15	
364—359	·25	·	·02	·	·	·07			·	·21			·	·03			·	·01			—
	3·10	4·32	3·19	2·70	2·46	1·49	0·87	1·36	0·94	1·55	1·28	1·15	1·25	1·30	1·47	1·48	1·88	0·88	1·44	1·15	—
Total *a*	−1·22		+0·49		+0·97		−0·49		−0·61		+0.13		−0·05		−0·01		+1·00		+0·29		
20 to 14	·10	·	·	·13	·	·09	·03	·			·11	·			·27	·					·39
21— 27	·36	·	·40 ·35	· ·			·38	·			·43	·			·39	·			·28	·	
48— 42	·43 ·29	· ·	·54	·			·58	·	·81	·			·98	·					·92	·	—
			&c.												&c.						
325—330	·19	·	·27	·			·02	·	·	·10			·	·25					·	·31	
351—345	·	·05	· ·	·33 ·19			·	·39			·	·38			·	·31			·	·19	—
352—358	·02	·	·06	·	·03	·	·02	·			·09	·			·17	·					·11
	2·37	4·13	2·16	3·84	0·98	2·02	1·51	2·15	1·83	0·99	0·92	2·60	2·40	0·71	0·99	1·78	1·53	0·56	1·56	1·32	
Total *b*	−1·76		−1·68		−1·04		−0·64		+0·84		−1·68		+1·69		−0·79		+0·97		+0·24		
a−*b*	+0·54		+2·17		+2·01		+0·15		−1·45		+1·81		−1·74		+0·78		+0·03		+0·05		
(*a*−*b*) × Mult	+0·54 − ·		+1·95 − ·		+1·61 − ·		+0·11 − ·		+ · −0·87		+0·91 − ·		+ · −0·70		+0·23 − ·		+0·01 − ·		+0·01 − ·		

$\Sigma(a-b) \times \text{Mult.} = +5{\cdot}37$. $\Sigma(a-b) \times \text{Mult.} = -1{\cdot}57$. $\Sigma dh \sin(\sigma-\varpi)t = \Sigma_{0}^{364} (a-b) \times \text{Mult.} = +3{\cdot}80$.

NOTE.—The arrows show the direction of the sequence of the entries of *dh* in the columns in which points are inserted, the values being entered under their proper signs—*e.g.*, in the first row (marked 7 to 0) with arrow from right to left, the entry (irrespective of sign) for day 0 is to be entered in column 0, for day 1 in column ·2, for day 2 in column ·4, and so on. After filling in the first two rows of the upper half, the first two rows of the lower half are to be filled, and so on alternately, the alternation of entry being indicated by the curved arrows. In the spaces containing double points two successive entries are to be filled in—the first entry above, the second below: *e.g.*, in row 7 to 0, column 1·0, there are double points, and the entry of *dh* for day 6 is to be made above the line and to left or right according as it is + or −, and for day 7 it is to be made below the line, and to left or right according as it is + or −.

Computed by ______________________ *Compared by* ______________________

LONG-PERIOD TIDES. [KARACHI, 1883-84.

FORM X.—SUMMATIONS AND EVALUATION OF LONG-PERIOD TIDES.—CLEARANCE FROM EFFECTS OF TIDES OF SHORT-PERIOD.

NOTE.—A *and* B *to be extracted from harmonic analysis for Tides of Short-Period.*

Tide Mm or $(\sigma-\varpi)$.

Cosine Series.

	Cosine Series.		Products.
M_2	A_2 =	− 2·378	= + 0·13
	multiplier =	− 0·0556	
	B_2 =	+ 1·098	= − 0·19
	multiplier =	− 0·1704	
N	A =	− 0·035	= + 0·00
	multiplier =	− 0·0588	
	B =	+ 0·599	= − 0·05
	multiplier =	− 0·0776	
O	A =	− 0·382	= + 0·02
	multiplier =	− 0·0649	
	B =	− 0·405	= + 0·14
	multiplier =	− 0·3477	

Total + = + 0·29
Total − = − 0·24

Total clearance = + 0·05
Uncleared $\Sigma dh \cos(\sigma-\varpi)t$ = + 1·29

$\Sigma dh \cos(\sigma-\varpi)t$ = + 1·34

Divisor 183·05. 1st approx. A = + 0·007

Sine Series.

	Sine Series.		Products.
M_2	A_2 =	− 2·378	= + 0·41
	multiplier =	− 0·1708	
	B^2 =	+ 1·098	= + 0·05
	multiplier =	+ 0·0441	
N	A =	− 0·035	= + 0·00
	multiplier =	− 0·0206	
	B =	+ 0·599	= + 0·07
	multiplier =	+ 0·1138	
O	A =	− 0·382	= + 0·13
	multiplier =	− 0·3452	
	B =	− 0·405	= − 0·02
	multiplier =	+ 0·0405	

Total + = + 0·66
Total − = − 0·02

Total clearance = + 0·64
Uncleared $\Sigma dh \sin(\sigma-\varpi)t$ = + 3·80

$\Sigma dh \sin(\sigma-\varpi)t$ = + 4·44

Divisor 181·95. 1st approx. B = + 0·024

Tide Mf or 2σ.

Cosine Series.

	Cosine Series.		Products.
M_2	A_2 =	− 2·378	= − 0·01
	multiplier =	+ 0·0030	
	B_2 =	+ 1·098	= − 0·04
	multiplier =	− 0·0377	
N	A =	− 0·035	= − 0·00
	multiplier =	+ 0·0368	
	B =	+ 0·599	= − 0·13
	multiplier =	− 0·2234	
O	A =	− 0·382	= − 0·01
	multiplier =	+ 0·0167	
	B =	− 0·405	= + 0·03
	multiplier =	− 0·0779	

Total + = + 0·03
Total − = − 0·19

Total clearance = − 0·16
Uncleared $\Sigma dh \cos 2\sigma t$ = + 7·10

$\Sigma dh \cos 2\sigma t$ = + 6·94

Divisor 183·18. 1st approx. C = + 0·038

Sine Series.

	Sine Series.		Products.
M_2	A_2 =	− 2·378	= − 0·10
	multiplier =	+ 0·0417	
	B_2 =	+ 1·098	= + 0·01
	multiplier =	+ 0·0105	
N	A =	− 0·035	= + 0·01
	multiplier =	− 0·1525	
	B =	+ 0·599	= − 0·05
	multiplier =	− 0·0854	
O	A =	− 0·382	= − 0·03
	multiplier =	+ 0·0842	
	B =	− 0·405	= − 0·01
	multiplier =	+ 0·0338	

Total + = + 0·02
Total − = − 0·19

Total clearance = − 0·17
Uncleared $\Sigma dh \sin 2\sigma t$ = + 2·14

$\Sigma dh \sin 2\sigma t$ = + 1·97

Divisor 181·82. 1st approx. D = + 0·011

The 1st approximations of the constants for the other three tides are deduced in the same way, and are for MSf, C′ = − 0·001, D′ = − 0·009; for Sa, E = + 0·089, F = − 0·001; and for Ssa, G = − 0·003, H = + 0·189.

Computed by ________ *Checked by* ________ *Compared by* ________

LONG-PERIOD TIDES. [KARACHI, 1883-84.

FORM XI.—EVALUATION OF LONG-PERIOD TIDES.

$\Sigma_0^{364} dh.\cos(\sigma-\varpi)t =$	+1·34 =	+183·05 A +	2·14 B +	0·73 C +	4·29 D +	0·77 C′ +	5·04 D′ +	4·88 E −	0·43 F +	4·96 G −	0·69 H.
	+1·34 =	183·05 A +	·05 +	·03 +	·05 −	·00 −	·05 +	·43 +	·00 −	·01 −	·13
	+0·97 =	183·05 A									
	+0·005 =	A									
$\Sigma_0^{364} dh.\sin(\sigma-\varpi)t =$	+4·44 = +	2·14 A +	181·95 B −	4·15 C +	1·02 D −	4·90 C′ +	1·07 D′ +	3·80 E +	0·34 F +	3·88 G +	0·69 H.
	+4·44 = +	·01	+181·95 B −	·16 +	·01 +	·00 −	·01 +	·34 −	·00 −	·01 +	·13
	+4·13 =	181·95 B									
	+0·023 =	B									
$\Sigma_0^{364} dh.\cos 2\sigma t$	= +6·94 = +	0·73 A −	4·15 B +	183·18 C +	0·88 D +	0·61 C′ +	0·92 D′ −	1·50 E −	0·10 F −	1·51 G −	0·19 H.
	+6·94 = +	·00 −	·10	+183·18 C +	·01 −	·00 −	·01 −	·13 +	·00 +	·00 −	·04
	+7·21 =	183·18 C									
	+0·039 =	C									
	&c.				&c.				&c.		

LUNAR MONTHLY Mm.

Log B	= +8·36173
Log A	= +7·69897
L. tan ζ_1	= +0·66276
ζ_1	= 77°·735
Motion for 11½h ...	= 6·260
ζ	= 83·995
V_0+u	= 314·013
κ	= 38·008
B^2	= 0·000529
A^2	= 0· 25
Sum = R^2	= 0·000554
R	= 0·024
$1/f$	= 0·900
H	= 0·022

LUNAR FORTNIGHTLY Mf.

Log D	= +7·95424
Log C	= +8·59106
L. tan ζ_1	= +9·36318
ζ_1	= 12°·995
Motion for 11½h ...	= 12·627
ζ	= 25·622
V_0+u	= 315·574
κ	= 351·196
D^2	= 0·000081
C^2	= 0· 1521
Sum = R^2	= 0·001602
R	= 0·040
$1/f$	= 1·449
H	= 0·058

LUNI-SOLAR FORTNIGHTLY MSf.

Log D′	= −8·04139
Log C′	= +7·00000
L. tan ζ_1	= −1·04139
ζ_1	= 275°·194
Motion for 11½h ...	= 11·683
ζ	= 286·877
V_0+u	= 221·282
κ	= 148·159
$(D')^2$	= 0·000121
$(C')^2$	= 0· 1
Sum = R^2	= 0·000122
R	= 0·011
$1/f$	= 0·969
H	= 0·011

SOLAR ANNUAL Sa.

Log F	= −7·00000
Log E	= +8·94448
Tan ζ_1	= −8·05552
ζ_1	= 359°·349
Motion for 11½h ...	= 0·472
ζ	= 359·821
V_0+u	= 38·890
κ	= 38·711
F^2	= 0·000001
E^2	= · 1444
Sum = R^2	= 0·001445
R = H	= 0·088

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Log H	= +9·27646
Log G	= −7·60206
Tan ζ_1	= −1·67440
ζ_1	= 91°·212
Motion for 11½h ...	= 0·945
ζ	= 92·157
V_0+u	= 77·780
κ	= 169·937
H^2	= 0·035721
G^2	= · 16
Sum = R^2	= 0·035737
R = H	= 0·189

Computed by ________ *Checked by* ________ *Compared by* ________

Auxiliary tables.

34. These tables have been constructed to facilitate the computations required for the harmonic analysis of the tidal observations. The following explanations and examples are given to illustrate the use of the tables.

Table I.

35. This table, for converting decimals of a degree into their corresponding values of minutes and seconds of arc, enters frequently into the computations, more especially in taking out the values of the trigonometrical functions from Shortrede's Logarithm Tables. Its use hardly requires explanation.

Table II.

36. This table is the converse of Table 1, but had to be made out somewhat differently. The correct value to three places of decimals of a degree is all that is generally required in the computations. This is given in the 4th column, and it will be observed that the figures in the 4th column are arranged midway between those in the 3rd and also midway between those in the 5th column, where the actual values corresponding to the angles in the 3rd column are given.

It will also be observed that the table is divided into six groups. The reason of this is as follows:—6 minutes = ·1 of a degree, 12 minutes = ·2, 18 minutes = ·3, and so on. Therefore, any number of seconds added to 6 minutes will give the same figures in the second and third place of decimals (in the equivalent value of a degree expressed in decimals), as the similar number of seconds added to 12 minutes or 18 minutes would give. Similarly, 1 minute, or 7 minutes, or 13 minutes, and so many seconds would each have for the second and third place of decimals the same figures in expressing their corresponding values in decimals of a degree (the first figure of the decimals of course alters).

To use the tables, first look for the minutes in one of the groups of column 1; opposite it in the 2nd column will be found the first figure of the equivalent value in decimals of a degree. *Keeping to the same group*, look in the 3rd column for the seconds (most probably the exact number will not be found), entering this column at the *space* between the number of seconds next less and the number of seconds next greater than that looked for, in the *4th column opposite this space* the second and third figures of the corresponding value of the decimals of a degree will be found. If the actual number of seconds looked for is found in the third column, then the corresponding value is obtained in column 4 *opposite the space in column 3 below* the number of seconds.

Example.—Required the decimals of a degree corresponding to 18′ 25″:

Column 1, group 1, 18′	... ... =	·3	in col. 2
" 3, " 1, 25″ (between 23″·4 and 27″·0)	=	·007	in col. 4
Value required	=	·307	

Again:—If the decimals of a degree corresponding to 40′ 0″ are required: then as in the above example, 40′ 3″ would just equal ·668, and anything less than 40′ 3″ must be less than ·668. Again, 39′ 59″·4 would just equal ·667, and anything greater than 39′ 59″·4 must be equal to ·667 or more. Therefore 40′ 0″ would equal ·667.

37. This is extracted and deduced from Hansen's *Tables de la Lune*, pages 299 and 300. The values of p, the mean longitude of the moon's perigee (or π as it is written in the tidal computation forms) is given for what is called January 0 of each year; but in the Preface to Hansen's Tables it will be found that January 0 and December the 31st mean the same date; therefore the values given in the tables are for 0 hours December 31st of the preceding year. But this is not the case for leap-years; the values given in the tables as regards those years are for 0 hours January 1st.

Table III.

If the value of π for 0 hours January 1st is required for any year which is not leap-year, take the value of π given opposite in the next column, and *add* one day's motion (Table V); if it is leap-year, the one day's motion has not to be added.

Example.—Required π for 0 hours January 1st, 1891.

In Table III opposite 1891	...	328°·00632
In Table V one day's motion for π	...	0·11140
π for 0 hours January 1st, 1891	=	328°·11772

Again :—Required π for 0 hours January 1st, 1892.

Opposite 1892 in Table III is $8^\circ\cdot78020$, which is the value required, for 1892 being leap-year, the one day's motion is not added.

Tables IV and V.

38. These tables are to be employed together; in using Table V great care must be taken that the proper number of days from January 0 is taken by means of Table IV. The motion of p or π is given for every day from 1 to 366.

Table VI.

39. This table gives the correction to the value of p or π on account of difference of longitude from Greenwich. The correction is required only to three places of decimals of a degree, as far as the tidal computations are concerned, and the table is constructed on the principle explained in Table II. Column 3 shows the exact correction for the difference of longitude given immediately opposite in column 1; and column 2 gives the correction which has to be used, for all values occurring *between the longitudes* given in column 1.

Example.—Required correction for π for $30^\circ\ 30'$ E. longitude. By Table II $30' = \cdot500$ of a degree.

From Table VI.—$30^\circ\cdot500$ lies between $27^\circ\cdot468$ and $30^\circ\cdot700$ in column 1, and in column 2 opposite the space between those two longitudes is ·009, which is the correction required, and the sign is −

Table VII.

40. This table will be found much more convenient than Crélle's for the particular multiplications required, and admits of much more rapid computation. The three augmenting factors R_3, R_6, and R_8 occur so seldom that it has not been thought necessary to extend the tables on their account.

The usual multiples of the factors are one integer and three places of decimals, or two figures only preceded by 0; for instance, 1·412, or 0·026. Sometimes, however, two integers and three places of decimals have to be multiplied by the augmenting factor. The use of the table hardly requires explanation, care being taken to put down the decimal point correctly. The values in the tables are the products of the factors by whole numbers.

41. These tables require no explanation. They give the value of S_1, S_3, S_4, and S_5 multiplied by every number between ·001 and ·999: from which the products of these factors with other numbers can be rapidly obtained, care being taken about the decimal point.

Tables VIII, IX, X, and XI.

42. This table has been made up in order to permit of the natural numbers corresponding to logarithms, with indices 6, 7, and 8, being taken out much more rapidly than could be done with Hutton's Logarithm Tables. The natural number corresponding to the given logarithm is only required correct to three places of decimals; the table has been made up on the same principle as explained in Table 11.

Table XII.

43. This table gives the values of N (longitude of moon's ascending node) for 0 hour January 1, Greenwich mean time, for each year from 1850 up to 1949.

Table XIII.

44. This table shows the amount to be subtracted from the values given in Table XIII, to obtain N at any particular date. The mean value of N to be used in the tidal reductions is the value at mid-year of the observations: and as half a year after 0 hour of the first day under analysis falls at midnight, the values in Table XIV are computed for each midnight.

Table XIV.

45. This table gives the value of the solar perigee for 0 hour January 1, from 1850 to 1949.

Table XV.

46. This table shows the increment to be added to the quantities given in Table XV to obtain the value on certain days of the year, as the value of the solar perigee (p_1) is required for mid-year of the observations.

Table XVI.

47. This table gives the values of I (inclination of the lunar orbit to the equator), ν (the right ascension of the intersection of the lunar orbit and the equator) and ξ (the longitude 'in the moon's orbit' of the intersection) corresponding to each half degree of N (the longitude of the moon's ascending node) from 0° up to 180°.

Table XVII.

When N is negative I has the same value as when N is positive; but ν and ξ change sign with N.

The values of I, ν and ξ corresponding to N at mid-year will be easily found by interpolation between the two nearest half degrees.

48. This table is subdivided into seven parts (1), (2), &c., and is used for the determination of the factors $1/f$ and f required in calculating H from R and *vice versâ*. The values are given corresponding to each 0°·1 of I, the inclination of the lunar orbit to the equator. The values required in the computation are those corresponding to I for mid-year; so that I is first obtained from Table XVII to correspond to N at mid-year and then the $1/f$ or f will be easily calculated by interpolation from the particular part of this table.

Table XVIII.

49. This table gives the values of ν' corresponding to each 0°·1 of I. The ν' is required in computing the values of $h_0 - \nu' - \frac{1}{2}\pi$, the initial argument for the tide K_1.

Table XIX.

50. This table is similar to Table XIX, and gives the value of $2\nu''$ employed in determining the initial argument, $2h_0 - 2\nu''$, for the tide K_2.

Table XX.

Attention should be paid to the notes at the foot of these tables.

TABLE I.—For converting Decimals of a Degree into Minutes and Seconds.

1

Parts of a Degree	Minutes
·1	6
·2	12
·3	18
·4	24
·5	30
·6	36
·7	42
·8	48
·9	54

2

Parts of a Degree	Minutes and Seconds	Parts of a Degree	Minutes and Seconds	Parts of a Degree	Minutes and Seconds
·001	0 3·6	·034	2 2·4	·067	4 1·2
2	0 7·2	35	2 6·0	68	4 4·8
3	0 10·8	36	2 9·6	69	4 8·4
4	0 14·4	37	2 13·2	·070	4 12·0
5	0 18·0	38	2 16·8	71	4 15·6
6	0 21·6	39	2 20·4	72	4 19·2
7	0 25·2	·040	2 24·0	73	4 22·8
8	0 28·8	41	2 27·6	74	4 26·4
9	0 32·4	42	2 31·2	75	4 30·0
·010	0 36·0	43	2 34·8	76	4 33·6
11	0 39·6	44	2 38·4	77	4 37·2
12	0 43·2	45	2 42·0	78	4 40·8
13	0 46·8	46	2 45·6	79	4 44·4
14	0 50·4	47	2 49·2	·080	4 48·0
15	0 54·0	48	2 52·8	81	4 51·6
16	0 57·6	49	2 56·4	82	4 55·2
17	1 1·2	·050	3 0·0	83	4 58·8
18	1 4·8	51	3 3·6	84	5 2·4
19	1 8·4	52	3 7·2	85	5 6·0
·020	1 12·0	53	3 10·8	86	5 9·6
21	1 15·6	54	3 14·4	87	5 13·2
22	1 19·2	55	3 18·0	88	5 16·8
23	1 22·8	56	3 21·6	89	5 20·4
24	1 26·4	57	3 25·2	·090	5 24·0
25	1 30·0	58	3 28·8	91	5 27·6
26	1 33·6	59	3 32·4	92	5 31·2
27	1 37·2	·060	3 36·0	93	5 34·8
28	1 40·8	61	3 39·6	94	5 38·4
29	1 44·4	62	3 43·2	95	5 42·0
·030	1 48·0	63	3 46·8	96	5 45·6
31	1 51·6	64	3 50·4	97	5 49·2
32	1 55·2	65	3 54·0	98	5 52·8
33	1 58·8	66	3 57·6	99	5 56·4

Example.—Required the value of 0°·875 in minutes and seconds:

From Part 1 ... ·8 = 48′

,, 2 ... ·075 = 4′ 30″

·875 = 52′ 30″

TABLE II.—For converting Minutes and Seconds into Decimals of a Degree.

Minutes	Decimals of a Degree	Seconds and Decimals of Seconds	Decimals of a Degree correct to three places	Actual value of Decimals of a Degree	Minutes	Decimals of a Degree	Seconds and Decimals of Seconds	Decimals of a Degree correct to three places	Actual value of Decimals of a Degree
0	·0	0·0	·000	·0000	3	·0	1·8	·051	·0505
6	·1	1·8	·001	·0005	9	·1	5·4	·052	·0515
12	·2	5·4	·002	·0015	15	·2	9·0	·053	·0525
18	·3	9·0	·003	·0025	21	·3	12·6	·054	·0535
24	·4	12·6	·004	·0035	27	·4	16·2	·055	·0545
30	·5	16·2	·005	·0045	33	·5	19·8	·056	·0555
36	·6	19·8	·006	·0055	39	·6	23·4	·057	·0565
42	·7	23·4	·007	·0065	45	·7	27·0	·058	·0575
48	·8	27·0	·008	·0075	51	·8	30·6	·059	·0585
54	·9	30·6	·009	·0085	57	·9	34·2	·060	·0595
		34·2	·010	·0095			37·8	·061	·0605
		37·8	·011	·0105			41·4	·062	·0615
		41·4	·012	·0115			45·0	·063	·0625
		45·0	·013	·0125			48·6	·064	·0635
		48·6	·014	·0135			52·2	·065	·0645
		52·2	·015	·0145			55·8	·066	·0655
		55·8	·016	·0155			59·4		·0665
		59·4		·0165				·067	
			·017		4	·0	3·0	·068	·0675
1	·0	3·0	·018	·0175	10	·1	6·6	·069	·0685
7	·1	6·6	·019	·0185	16	·2	10·2	·070	·0695
13	·2	10·2	·020	·0195	22	·3	13·8	·071	·0705
19	·3	13·8	·021	·0205	28	·4	17·4	·072	·0715
25	·4	17·4	·022	·0215	34	·5	21·0	·073	·0725
31	·5	21·0	·023	·0225	40	·6	24·6	·074	·0735
37	·6	24·6	·024	·0235	46	·7	28·2	·075	·0745
43	·7	28·2	·025	·0245	52	·8	31·8	·076	·0755
49	·8	31·8	·026	·0255	58	·9	35·4	·077	·0765
55	·9	35·4	·027	·0265			39·0	·078	·0775
		39·0	·028	·0275			42·6	·079	·0785
		42·6	·029	·0285			46·2	·080	·0795
		46·2	·030	·0295			49·8	·081	·0805
		49·8	·031	·0305			53·4	·082	·0815
		53·4	·032	·0315			57·0		·0825
		57·0		·0325				·083	
			·033		5	·0	0·6	·084	·0835
2	·0	0·6	·034	·0335	11	·1	4·2	·085	·0845
8	·1	4·2	·035	·0345	17	·2	7·8	·086	·0855
14	·2	7·8	·036	·0355	23	·3	11·4	·087	·0865
20	·3	11·4	·037	·0365	29	·4	15·0	·088	·0875
26	·4	15·0	·038	·0375	35	·5	18·6	·089	·0885
32	·5	18·6	·039	·0385	41	·6	22·2	·090	·0895
38	·6	22·2	·040	·0395	47	·7	25·8	·091	·0905
44	·7	25·8	·041	·0405	53	·8	29·4	·092	·0915
50	·8	29·4	·042	·0415	59	·9	33·0	·093	·0925
56	·9	33·0	·043	·0425			36·6	·094	·0935
		36·6	·044	·0435			40·2	·095	·0945
		40·2	·045	·0445			43·8	·096	·0955
		43·8	·046	·0455			47·4	·097	·0965
		47·4	·047	·0465			51·0	·098	·0975
		51·0	·048	·0475			54·6	·099	·0985
		54·6	·049	·0485			58·2	·100	·0995
		58·2	·050	·0495		·1	1·8		·1005
3	·0	1·8		·0505					

TABLE III.—Values of p the Mean Longitude of the Moon's Perigee (the π of the computation forms) for every year from 1850 to 1949.

Year	p or π	Year	p or π	Year	p or π	Year	p or π
	°		°		°		°
1850	99·72920	1875	36·96088	1900	334·19130	1925	271·42549
51	140·39176	76	77·73479	1	14·85396	26	312·08810
52	181·16571	77	118·39729	2	55·51661	27	352·75070
53	221·82826	78	159·05979	3	96·17926	28	33·52471
54	262·49081	79	199·72229	4	136·95331	29	74·18731
1855	303·15336	1880	240·49619	1905	177·61596	1930	114·84990
56	343·92731	81	281·15868	6	218·27861	31	155·51250
57	24·58985	82	321·82118	7	258·94125	32	196·28649
58	65·25239	83	2·48367	8	299·71529	33	236·94908
59	105·91493	84	43·25756	9	340·37793	34	277·61167
1860	146·68887	1885	83·92004	1910	21·04057	1935	318·27426
61	187·35140	86	124·58253	11	61·70320	36	359·04825
62	228·01393	87	165·24501	12	102·47724	37	39·71083
63	268·67646	88	206·01889	13	143·13987	38	80·37341
64	309·45040	89	246·68137	14	183·80250	39	121·03599
1865	350·11292	1890	287·34385	1915	224·46513	1940	161·80997
66	30·77545	91	328·00632	16	265·23915	41	202·47254
67	71·43797	92	8·78020	17	305·90178	42	243·13512
68	112·21189	93	49·44267	18	346·56440	43	283·79769
69	152·87441	94	90·10514	19	27·22702	44	324·57166
1870	193·53693	1895	130·76760	1920	68·00104	1945	5·23423
71	234·19944	96	171·54147	21	108·66365	46	45·89679
72	274·97336	97	212·20393	22	149·32626	47	86·55935
73	315·63587	98	252·86639	23	189·98888	48	127·33332
74	356·29838	99	293·52885	24	230·76289	49	167·99588

These values are for January 0 (*i. e.*, noon December 31st of preceding year), except in the case of leap-years, when the values are for 0 hour January 1st.

The values given in the above table require 0°·136 to be added to give the true values of p or π (see page 36, Preface to Hansen's Tables), but as the form for the computation of tidal observations has been constructed showing the constant 0°·136 to be added, it has been thought advisable not to make this correction in the above table.

TABLE IV.—Number of Days from January 0.

Month	Common year	Leap-year	Month	Common year	Leap-year
January 0 ...	0	−1	July 0 ...	181	181
February 0 ...	31	30	August 0 ...	212	212
March 0 ...	59	59	September 0 ...	243	243
April 0 ...	90	90	October 0 ...	273	273
May 0 ...	120	120	November 0 ...	304	304
June 0 ...	151	151	December 0 ...	334	334

TABLE V.—Value of Movement of p or π for 1 to 366 Days.

Days	π	Days	π	Days	π	Days	π	Days	π	Days	π
	°		°		°		°		°		°
1	0·11140	62	6·90708	123	13·70276	184	20·49844	245	27·29411	306	34·08979
2	0·22281	63	7·01849	124	13·81416	185	20·60984	246	27·40552	307	34·20119
3	0·33421	64	7·12989	125	13·92557	186	20·72124	247	27·51692	308	34·31260
4	0·44562	65	7·24130	126	14·03697	187	20·83265	248	27·62833	309	34·42400
5	0·55702	66	7·35270	127	14·14838	188	20·94405	249	27·73973	310	34·53541
6	0·66843	67	7·46410	128	14·25978	189	21·05546	250	27·85113	311	34·64681
7	0·77983	68	7·57551	129	14·37119	190	21·16686	251	27·96254	312	34·75822
8	0·89124	69	7·68691	130	14·48259	191	21·27827	252	28·07394	313	34·86962
9	1·00264	70	7·79832	131	14·59399	192	21·38967	253	28·18535	314	34·98102
10	1·11405	71	7·90972	132	14·70540	193	21·50108	254	28·29675	315	35·09243
11	1·22545	72	8·02113	133	14·81680	194	21·61248	255	28·40816	316	35·20383
12	1·33685	73	8·13253	134	14·92821	195	21·72389	256	28·51956	317	35·31524
13	1·44826	74	8·24394	135	15·03961	196	21·83529	257	28·63097	318	35·42664
14	1·55966	75	8·35534	136	15·15102	197	21·94669	258	28·74237	319	35·53805
15	1·67107	76	8·46675	137	15·26242	198	22·05810	259	28·85378	320	35·64945
16	1·78247	77	8·57815	138	15·37383	199	22·16950	260	28·96518	321	35·76086
17	1·89388	78	8·68955	139	15·48523	200	22·28091	261	29·07658	322	35·87226
18	2·00528	79	8·80096	140	15·59664	201	22·39231	262	29·18799	323	35·98367
19	2·11669	80	8·91236	141	15·70804	202	22·50372	263	29·29939	324	36·09507
20	2·22809	81	9·02377	142	15·81944	203	22·61512	264	29·41080	325	36·20647
21	2·33950	82	9·13517	143	15·93085	204	22·72653	265	29·52220	326	36·31788
22	2·45090	83	9·24658	144	16·04225	205	22·83793	266	29·63361	327	36·42928
23	2·56230	84	9·35798	145	16·15366	206	22·94933	267	29·74501	328	36·54069
24	2·67371	85	9·46939	146	16·26506	207	23·06074	268	29·85642	329	36·65209
25	2·78511	86	9·58079	147	16·37647	208	23·17214	269	29·96782	330	36·76350
26	2·89652	87	9·69219	148	16·48787	209	23·28355	270	30·07923	331	36·87490
27	3·00792	88	9·80360	149	16·59928	210	23·39495	271	30·19063	332	36·98631
28	3·11933	89	9·91500	150	16·71068	211	23·50636	272	30·30203	333	37·09771
29	3·23073	90	10·02641	151	16·82209	212	23·61776	273	30·41344	334	37·20912
30	3·34214	91	10·13781	152	16·93349	213	23·72917	274	30·52484	335	37·32052
31	3·45354	92	10·24922	153	17·04489	214	23·84057	275	30·63625	336	37·43192
32	3·56495	93	10·36062	154	17·15630	215	23·95198	276	30·74765	337	37·54333
33	3·67635	94	10·47203	155	17·26770	216	24·06338	277	30·85906	338	37·65473
34	3·78775	95	10·58343	156	17·37911	217	24·17478	278	30·97046	339	37·76614
35	3·89916	96	10·69484	157	17·49051	218	24·28619	279	31·08187	340	37·87754
36	4·01056	97	10·80624	158	17·60192	219	24·39759	280	31·19327	341	37·98895
37	4·12197	98	10·91765	159	17·71332	220	24·50900	281	31·30468	342	38·10035
38	4·23337	99	11·02905	160	17·82473	221	24·62040	282	31·41608	343	38·21176
39	4·34478	100	11·14045	161	17·93613	222	24·73181	283	31·52748	344	38·32316
40	4·45618	101	11·25186	162	18·04754	223	24·84321	284	31·63889	345	38·43457
41	4·56759	102	11·36326	163	18·15894	224	24·95462	285	31·75029	346	38·54597
42	4·67899	103	11·47467	164	18·27034	225	25·06602	286	31·86170	347	38·65737
43	4·79040	104	11·58607	165	18·38175	226	25·17743	287	31·97310	348	38·76878
44	4·90180	105	11·69748	166	18·49315	227	25·28883	288	32·08451	349	38·88018
45	5·01320	106	11·80888	167	18·60456	228	25·40023	289	32·19591	350	38·99159
46	5·12461	107	11·92029	168	18·71596	229	25·51164	290	32·30732	351	39·10299
47	5·23601	108	12·03169	169	18·82737	230	25·62304	291	32·41872	552	39·21440
48	5·34742	109	12·14309	170	18·93877	231	25·73445	292	32·53012	353	39·32580
49	5·45882	110	12·25450	171	19·05018	232	25·84585	293	32·64153	354	39·43721
50	5·57023	111	12·36590	172	19·16158	233	25·95726	294	32·75293	355	39·54861
51	5·68163	112	12·47731	173	19·27299	234	26·06866	295	32·86434	356	39·66002
52	5·79304	113	12·58871	174	19·38439	235	26·18007	296	32·97574	357	39·77142
53	5·90444	114	12·70012	175	19·49579	236	26·29147	297	33·08715	358	39·88282
54	6·01585	115	12·81152	176	19·60720	237	26·40288	298	33·19855	359	39·99423
55	6·12725	116	12·92293	177	19·71860	238	26·51428	299	33·30996	360	40·10563
56	6·23865	117	13·03433	178	19·83001	239	26·62568	300	33·42136	361	40·21704
57	6·35006	118	13·14574	179	19·94141	240	26·73709	301	33·53277	362	40·32844
58	6·46146	119	13·25714	180	20·05282	241	26·84849	302	33·64417	363	40·43985
59	6·57287	120	13·36854	181	20·16422	242	26·95990	303	33·75557	364	40·55125
60	6·68427	121	13·47995	182	20·27563	243	27·07130	304	33·86698	365	40·66266
61	6·79568	122	13·59135	183	20·38703	244	27·18271	305	33·97838	366	40·77406

TABLE VI.—Value of the Movement of p or π for differences of Longitude Greenwich.

Difference of Longitude	Value of π	Actual values of π corresponding to Degrees in Column I	Difference of Longitude	Value of π	Actual values of π corresponding to Degrees in Column I
°	°	°	°	°	°
0·000	·000	·0000	88·865	·028	·0275
1·616	·001	·0005	92·097	·029	·0285
4·847	·002	·0015	95·328	·030	·0295
8·079	·003	·0025	98·560	·031	·0305
11·310	·004	·0035	101·791	·032	·0315
14·542	·005	·0045	105·023	·033	·0325
17·773	·006	·0055	108·254	·034	·0335
21·005	·007	·0065	111·486	·035	·0345
24·236	·008	·0075	114·717	·036	·0355
27·467	·009	·0085	117·949	·037	·0365
30·699	·010	·0095	121·180	·038	·0375
33·930	·011	·0105	124·411	·039	·0385
37·162	·012	·0115	127·643	·040	·0395
40·393	·013	·0125	130·874	·041	·0405
43·625	·014	·0135	134·106	·042	·0415
46·856	·015	·0145	137·337	·043	·0425
50·088	·016	·0155	140·569	·044	·0435
53·319	·017	·0165	143·800	·045	·0445
56·551	·018	·0175	147·032	·046	·0455
59·782	·019	·0185	150·263	·047	·0465
63·014	·020	·0195	153·495	·048	·0475
66·245	·021	·0205	156·726	·049	·0485
69·477	·022	·0215	159·958	·050	·0495
72·708	·023	·0225	163·189	·051	·0505
75·939	·024	·0235	166·421	·052	·0515
79·171	·025	·0245	169·652	·053	·0525
82·402	·026	·0255	172·884	·054	·0535
85·634	·027	·0265	176·115	·055	·0545
88·865		·0275	179·346	·056	·0555
			182·578		·0565

Correction for E. Longitude −, for W. Longitude +.

TABLE VII.—Products of Augmenting Factors R_1, R_2, and R_4 multiplied by 1 to 99.

$R_1 = \cdot 0028.$

	0	10	20	30	40	50	60	70	80	90	
0	·0000	·0280	·0560	·0840	·1120	·1400	·1680	·1960	·2240	·2520	0
1	·0028	·0308	·0588	·0868	·1148	·1428	·1708	·1988	·2268	·2548	1
2	·0056	·0336	·0616	·0896	·1176	·1456	·1736	·2016	·2296	·2576	2
3	·0084	·0364	·0644	·0924	·1204	·1484	·1764	·2044	·2324	·2604	3
4	·0112	·0392	·0672	·0952	·1232	·1512	·1792	·2072	·2352	·2632	4
5	·0140	·0420	·0700	·0980	·1260	·1540	·1820	·2100	·2380	·2660	5
6	·0168	·0448	·0728	·1008	·1288	·1568	·1848	·2128	·2408	·2688	6
7	·0196	·0476	·0756	·1036	·1316	·1596	·1876	·2156	·2436	·2716	7
8	·0224	·0504	·0784	·1064	·1344	·1624	·1904	·2184	·2464	·2744	8
9	·0252	·0532	·0812	.1092	·1372	·1652	·1932	·2212	·2492	·2772	9

$R_2 = \cdot 0115.$

	0	10	20	30	40	50	60	70	80	90	
0	·0000	·1150	·2300	·3450	·4600	·5750	·6900	·8050	·9200	1·0350	0
1	·0115	·1265	·2415	·3565	·4715	·5865	·7015	·8165	·9315	1·0465	1
2	·0230	·1380	·2530	·3680	·4830	·5980	·7130	·8280	·9430	1·0580	2
3	·0345	·1495	·2645	·3795	·4945	·6095	·7245	·8395	·9545	1·0695	3
4	·0460	·1610	·2760	·3910	·5060	·6210	·7360	·8510	·9660	1·0810	4
5	·0575	·1725	·2875	·4025	·5175	·6325	·7475	·8625	·9775	1·0925	5
6	·0690	·1840	·2990	·4140	·5290	·6440	·7590	·8740	·9890	1·1040	6
7	·0805	·1955	·3105	·4255	·5405	·6555	·7705	·8855	1·0005	1·1155	7
8	·0920	·2070	·3220	·4370	·5520	·6670	·7820	·8970	1·0120	1·1270	8
9	·1035	·2185	·3335	·4485	·5635	·6785	·7935	·9085	1·0235	1·1385	9

$R_4 = \cdot 0472.$

	0	10	20	30	40	50	60	70	80	90	
0	·0000	·4720	·9440	1·4160	1 8880	2·3600	2·8320	3·3040	3·7760	4·2480	0
1	·0472	·5192	·9912	1·4632	1·9352	2·4072	2·8792	3·3512	3·8232	4·2952	1
2	·0944	·5664	1·0384	1·5104	1·9824	2·4544	2·9264	3·3984	3·8704	4·3424	2
3	·1416	·6136	1·0856	1·5576	2·0296	2·5016	2·9736	3·4456	3·9176	4·3896	3
4	·1888	·6608	1·1328	1·6048	2·0768	2·5488	3·0208	3·4928	3·9648	4·4368	4
5	·2360	·7080	1·1800	1·6520	2·1240	2·5960	3·0680	3·5400	4·0120	4·4840	5
6	·2832	·7552	1·2272	1·6992	2·1712	2·6432	3·1152	3·5872	4·0592	4·5312	6
7	·3304	·8024	1·2744	1·7464	2·2184	2·6904	3·1624	3·6344	4·1064	4·5784	7
8	·3776	·8496	1·3216	1·7936	2·2656	2·7376	3·2096	3·6816	4·1536	4·6256	8
9	·4248	·8968	1·3688	1·8408	2·3128	2·7848	3·2568	3·7288	4·2008	4·6728	9

TABLE VIII.—Products of $S_1 \times \cdot 001$ up to $S_1 \times 1 \cdot 000$.

$S_1 = \sin 15° = \cdot 25882$.

No.	·000	·001	·002	·003	·004	·005	·006	·007	·008	·009	No.
·00	·000000	·000259	·000518	·000776	·001035	·001294	·001553	·001812	·002071	·002329	·00
·01	·002588	·002847	·003106	·003365	·003623	·003882	·004141	·004400	·004659	·004918	·01
·02	·005176	·005435	·005694	·005953	·006212	·006471	·006729	·006988	·007247	·007506	·02
·03	·007765	·008023	·008282	·008541	·008800	·009059	·009318	·009576	·009835	·010094	·03
·04	·010353	·010612	·010870	·011129	·011388	·011647	·011906	·012165	·012423	·012682	·04
·05	·012941	·013200	·013459	·013717	·013976	·014235	·014494	·014753	·015012	·015270	·05
·06	·015529	·015788	·016047	·016306	·016564	·016823	·017082	·017341	·017600	·017859	·06
·07	·018117	·018376	·018635	·018894	·019153	·019412	·019670	·019929	·020188	·020447	·07
·08	·020706	·020964	·021223	·021482	·021741	·022000	·022259	·022517	·022776	·023035	·08
·09	·023294	·023553	·023811	·024070	·024329	·024588	·024847	·025106	·025364	·025623	·09
·10	·025882	·026141	·026400	·026658	·026917	·027176	·027435	·027694	·027953	·028211	·10
·11	·028470	·028729	·028988	·029247	·029505	·029764	·030023	·030282	·030541	·030800	·11
·12	·031058	·031317	·031576	·031835	·032094	·032353	·032611	·032870	·033129	·033388	·12
·13	·033647	·033905	·034164	·034423	·034682	·034941	·035200	·035458	·035717	·035976	·13
·14	·036235	·036494	·036752	·037011	·037270	·037529	·037788	·038047	·038305	·038564	·14
·15	·038823	·039082	·039341	·039599	·039858	·040117	·040376	·040635	·040894	·041152	·15
·16	·041411	·041670	·041929	·042188	·042446	·042705	·042964	·043223	·043482	·043741	·16
·17	·043999	·044258	·044517	·044776	·045035	·045294	·045552	·045811	·046070	·046329	·17
·18	·046588	·046846	·047105	·047364	·047623	·047882	·048141	·048399	·048658	·048917	·18
·19	·049176	·049435	·049693	·049952	·050211	·050470	·050729	·050988	·051246	·051505	·19
·20	·051764	·052023	·052282	·052540	·052799	·053058	·053317	·053576	·053835	·054093	·20
·21	·054352	·054611	·054870	·055129	·055387	·055646	·055905	·056164	·056423	·056682	·21
·22	·056940	·057199	·057458	·057717	·057976	·058235	·058493	·058752	·059011	·059270	·22
·23	·059529	·059787	·060046	·060305	·060564	·060823	·061082	·061340	·061599	·061858	·23
·24	·062117	·062376	·062634	·062893	·063152	·063411	·063670	·063929	·064187	·064446	·24
·25	·064705	·064964	·065223	·065481	·065740	·065999	·066258	·066517	·066776	·067034	·25
·26	·067293	·067552	·067811	·068070	·068328	·068587	·068846	·069105	·069364	·069623	·26
·27	·069881	·070140	·070399	·070658	·070917	·071176	·071434	·071693	·071952	·072211	·27
·28	·072470	·072728	·072987	·073246	·073505	·073764	·074023	·074281	·074540	·074799	·28
·29	·075058	·075317	·075575	·075834	·076093	·076352	·076611	·076870	·077128	·077387	·29
·30	·077646	·077905	·078164	·078422	·078681	·078940	·079199	·079458	·079717	·079975	·30
·31	·080234	·080493	·080752	·081011	·081269	·081528	·081787	·082046	·082305	·082564	·31
·32	·082822	·083081	·083340	·083599	·083858	·084117	·084375	·084634	·084893	·085152	·32
·33	·085411	·085669	·085928	·086187	·086446	·086705	·086964	·087222	·087481	·087740	·33
·34	·087999	·088258	·088516	·088775	·089034	·089293	·089552	·089811	·090069	·090328	·34
·35	·090587	·090846	·091105	·091363	·091622	·091881	·092140	·092399	·092658	·092916	·35
·36	·093175	·093434	·093693	·093952	·094210	·094469	·094728	·094987	·095246	·095505	·36
·37	·095763	·096022	·096281	·096540	·096799	·097058	·097316	·097575	·097834	·098093	·37
·38	·098352	·098610	·098869	·099128	·099387	·099646	·099905	·100163	·100422	·100681	·38
·39	·100940	·101199	·101457	·101716	·101975	·102234	·102493	·102752	·103010	·103269	·39
·40	·103528	·103787	·104046	·104304	·104563	·104822	·105081	·105340	·105599	·105857	·40
·41	·106116	·106375	·106634	·106893	·107151	·107410	·107669	·107928	·108187	·108446	·41
·42	·108704	·108963	·109222	·109481	·109740	·109999	·110257	·110516	·110775	·111034	·42
·43	·111293	·111551	·111810	·112069	·112328	·112587	·112846	·113104	·113363	·113622	·43
·44	·113881	·114140	·114398	·114657	·114916	·115175	·115434	·115693	·115951	·116210	·44
·45	·116469	·116728	·116987	·117245	·117504	·117763	·118022	·118281	·118540	·118798	·45
·46	·119057	·119316	·119575	·119834	·120092	·120351	·120610	·120869	·121128	·121387	·46
·47	·121645	·121904	·122163	·122422	·122681	·122940	·123198	·123457	·123716	·123975	·47
·48	·124234	·124492	·124751	·125010	·125269	·125528	·125787	·126045	·126304	·126563	·48
·49	·126822	·127081	·127339	·127598	·127857	·128116	·128375	·128634	·128892	·129151	·49
·50	·129410	·129669	·129928	·130186	·130445	·130704	·130963	·131222	·131481	·131739	·50

TABLE VIII.—Products of $S_1 \times$ ·001 up to $S_1 \times$ 1·000.

$S_1 = \sin 15° =$ ·25882.

No.	·000	·001	·002	·003	·004	·005	·006	·007	·008	·009	No.
·50	·129410	·129669	·129928	·130186	·130445	·130704	·130963	·131222	·131481	·131739	·50
·51	·131998	·132257	·132516	·132775	·133033	·133292	·133551	·133810	·134069	·134328	·51
·52	·134586	·134845	·135104	·135363	·135622	·135881	·136139	·136398	·136657	·136916	·52
·53	·137175	·137433	·137692	·137951	·138210	·138469	·138728	·138986	·139245	·139504	·53
·54	·139763	·140022	·140280	·140539	·140798	·141057	·141316	·141575	·141833	·142092	·54
·55	·142351	·142610	·142869	·143127	·143386	·143645	·143904	·144163	·144422	·144680	·55
·56	·144939	·145198	·145457	·145716	·145974	·146233	·146492	·146751	·147010	·147269	·56
·57	·147527	·147786	·148045	·148304	·148563	·148822	·149080	·149339	·149598	·149857	·57
·58	·150116	·150374	·150633	·150892	·151151	·151410	·151669	·151927	·152186	·152445	·58
·59	·152704	·152963	·153221	·153480	·153739	·153998	·154257	·154516	·154774	·155033	·59
·60	·155292	·155551	·155810	·156068	·156327	·156586	·156845	·157104	·157363	·157621	·60
·61	·157880	·158139	·158398	·158657	·158915	·159174	·159433	·159692	·159951	·160210	·61
·62	·160468	·160727	·160986	·161245	·161504	·161763	·162021	·162280	·162539	·162798	·62
·63	·163057	·163315	·163574	·163833	·164092	·164351	·164610	·164868	·165127	·165386	·63
·64	·165645	·165904	·166162	·166421	·166680	·166939	·167198	·167457	·167715	·167974	·64
·65	·168233	·168492	·168751	·169009	·169268	·169527	·169786	·170045	·170304	·170562	·65
·66	·170821	·171080	·171339	·171598	·171856	·172115	·172374	·172633	·172892	·173151	·66
·67	·173409	·173668	·173927	·174186	·174445	·174704	·174962	·175221	·175480	·175739	·67
·68	·175998	·176256	·176515	·176774	·177033	·177292	·177551	·177809	·178068	·178327	·68
·69	·178586	·178845	·179103	·179362	·179621	·179880	·180139	·180398	·180656	·180915	·69
·70	·181174	·181433	·181692	·181950	·182209	·182468	·182727	·182986	·183245	·183503	·70
·71	·183762	·184021	·184280	·184539	·184797	·185056	·185315	·185574	·185833	·186092	·71
·72	·186350	·186609	·186868	·187127	·187386	·187645	·187903	·188162	·188421	·188680	·72
·73	·188939	·189197	·189456	·189715	·189974	·190233	·190492	·190750	·191009	·191268	·73
·74	·191527	·191786	·192044	·192303	·192562	·192821	·193080	·193339	·193597	·193856	·74
·75	·194115	·194374	·194633	·194891	·195150	·195409	·195668	·195927	·196186	·196444	·75
·76	·196703	·196962	·197221	·197480	·197738	·197997	·198256	·198515	·198774	·199033	·76
·77	·199291	·199550	·199809	·200068	·200327	·200586	·200844	·201103	·201362	·201621	·77
·78	·201880	·202138	·202397	·202656	·202915	·203174	·203433	·203691	·203950	·204209	·78
·79	·204468	·204727	·204985	·205244	·205503	·205762	·206021	·206280	·206538	·206797	·79
·80	·207056	·207315	·207574	·207832	·208091	·208350	·208609	·208868	·209127	·209385	·80
·81	·209644	·209903	·210162	·210421	·210679	·210938	·211197	·211456	·211715	·211974	·81
·82	·212232	·212491	·212750	·213009	·213268	·213527	·213785	·214044	·214303	·214562	·82
·83	·214821	·215079	·215338	·215597	·215856	·216115	·216374	·216632	·216891	·217150	·83
·84	·217409	·217668	·217926	·218185	·218444	·218703	·218962	·219221	·219470	·219738	·84
·85	·219997	·220256	·220515	·220773	·221032	·221291	·221550	·221809	·222068	·222326	·85
·86	·222585	·222844	·223103	·223362	·223620	·223879	·224138	·224397	·224656	·224915	·86
·87	·225173	·225432	·225691	·225950	·226209	·226468	·226726	·226985	·227244	·227503	·87
·88	·227762	·228020	·228279	·228538	·228797	·229056	·229315	·229573	·229832	·230091	·88
·89	·230350	·230609	·230867	·231126	·231385	·231644	·231903	·232162	·232420	·232679	·89
·90	·232938	·233197	·233456	·233714	·233973	·234232	·234491	·234750	·235009	·235267	·90
·91	·235526	·235785	·236044	·236303	·236561	·236820	·237079	·237338	·237597	·237856	·91
·92	·238114	·238373	·238632	·238891	·239150	·239409	·239667	·239926	·240185	·240444	·92
·93	·240703	·240961	·241220	·241479	·241738	·241997	·242256	·242514	·242773	·243032	·93
·94	·243291	·243550	·243808	·244067	·244326	·244585	·244844	·245103	·245361	·245620	·94
·95	·245879	·246138	·246397	·246655	·246914	·247173	·247432	·247691	·247950	·248208	·95
·96	·248467	·248726	·248985	·249244	·249502	·249761	·250020	·250279	·250538	·250797	·96
·97	·251055	·251314	·251573	·251832	·252091	·252350	·252608	·252867	·253126	·253385	·97
·98	·253644	·253902	·254161	·254420	·254679	·254938	·255197	·255455	·255714	·255973	·98
·99	·256232	·256491	·256749	·257008	·257267	·257526	·257785	·258044	·258302	·258561	·99
1·00	·258820										1·00

TABLE IX.—Products of $S_3 \times \cdot 001$ up to $S_3 \times 1\cdot 000$.

$S_3 = \sin 45° = \cdot 70711$.

No.	·000	·001	·002	·003	·004	·005	·006	·007	·008	·009	No.
·00	·000000	·000707	·001414	·002121	·002828	·003536	·004243	·004950	·005657	·006364	·00
·01	·007071	·007778	·008485	·009192	·009900	·010607	·011314	·012021	·012728	·013435	·01
·02	·014142	·014849	·015556	·016264	·016971	·017678	·018385	·019092	·019799	·020506	·02
·03	·021213	·021920	·022628	·023335	·024042	·024749	·025456	·026163	·026870	·027577	·03
·04	.028284	·028992	·029699	·030406	·031113	·031820	·032527	·033234	·033941	·034648	·04
·05	·035356	·036063	·036770	·037477	·038184	·038891	·039598	·040305	·041012	·041719	·05
·06	·042427	·043134	·043841	·044548	·045255	·045962	·046669	·047376	·048083	·048791	·06
·07	·049498	·050205	·050912	051619	·052326	·053033	·053740	·054447	·055155	·055862	·07
·08	·056569	·057276	·057983	058690	·059397	·060104	·060811	·061519	·062226	·062933	·08
·09	·063640	·064347	·065054	·065761	·066468	·067175	·067883	·068590	·069297	·070004	·09
·10	·070711	·071418	·072125	·072832	·073539	·074247	·074954	·075661	·076368	·077075	·10
·11	·077782	·078489	·079196	·079903	·080611	·081318	·082025	·082732	·083439	·084146	·11
·12	·084853	·085560	·086267	086975	087682	·088389	·089096	·089803	·090510	·091217	·12
·13	·091924	·092631	·093339	·094046	·094753	·095460	·096167	·096874	·097581	·098288	·13
·14	·098995	·099703	·100410	·101117	·101824	·102531	·103238	·103945	·104652	·105359	·14
·15	·106067	·106774	·107481	·108188	·108895	·109602	·110309	·111016	·111723	·112430	·15
·16	·113138	·113845	·114552	·115259	·115966	·116673	·117380	·118087	·118794	·119502	·16
·17	·120209	·120916	·121623	·122330	·123037	·123744	·124451	·125158	·125866	·126573	·17
·18	·127280	·127987	·128694	·129401	·130108	·130815	·131522	·132230	·132937	·133644	·18
·19	·134351	·135058	·135765	·136472	·137179	·137886	·138594	·139301	·140008	·140715	·19
·20	·141422	·142129	·142836	·143543	·144250	·144958	·145665	·146372	·147079	·147786	·20
·21	·148493	·149200	·149907	·150614	·151322	·152029	·152736	·153443	·154150	·154857	·21
·22	·155564	·156271	·156978	·157686	·158393	·159100	·159807	·160514	·161221	·161928	·22
·23	·162635	·163342	·164050	·164757	·165464	·166171	·166878	·167585	·168292	·168999	·23
·24	·169706	·170414	·171121	·171828	·172535	·173242	·173949	·174656	·175363	·176070	·24
·25	·176778	·177485	·178192	·178899	·179606	·180313	·181020	·181727	·182434	·183141	·25
·26	·183849	·184556	·185263	·185970	·186677	·187384	·188091	·188798	·189505	·190213	·26
·27	·190920	·191627	·192334	·193041	·193748	·194455	·195162	·195869	·196577	·197284	·27
·28	·197991	·198698	·199405	·200112	·200819	·201526	·202233	·202941	·203648	·204355	·28
·29	·205062	·205769	·206476	·207183	·207890	·208597	·209305	·210012	·210719	·211426	·29
·30	·212133	·212840	·213547	214254	·214961	·215669	·216376	·217083	·217790	·218497	·30
·31	·219204	·219911	·220618	·221325	·222033	·222740	·223447	·224154	·224861	·225568	·31
·32	·226275	·226982	·227689	·228397	·229104	·229811	230518	231225	·231932	·232639	·32
·33	·233346	·234053	·234761	·235468	·236175	236882	·237589	238296	·239003	·239710	·33
·34	·240417	·241125	·241832	·242539	·243246	·243953	·244660	245367	·246074	·246781	·34
·35	·247489	·248196	·248903	·249610	·250317	·251024	·251731	·252438	·253145	·253852	·35
·36	·254560	·255267	·255974	·256681	·257388	·258095	·258802	·259509	·260216	·260924	·36
·37	·261631	·262338	·263045	·263752	·264459	·265166	·265873	·266580	267288	·267995	·37
·38	·268702	·269409	·270116	·270823	·271530	·272237	·272944	·273652	·274359	·275066	·38
·39	·275773	·276480	·277187	·277894	·278601	·279308	·280016	·280723	·281430	·282137	·39
·40	·282844	·283551	·284258	·284965	·285672	·286380	·287087	·287794	·288501	·289208	·40
·41	·289915	·290622	·291329	·292036	·292744	·293451	·294158	·294865	·295572	·296279	·41
·42	·296986	·297693	·298400	·299108	·299815	·300522	·301229	·301936	·302643	·303350	·42
·43	·304057	·304764	·305472	·306179	·306886	·307593	·308300	·309007	·309714	·310421	·43
.44	·311128	·311836	·312543	·313250	·313957	·314664	·315371	·316078	·316785	·317492	·44
·45	·318200	·318907	·319614	·320321	·321028	·321735	·322442	·323149	·323856	·324563	·45
·46	·325271	·325978	·326685	·327392	·328099	·328806	·329513	·330220	·330927	·331635	·46
·47	·332342	·333049	·333756	·334463	·335170	·335877	·336584	·337291	·337999	·338706	·47
·48	·339413	·340120	·340827	·341534	·342241	·342948	·343655	·344363	·345070	·345777	·48
·49	·346484	·347191	·347898	·348605	·349312	·350019	·350727	·351434	·352141	·352848	·49
·50	·353555	·354262	·354969	·355676	·356383	·357091	·357798	·358505	·359212	·359919	·50

TABLE IX.—Products of $S_3 \times \cdot 001$ up to $S_3 \times 1\cdot 000$.
$S_3 = \sin 45° = \cdot 70711$.

No.	·000	·001	·002	·003	·004	·005	·006	·007	·008	·009	No.
·50	·353555	·354262	·354969	·355676	·356383	·357091	·357798	·358505	·359212	·359919	·50
·51	·360626	·361333	·362040	·362747	·363455	·364162	·364869	·365576	·366283	·366990	·51
·52	·367697	·368404	·369111	·369819	·370526	·371233	·371940	·372647	·373354	·374061	·52
·53	·374768	·375475	·376183	·376890	·377597	·378304	·379011	·379718	·380425	·381132	·53
·54	·381839	·382547	·383254	·383961	·384668	·385375	·386082	·386789	·387496	·388203	·54
·55	·388911	·389618	·390325	·391032	·391739	·392446	·393153	·393860	·394567	·395274	·55
·56	·395982	·396689	·397396	·398103	·398810	·399517	·400224	·400931	·401638	·402346	·56
·57	·403053	·403760	·404467	·405174	·405881	·406588	·407295	·408002	·408710	·409417	·57
·58	·410124	·410831	·411538	·412245	·412952	·413659	·414366	·415074	·415781	·416488	·58
·59	·417195	·417902	·418609	·419316	·420023	·420730	·421438	·422145	·422852	·423559	·59
·60	·424266	·424973	·425680	·426387	·427094	·427802	·428509	·429216	·429923	·430630	·60
·61	·431337	·432044	·432751	·433458	·434166	·434873	·435580	·436287	·436994	·437701	·61
·62	·438408	·439115	·439822	·440530	·441237	·441944	·442651	·443358	·444065	·444772	·62
·63	·445479	·446186	·446894	·447601	·448308	·449015	·449722	·450429	·451136	·451843	·63
·64	·452550	·453258	·453965	·454672	·455379	·456086	·456793	·457500	·458207	·458914	·64
·65	·459622	·460329	·461036	·461743	·462450	·463157	·463864	·464571	·465278	·465985	·65
·66	·466693	·467400	·468107	·468814	·469521	·470228	·470935	·471642	·472349	·473057	·66
·67	·473764	·474471	·475178	·475885	·476592	·477299	·478006	·478713	·479421	·480128	·67
·68	·480835	·481542	·482249	·482956	·483663	·484370	·485077	·485785	·486492	·487199	·68
·69	·487906	·488613	·489320	·490027	·490734	·491441	·492149	·492856	·493563	·494270	·69
·70	·494977	·495684	·496391	·497098	·497805	·498513	·499220	·499927	·500634	·501341	·70
·71	·502048	·502755	·503462	·504169	·504877	·505584	·506291	·506998	·507705	·508412	·71
·72	·509119	·509826	·510533	·511241	·511948	·512655	·513362	·514069	·514776	·515483	·72
·73	·516190	·516897	·517605	·518312	·519019	·519726	·520433	·521140	·521847	·522554	·73
·74	·523261	·523969	·524676	·525383	·526090	·526797	·527504	·528211	·528918	·529625	·74
·75	·530333	·531040	·531747	·532454	·533161	·533868	·534575	·535282	·535989	·536696	·75
·76	·537404	·538111	·538818	·539525	·540232	·540939	·541646	·542353	·543060	·543768	·76
·77	·544475	·545182	·545889	·546596	·547303	·548010	·548717	·549424	·550132	·550839	·77
·78	·551546	·552253	·552960	·553667	·554374	·555081	·555788	·556496	·557203	·557910	·78
·79	·558617	·559324	·560031	·560738	·561445	·562152	·562860	·563567	·564274	·564981	·79
·80	·565688	·566395	·567102	·567809	·568516	·569224	·569931	·570638	·571345	·572052	·80
·81	·572759	·573466	·574173	·574880	·575588	·576295	·577002	·577709	·578416	·579123	·81
·82	·579830	·580537	·581244	·581952	·582659	·583366	·584073	·584780	·585487	·586194	·82
·83	·586901	·587608	·588316	·589023	·589730	·590437	·591144	·591851	·592558	·593265	·83
·84	·593972	·594680	·595387	·596094	·596801	·597508	·598215	·598922	·599629	·600336	·84
·85	·601044	·601751	·602458	·603165	·603872	·604579	·605286	·605993	·606700	·607407	·85
·86	·608115	·608822	·609529	·610236	·610943	·611650	·612357	·613064	·613771	·614479	·86
·87	·615186	·615893	·616600	·617307	·618014	·618721	·619428	·620135	·620843	·621550	·87
·88	·622257	·622964	·623671	·624378	·625085	·625792	·626499	·627207	·627914	·628621	·88
·89	·629328	·630035	·630742	·631449	·632156	·632863	·633571	·634278	·634985	·635692	·89
·90	·636399	·637106	·637813	·638520	·639227	·639935	·640642	·641349	·642056	·642763	·90
·91	·643470	·644177	·644884	·645591	·646299	·647006	·647713	·648420	·649127	·649834	·91
·92	·650541	·651248	·651955	·652663	·653370	·654077	·654784	·655491	·656198	·656905	·92
·93	·657612	·658319	·659027	·659734	·660441	·661148	·661855	·662562	·663269	·663976	·93
·94	·664683	·665391	·666098	·666805	·667512	·668219	·668926	·669633	·670340	·671047	·94
·95	·671755	·672462	·673169	·673876	·674583	·675290	·675997	·676704	·677411	·678118	·95
·96	·678826	·679533	·680240	·680947	·681654	·682361	·683068	·683775	·684482	·685190	·96
·97	·685897	·686604	·687311	·688018	·688725	·689432	·690139	·690846	·691554	·692261	·97
·98	·692968	·693675	·694382	·695089	·695796	·696503	·697210	·697918	·698625	·699332	·98
·99	·700039	·700746	·701453	·702160	·702867	·703574	·704282	·704989	·705696	·706403	·99
1·00	·707110										1·00

TABLE X.—Products of $S_4 \times \cdot 001$ up to $S_4 \times 1\cdot 000$.

$S_4 = \sin 60° = \cdot 86603$.

No.	·000	·001	·002	·003	·004	·005	·006	·007	·008	·009	No.
·00	·000000	·000866	·001732	·002598	·003464	·004330	·005196	·006062	·006928	·007794	·00
·01	·008660	·009526	·010392	·011258	·012124	·012990	·013856	·014723	·015589	·016455	·01
·02	·017321	·018187	·019053	·019919	·020785	·021651	·022517	·023383	·024249	·025115	·02
·03	·025981	·026847	·027713	·028579	·029445	·030311	·031177	·032043	·032909	·033775	·03
·04	·034641	·035507	·036373	·037239	·038105	·038971	·039837	·040703	·041569	·042435	·04
·05	·043302	·044168	·045034	·045900	·046766	·047632	·048498	·049364	·050230	·051096	·05
·06	·051962	·052828	·053694	·054560	·055426	·056292	·057158	·058024	·058890	·059756	·06
·07	·060622	·061488	·062354	·063220	·064086	·064952	·065818	·066684	·067550	·068416	·07
·08	·069282	·070148	·071014	·071880	·072747	·073613	·074479	·075345	·076211	·077077	·08
·09	·077943	·078809	·079675	·080541	·081407	·082273	·083139	·084005	·084871	·085737	·09
·10	·086603	·087469	·088335	·089201	·090067	·090933	·091799	·092665	·093531	·094397	·10
·11	·095263	·096129	·096995	·097861	·098727	·099593	·100459	·101326	·102192	·103058	·11
·12	·103924	·104790	·105656	·106522	·107388	·108254	·109120	·109986	·110852	·111718	·12
·13	·112584	·113450	·114316	·115182	·116048	·116914	·117780	·118646	·119512	·120378	·13
·14	·121244	·122110	·122976	·123842	·124708	·125574	·126440	·127306	·128172	·129038	·14
·15	·129905	·130771	·131637	·132503	·133369	·134235	·135101	·135967	·136833	·137699	·15
·16	·138565	·139431	·140297	·141163	·142029	·142895	·143761	·144627	·145493	·146359	·16
·17	·147225	·148091	·148957	·149823	·150689	·151555	·152421	·153287	·154153	·155019	·17
·18	·155885	·156751	·157617	·158483	·159350	·160216	·161082	·161948	·162814	·163680	·18
·19	·164546	·165412	·166278	·167144	·168010	·168876	·169742	·170608	·171474	·172340	·19
·20	·173206	·174072	·174938	·175804	·176670	·177536	·178402	·179268	·180134	·181000	·20
·21	·181866	·182732	·183598	·184464	·185330	·186196	·187062	·187929	·188795	·189661	·21
·22	·190527	·191393	·192259	·193125	·193991	·194857	·195723	·196589	·197455	·198321	·22
·23	·199187	·200053	·200919	·201785	·202651	·203517	·204383	·205249	·206115	·206981	·23
·24	·207847	·208713	·209579	·210445	·211311	·212177	·213043	·213909	·214775	·215641	·24
·25	·216508	·217374	·218240	·219106	·219972	·220838	·221704	·222570	·223436	·224302	·25
·26	·225168	·226034	·226900	·227766	·228632	·229498	·230364	·231230	·232096	·232962	·26
·27	·233828	·234694	·235560	·236426	·237292	·238158	·239024	·239890	·240756	·241622	·27
·28	·242488	·243354	·244220	·245086	·245953	·246819	·247685	·248551	·249417	·250283	·28
·29	·251149	·252015	·252881	·253747	·254613	·255479	·256345	·257211	·258077	·258943	·29
·30	·259809	·260675	·261541	·262407	·263273	·264139	·265005	·265871	·266737	·267603	·30
·31	·268469	·269335	·270201	·271067	·271933	·272799	·273665	·274532	·275398	·276264	·31
·32	·277130	·277996	·278862	·279728	·280594	·281460	·282326	·283192	·284058	·284924	·32
·33	·285790	·286656	·287522	·288388	·289254	·290120	·290986	·291852	·292718	·293584	·33
·34	·294450	·295316	·296182	·297048	·297914	·298780	·299646	·300512	·301378	·302244	·34
·35	·303111	·303977	·304843	·305709	·306575	·307441	·308307	·309173	·310039	·310905	·35
·36	·311771	·312637	·313503	·314369	·315235	·316101	·316967	·317833	·318699	·319565	·36
·37	·320431	·321297	·322163	·323029	·323895	·324761	·325627	·326493	·327359	·328225	·37
·38	·329091	·329957	·330823	·331689	·332556	·333422	·334288	·335154	·336020	·336886	·38
·39	·337752	·338618	·339484	·340350	·341216	·342082	·342948	·343814	·344680	·345546	·39
·40	·346412	·347278	·348144	·349010	·349876	·350742	·351608	·352474	·353340	·354206	·40
·41	·355072	·355938	·356804	·357670	·358536	·359402	·360268	·361135	·362001	·362867	·41
·42	·363733	·364599	·365465	·366331	·367197	·368063	·368929	·369795	·370661	·371527	·42
·43	·372393	·373259	·374125	·374991	·375857	·376723	·377589	·378455	·379321	·380187	·43
·44	·381053	·381919	·382785	·383651	·384517	·385383	·386249	·387115	·387981	·388847	·44
·45	·389714	·390580	·391446	·392312	·393178	·394044	·394910	·395776	·396642	·397508	·45
·46	·398374	·399240	·400106	·400972	·401838	·402704	·403570	·404436	·405302	·406168	·46
·47	·407034	·407900	·408766	·409632	·410498	·411364	·412230	·413096	·413962	·414828	·47
·48	·415694	·416560	·417426	·418292	·419159	·420025	·420891	·421757	·422623	·423489	·48
·49	·424355	·425221	·426087	·426953	·427819	·428685	·429551	·430417	·431283	·432149	·49
·50	·433015	·433881	·434747	·435613	·436479	·437345	·438211	·439077	·439943	·440809	·50

TABLE X.—Products of $S_4 \times \cdot 001$ up to $S_4 \times 1\cdot 000$.

$S_4 = \sin 60° = \cdot 86603$.

No.	·000	·001	·002	·003	·004	·005	·006	·007	·008	·009	No.
·50	·433015	·433881	·434747	·435613	·436479	·437345	·438211	·439077	·439943	·440809	·50
·51	·441675	·442541	·443407	·444273	·445139	·446005	·446871	·447738	·448604	·449470	·51
·52	·450336	·451202	·452068	·452934	·453800	·454666	·455532	·456398	·457264	·458130	·52
·53	·458996	·459862	·460728	·461594	·462460	·463326	·464192	·465058	·465924	·466790	·53
·54	·467656	·468522	·469388	·470254	·471120	·471986	·472852	·473718	·474584	·475450	·54
·55	·476317	·477183	·478049	·478915	·479781	·480647	·481513	·482379	·483245	·484111	·55
·56	·484977	·485843	·486709	·487575	·488441	·489307	·490173	·491039	·491905	·492771	·56
·57	·493637	·494503	·495369	·496235	·497101	·497967	·498833	·499699	·500565	·501431	·57
·58	·502297	·503163	·504029	·504895	·505762	·506628	·507494	·508360	·509226	·510092	·58
·59	·510958	·511824	·512690	·513556	·514422	·515288	·516154	·517020	·517886	·518752	·59
·60	·519618	·520484	·521350	·522216	·523082	·523948	·524814	·525680	·526546	·527412	·60
·61	·528278	·529144	·530010	·530876	·531742	·532608	·533474	·534341	·535207	·536073	·61
·62	·536939	·537805	·538671	·539537	·540403	·541269	·542135	·543001	·543867	·544733	·62
·63	·545599	·546465	·547331	·548197	·549063	·549929	·550795	·551661	·552527	·553393	·63
·64	·554259	·555125	·555991	·556857	·557723	·558589	·559455	·560321	·561187	·562053	·64
·65	·562920	·563786	·564652	·565518	·566384	·567250	·568116	·568982	·569848	·570714	·65
·66	·571580	·572446	·573312	·574178	·575044	·575910	·576776	·577642	·578508	·579374	·66
·67	·580240	·581106	·581972	·582838	·583704	·584570	·585436	·586302	·587168	·588034	·67
·68	·588900	·589766	·590632	·591498	·592365	·593231	·594097	·594963	·595829	·596695	·68
·69	·597561	·598427	·599293	·600159	·601025	·601891	·602757	·603623	·604489	·605355	·69
·70	·606221	·607087	·607953	·608819	·609685	·610551	·611417	·612283	·613149	·614015	·70
·71	·614881	·615747	·616613	·617479	·618345	·619211	·620077	·620944	·621810	·622676	·71
·72	·623542	·624408	·625274	·626140	·627006	·627872	·628738	·629604	·630470	·631336	·72
·73	·632202	·633068	·633934	·634800	·635666	·636532	·637398	·638264	·639130	·639996	·73
·74	·640862	·641728	·642594	·643460	·644326	·645192	·646058	·646924	·647790	·648656	·74
·75	·649523	·650389	·651255	·652121	·652987	·653853	·654719	·655585	·656451	·657317	·75
·76	·658183	·659049	·659915	·660781	·661647	·662513	·663379	·664245	·665111	·665977	·76
·77	·666843	·667709	·668575	·669441	·670307	·671173	·672039	·672905	·673771	·674637	·77
·78	·675503	·676369	·677235	·678101	·678968	·679834	·680700	·681566	·682432	·683298	·78
·79	·684164	·685030	·685896	·686762	·687628	·688494	·689360	·690226	·691092	·691958	·79
·80	·692824	·693690	·694556	·695422	·696288	·697154	·698020	·698886	·699752	·700618	·80
·81	·701484	·702350	·703216	·704082	·704948	·705814	·706680	·707547	·708413	·709279	·81
·82	·710145	·711011	·711877	·712743	·713609	·714475	·715341	·716207	·717073	·717939	·82
·83	·718805	·719671	·720537	·721403	·722269	·723135	·724001	·724867	·725733	·726599	·83
·84	·727465	·728331	·729197	·730063	·730929	·731795	·732661	·733527	·734393	·735259	·84
·85	·736126	·736992	·737858	·738724	·739590	·740456	·741322	·742188	·743054	·743920	·85
·86	·744786	·745652	·746518	·747384	·748250	·749116	·749982	·750848	·751714	·752580	·86
·87	·753446	·754312	·755178	·756044	·756910	·757776	·758642	·759508	·760374	·761240	·87
·88	·762106	·762972	·763838	·764704	·765571	·766437	·767303	·768169	·769035	·769901	·88
·89	·770767	·771633	·772499	·773365	·774231	·775097	·775963	·776829	·777695	·778561	·89
·90	·779427	·780293	·781159	·782025	·782891	·783757	·784623	·785489	·786355	·787221	·90
·91	·788087	·788953	·789819	·790685	·791551	·792417	·793283	·794150	·795016	·795882	·91
·92	·796748	·797614	·798480	·799346	·800212	·801078	·801944	·802810	·803676	·804542	·92
·93	·805408	·806274	·807140	·808006	·808872	·809738	·810604	·811470	·812336	·813202	·93
·94	·814068	·814934	·815800	·816666	·817532	·818398	·819264	·820130	·820996	·821862	·94
·95	·822729	·823595	·824461	·825327	·826193	·827059	·827925	·828791	·829657	·830523	·95
·96	·831389	·832255	·833121	·833987	·834853	·835719	·836585	·837451	·838317	·839183	·96
·97	·840049	·840915	·841781	·842647	·843513	·844379	·845245	·846111	·846977	·847843	·97
·98	·848709	·849575	·850441	·851307	·852174	·853040	·853906	·854772	·855638	·856504	·98
·99	·857370	·858236	·859102	·859968	·860834	·861700	·862566	·863432	·864298	·865164	·99
1·00	·866030										1·00

TABLE XI.—Products of $S_5 \times \cdot001$ up to $S_5 \times 1\cdot000$.

$S_5 = \sin 75° = \cdot96593$.

No.	·000	·001	·002	·003	·004	·005	·006	·007	·008	·009	No.
·00	·000000	·000966	·001932	·002898	·003864	·004830	·005796	·006762	·007727	·008639	·00
·01	·009659	·010625	·011591	·012557	·013523	·014489	·015455	·016421	·017387	·018353	·01
·02	·019319	·020285	·021250	·022216	·023182	·024148	·025114	·026080	·027046	·028012	·02
·03	·028978	·029944	·030910	·031876	·032842	·033808	·034773	·035739	·036705	·037671	·03
·04	·038637	·039603	·040569	·041535	·042501	·043467	·044433	·045399	·046365	·047331	·04
·05	·048297	·049262	·050228	·051194	·052160	·053126	·054092	·055058	·056024	·056990	·05
·06	·057956	·058922	·059888	·060854	·061820	·062785	·063751	·064717	·065683	·066649	·06
·07	·067615	·068581	·069547	·070513	·071479	·072445	·073411	·074377	·075343	·076308	·07
·08	·077274	·078240	·079206	·080172	·081138	·082104	·083070	·084036	·085002	·085968	·08
·09	·086934	·087900	·088866	·089831	·090797	·091763	·092729	·093695	·094661	·095627	·09
·10	·096593	·097559	·098525	·099491	·100457	·101423	·102389	·103355	·104320	·105286	·10
·11	·106252	·107218	·108184	·109150	·110116	·111082	·112048	·113014	·113980	·114946	·11
·12	·115912	·116878	·117843	·118809	·119775	·120741	·121707	·122673	·123639	·124605	·12
·13	·125571	·126537	·127503	·128469	·129435	·130401	·131366	·132332	·133298	·134264	·13
·14	·135230	·136196	·137162	·138128	·139094	·140060	·141026	·141992	·142958	·143924	·14
·15	·144890	·145855	·146821	·147787	·148753	·149719	·150685	·151651	·152617	·153583	·15
·16	·154549	·155515	·156481	·157447	·158413	·159378	·160344	·161310	·162276	·163242	·16
·17	·164208	·165174	·166140	·167106	·168072	·169038	·170004	·170970	·171936	·172901	·17
·18	·173867	·174833	·175799	·176765	·177731	·178697	·179663	·180629	·181595	·182561	·18
·19	·183527	·184493	·185459	·186424	·187390	·188356	·189322	·190288	·191254	·192220	·19
·20	·193186	·194152	·195118	·196084	·197050	·198016	·198982	·199948	·200913	·201879	·20
·21	·202845	·203811	·204777	·205743	·206709	·207675	·208641	·209607	·210573	·211539	·21
·22	·212505	·213471	·214436	·215402	·216368	·217334	·218300	·219266	·220232	·221198	·22
·23	·222164	·223130	·224096	·225062	·226028	·226994	·227959	·228925	·229891	·230857	·23
·24	·231823	·232789	·233755	·234721	·235687	·236653	·237619	·238585	·239551	·240517	·24
·25	·241483	·242448	·243414	·244380	·245346	·246312	·247278	·248244	·249210	·250176	·25
·26	·251142	·252108	·253074	·254040	·255006	·255971	·256937	·257903	·258869	·259835	·26
·27	·260801	·261767	·262733	·263699	·264665	·265631	·266597	·267563	·268529	·269494	·27
·28	·270460	·271426	·272392	·273358	·274324	·275290	·276256	·277222	·278188	·279154	·28
·29	·280120	·281086	·282052	·283017	·283983	·284949	·285915	·286881	·287847	·288813	·29
·30	·289779	·290745	·291711	·292677	·293643	·294609	·295575	·296541	·297506	·298472	·30
·31	·299438	·300404	·301370	·302336	·303302	·304268	·305234	·306200	·307166	·308132	·31
·32	·309098	·310064	·311029	·311995	·312961	·313927	·314893	·315859	·316825	·317791	·32
·33	·318757	·319723	·320689	·321655	·322621	·323587	·324552	·325518	·326484	·327450	·33
·34	·328416	·329382	·330348	·331314	·332280	·333246	·334212	·335178	·336144	·337110	·34
·35	·338076	·339041	·340007	·340973	·341939	·342905	·343871	·344837	·345803	·346769	·35
·36	·347735	·348701	·349667	·350633	·351599	·352564	·353530	·354496	·355462	·356428	·36
·37	·357394	·358360	·359326	·360292	·361258	·362224	·363190	·364156	·365122	·366087	·37
·38	·367053	·368019	·368985	·369951	·370917	·371883	·372849	·373815	·374781	·375747	·38
·39	·376713	·377679	·378645	·379610	·380576	·381542	·382508	·383474	·384440	·385406	·39
·40	·386372	·387338	·388304	·389270	·390236	·391202	·392168	·393134	·394099	·395065	·40
·41	·396031	·396997	·397963	·398929	·399895	·400861	·401827	·402793	·403759	·404725	·41
·42	·405691	·406657	·407622	·408588	·409554	·410520	·411486	·412452	·413418	·414384	·42
·43	·415350	·416316	·417282	·418248	·419214	·420180	·421145	·422111	·423077	·424043	·43
·44	·425009	·425975	·426941	·427907	·428873	·429839	·430805	·431771	·432737	·433703	·44
·45	·434669	·435634	·436600	·437566	·438532	·439498	·440464	·441430	·442396	·443362	·45
·46	·444328	·445294	·446260	·447226	·448192	·449157	·450123	·451089	·452055	·453021	·46
·47	·453987	·454953	·455919	·456885	·457851	·458817	·459783	·460749	·461715	·462680	·47
·48	·463646	·464612	·465578	·466544	·467510	·468476	·469442	·470408	·471374	·472340	·48
·49	·473306	·474272	·475238	·476203	·477169	·478135	·479101	·480067	·481033	·481999	·49
·50	·482965	·483931	·484897	·485863	·486829	·487795	·488761	·489727	·490692	·491658	·50

TABLE XI.—Products of $S_5 \times$ ·001 up to $S_5 \times$ 1·000.

$S_5 = \sin 75° =$ ·96593.

No.	·000	·001	·002	·003	·004	·005	·006	·007	·008	·009	No.
·50	·482965	·483931	·484897	·485863	·486829	·487795	·488761	·489727	·490692	·491658	·50
·51	·492624	·493590	·494556	·495522	·496488	·497454	·498420	·499386	·500352	·501318	·51
·52	·502284	·503250	·504215	·505181	·506147	·507113	·508079	·509045	·510011	·510977	·52
·53	·511943	·512909	·513875	·514841	·515807	·516773	·517738	·518704	·519670	·520636	·53
·54	·521602	·522568	·523534	·524500	·525466	·526432	·527398	·528364	·529330	·530296	·54
·55	·531262	·532227	·533193	·534159	·535125	·536091	·537057	·538023	·538989	·539955	·55
·56	·540921	·541887	·542853	·543819	·544785	·545750	·546716	·547682	·548648	·549614	·56
·57	·550580	·551546	·552512	·553478	·554444	·555410	·556376	·557342	·558308	·559273	·57
·58	·560239	·561205	·562171	·563137	·564103	·565069	·566035	·567001	·567967	·568933	·58
·59	·569899	·570865	·571831	·572796	·573762	·574728	·575694	·576660	·577626	·578592	·59
·60	·579558	·580524	·581490	·582456	·583422	·584388	·585354	·586320	·587285	·588251	·60
·61	·589217	·590183	·591149	·592115	·593081	·594047	·595013	·595979	·596945	·597911	·61
·62	·598877	·599843	·600808	·601774	·602740	·603706	·604672	·605638	·606604	·607570	·62
·63	·608536	·609502	·610468	·611434	·612400	·613366	·614331	·615297	·616263	·617229	·63
·64	·618195	·619161	·620127	·621093	·622059	·623025	·623991	·624957	·625923	·626889	·64
·65	·627855	·628820	·629786	·630752	·631718	·632684	·633650	·634616	·635582	·636548	·65
·66	·637514	·638480	·639446	·640412	·641378	·642343	·643309	·644275	·645241	·646207	·66
·67	·647173	·648139	·649105	·650071	·651037	·652003	·652969	·653935	·654901	·655866	·67
·68	·656832	·657798	·658764	·659730	·660696	·661662	·662628	·663594	·664560	·665526	·68
·69	·666492	·667458	·668424	·669389	·670355	·671321	·672287	·673253	·674219	·675185	·69
·70	·676151	·677117	·678083	·679049	·680015	·680981	·681947	·682913	·683878	·684844	·70
·71	·685810	·686776	·687742	·688708	·689674	·690640	·691606	·692572	·693538	·694504	·71
·72	·695470	·696436	·697401	·698367	·699333	·700299	·701265	·702231	·703197	·704163	·72
·73	·705129	·706095	·707061	·708027	·708993	·709959	·710924	·711890	·712856	·713822	·73
·74	·714788	·715754	·716720	·717686	·718652	·719618	·720584	·721550	·722516	·723482	·74
·75	·724448	·725413	·726379	·727345	·728311	·729277	·730243	·731209	·732175	·733141	·75
·76	·734107	·735073	·736039	·737005	·737971	·738936	·739902	·740868	·741834	·742800	·76
·77	·743766	·744732	·745698	·746664	·747630	·748596	·749562	·750528	·751494	·752459	·77
·78	·753425	·754391	·755357	·756323	·757289	·758255	·759221	·760187	·761153	·762119	·78
·79	·763085	·764051	·765017	·765982	·766948	·767914	·768880	·769846	·770812	·771778	·79
·80	·772744	·773710	·774676	·775642	·776608	·777574	·778540	·779506	·780471	·781437	·80
·81	·782403	·783369	·784335	·785301	·786267	·787233	·788199	·789165	·790131	·791097	·81
·82	·792063	·793029	·793994	·794960	·795926	·796892	·797858	·798824	·799790	·800756	·82
·83	·801722	·802688	·803654	·804620	·805586	·806552	·807517	·808483	·809449	·810415	·83
·84	·811381	·812347	·813313	·814279	·815245	·816211	·817177	·818143	·819109	·820075	·84
·85	·821041	·822006	·822972	·823938	·824904	·825870	·826836	·827802	·828768	·829734	·85
·86	·830700	·831666	·832632	·833598	·834564	·835529	·836495	·837461	·838427	·839393	·86
·87	·840359	·841325	·842291	·843257	·844223	·845189	·846155	·847121	·848087	·849052	·87
·88	·850018	·850984	·851950	·852916	·853882	·854848	·855814	·856780	·857746	·858712	·88
·89	·859678	·860644	·861610	·862575	·863541	·864507	·865473	·866439	·867405	·868371	·89
·90	·869337	·870303	·871269	·872235	·873201	·874167	·875133	·876099	·877064	·878030	·90
·91	·878996	·879962	·880928	·881894	·882860	·883826	·884792	·885758	·886724	·887690	·91
·92	·888656	·889622	·890587	·891553	·892519	·893485	·894451	·895417	·896383	·897349	·92
·93	·898315	·899281	·900247	·901213	·902179	·903145	·904110	·905076	·906042	·907008	·93
·94	·907974	·908940	·909906	·910872	·911838	·912804	·913770	·914736	·915702	·916668	·94
·95	·917634	·918599	·919565	·920531	·921497	·922463	·923429	·924395	·925361	·926327	·95
·96	·927293	·928259	·929225	·930191	·931157	·932122	·933088	·934054	·935020	·935986	·96
·97	·936952	·937918	·938884	·939850	·940816	·941782	·942748	·943714	·944680	·945645	·97
·98	·946611	·947577	·948543	·949509	·950475	·951441	·952407	·953373	·954339	·955305	·98
·99	·956271	·957237	·958203	·959168	·960134	·961100	·962066	·963032	·963998	·964964	·99
1·00	·965930										1·00

TABLE XII.—Natural Numbers to three places of Decimals corresponding to Logarithms with Indices 6, 7 and 8.

Logarithms with Index 6 or $\bar{4}$.

Natural No.	Natural No. to 3 places of Decimals	Logarithms
·00000	·000	·0000000
·00050	·001	·6989700
·00099		·9999999

Logarithms with Index 7 or $\bar{3}$.

Nat. No.	Nat. No. to 3 places of Decimals	Logarithms	Nat. No.	Nat. No. to 3 places of Decimals	Logarithms	Nat. No.	Nat. No. to 3 places of Decimals	Logarithms
·0010	·001	·0000000	·0045	·005	·6532125	·0085	·009	·9294189
·0015	·002	·1760913	·0055	·006	·7403627	·0095	·010	·9777236
·0025	·003	·3979400	·0065	·007	·8129134	·0099		·9999999
·0035	·004	·5440680	·0075	·008	·8750613			
·0045		·6532125	·0085		·9294189			

Logarithms with Index 8 or $\bar{2}$.

Nat. No.	Nat. No. to 3 places of Decimals	Logarithms	Nat. No.	Nat. No. to 3 places of Decimals	Logarithms	Nat. No.	Nat. No. to 3 places of Decimals	Logarithms	Nat. No.	Nat. No. to 3 places of Decimals	Logarithms
·0100	·010	·0000000	·0325	·033	·5118834	·0555	·056	·7442930	·0785	·079	·8948697
·0105	·011	·0211893	·0335	·034	·5250448	·0565	·057	·7520484	·0795	·080	·9003671
·0115	·012	·0606978	·0345	·035	·5378191	·0575	·058	·7596678	·0805	·081	·9057959
·0125	·013	·0969100	·0355	·036	·5502284	·0585	·059	·7671559	·0815	·082	·9111576
·0135	·014	·1303338	·0365	·037	·5622929	·0595	·060	·7745170	·0825	·083	·9164539
·0145	·015	·1613680	·0375	·038	·5740313	·0605	·061	·7817554	·0835	·084	·9216865
·0155	·016	·1903317	·0385	·039	·5854607	·0615	·062	·7888751	·0845	·085	·9268567
·0165	·017	·2174839	·0395	·040	·5965971	·0625	·063	·7958800	·0855	·086	·9319661
·0175	·018	·2430380	·0405	·041	·6074550	·0635	·064	·8027737	·0865	·087	·9370161
·0185	·019	·2671717	·0415	·042	·6180481	·0645	·065	·8095597	·0875	·088	·9420081
·0195	·020	·2900346	·0425	·043	·6283889	·0655	·066	·8162413	·0885	·089	·9469433
·0205	·021	·3117539	·0435	·044	·6384893	·0665	·067	·8228216	·0895	·090	·9518230
·0215	·022	·3324385	·0445	·045	·6483600	·0675	·068	·8293038	·0905	·091	·9566486
·0225	·023	·3521825	·0455	·046	·6580114	·0685	·069	·8356906	·0915	·092	·9614211
·0235	·024	·3710679	·0465	·047	·6674530	·0695	·070	·8419848	·0925	·093	·9661417
·0245	·025	·3891661	·0475	·048	·6766936	·0705	·071	·8481891	·0935	·094	·9708116
·0255	·026	·4065402	·0485	·049	·6857417	·0715	·072	·8543060	·0945	·095	·9754318
·0265	·027	·4232459	·0495	·050	·6946052	·0725	·073	·8603380	·0955	·096	·9800034
·0275	·028	·4393327	·0505	·051	·7032914	·0735	·074	·8662873	·0965	·097	·9845273
·0285	·029	·4548449	·0515	·052	·7118072	·0745	·075	·8721563	·0975	·098	·9890046
·0295	·030	·4698220	·0525	·053	·7201593	·0755	·076	·8779470	·0985	·099	·9934362
·0305	·031	·4842998	·0535	·054	·7283538	·0765	·077	8836614	·0995	·100	·9978231
·0315	·032	·4983106	·0545	·055	·7363965	·0775	·078	·8893017	·0999		·9999999
·0325		·5118834	·0555		·7442930	·0785		·8948697			

TABLE XIII.—Values of N (Longitude of Moon's Ascending Node) for 0 hour Greenwich Mean Time, January 1.

Value on 0 hour G. M. T., January 1st, 1880 = 285°·956863.
Motion per Julian year in 1880 = 19°·34146248.
Motion for 365 days = 19°·32822387 and for 1 day = 0°·052954.

Year	N	Year	N	Year	N	Year	N
1850	146°·174466	1875	22°·650943	1900	259°·127704	1925	135°·604751
1	126·846228	6	3·322717	1	239·799490	6	116·276548
2	107·517991	7	343·941537	2	220·471276	7	96·948346
3	88·136801	8	324·613312	3	201·143062	8	77·620143
4	68·808565	9	305·285087	4	181·814849	9	58·238987
1855	49·480329	1880	285·956863	1905	162·433682	1930	38·910786
6	30·152094	1	266·575685	6	143·105470	1	19·582585
7	10·770905	2	247·247462	7	123·777258	2	0·254384
8	351·442671	3	227·919239	8	104·449046	3	340·873230
9	332·114437	4	208·591017	9	85·067881	4	321·545031
1860	312·786204	1885	189·209841	1910	65·739671	1935	302·216832
1	293·405017	6	169·881619	1	46·411461	6	282·888633
2	274·076784	7	150·553398	2	27·083251	7	263·507481
3	254·748553	8	131·225178	3	7·702088	8	244·179283
4	235·420321	9	111·844004	4	348·373879	9	224·851086
1865	216·039136	1890	92·515784	1915	329·045671	1940	205·522889
6	196·710905	1	73·187565	6	309·717463	1	186·141739
7	177·382675	2	53·859346	7	290·336302	2	166·813543
8	158·054446	3	34·478174	8	271·008095	3	147·485347
9	138·673262	4	15·149956	9	251·679888	4	128·157152
1870	119·345034	1895	355·821739	1920	232·351682	1945	108·776004
1	100·016805	6	336·493522	1	212·970523	6	89·447810
2	80·688577	7	317·112351	2	193·642318	7	70·119616
3	61·307396	8	297·784135	3	174·314113	8	50·791423
4	41·979169	9	278·455920	4	154·985909	9	31·410276

TABLE XIV.—Showing the Decrement of N (Longitude of Moon's Ascending Node) since 0 hour January 1 up to Midnight of each Day throughout the Year.

Daily Motion = 0°·0529541.

In Leap Years for all dates after February 28—March 1, use a mean value between the particular day and the day following.

Date	Decre.	Date	Decre.	Date	Decre.	Date	Decre.	Date	Decre.	Date	Decre.
JAN.	°	JAN.	°	JAN.	°	JAN	°	FEB.	°	FEB.	°
1- 2	0·0265	11-12	0·5560	21-22	1·0856	31-32	1·6151	9-10	2·0917	19-20	2·6212
2- 3	0·0794	12-13	0·6090	22-23	1·1385	FEB.		10-11	2·1446	20-21	2·6742
3- 4	0·1324	13-14	0·6619	23-24	1·1915	1- 2	1·6681	11-12	2·1976	21-22	2·7271
4- 5	0·1853	14-15	0·7149	24-25	1·2444	2- 3	1·7210	12-13	2·2505	22-23	2·7801
5- 6	0·2383	15-16	0·7678	25-26	1·2974	3- 4	1·7740	13-14	2·3035	23-24	2·8330
6- 7	0·2912	16-17	0·8208	26-27	1·3503	4- 5	1·8269	14-15	2·3565	24-25	2·8860
7- 8	0·3442	17-18	0·8737	27-28	1·4033	5- 6	1·8799	15-16	2·4094	25-26	2·9390
8- 9	0·3972	18-19	0·9267	28-29	1·4562	6- 7	1·9328	16-17	2·4624	26-27	2·9919
9-10	0·4501	19-20	0·9797	29-30	1·5092	7- 8	1·9858	17-18	2·5153	27-28	3·0449
10-11	0·5031	20-21	1·0326	30-31	1·5621	8- 9	2·0387	18-19	2·5683	28-29	3·0978

N.B.—In Table XIV. The middle of the year of observations will occur at noon or midnight according as the 29th February is included in the period of observations or not. If the *midnight* falls on a date in a common year, or before the 29th February in a leap year, then the Decrement for N as given in the Table is correct: If, however, the *midnight* falls after the 29th February in a leap year, then take the value as that given for the *succeeding date* in the Table.

If the *noon* falls in a common year or before the 29th February in a leap year, the value to be taken from

TABLE XIV.—Showing the Decrement of N (Longitude of Moon's Ascending Node) since 0 hour January 1 up to Midnight of each Day throughout the Year—(*Continued*).

Daily Motion = 0°·0529541.

In Leap Years for all dates after February 28—March 1, use a mean value between the particular day and the day following.

Date	Decre.	Date	Decre.	Date	Decre.	Date	Decre.	Date	Decre.	Date	Decre.
MAR.	°	APR.	°	JUNE	°	AUG.	°	SEPT.	°	NOV.	°
1- 2	3·1508	22-23	5·9044	12-13	8·6050	2- 3	11·3057	23-24	14·0593	13-14	16·7600
2- 3	3·2037	23-24	5·9573	13-14	8·6580	3- 4	11·3587	24-25	14·1123	14-15	16·8129
3- 4	3·2567	24-25	6·0103	14-15	8·7109	4- 5	11·4116	25-26	14·1652	15-16	16·8659
4- 5	3·3096	25-26	6·0632	15-16	8·7639	5- 6	11·4646	26-27	14·2182	16-17	16·9188
5- 6	3·3626	26-27	6·1162	16-17	8·8169	6- 7	11·5175	27-28	14·2711	17-18	16·9718
6- 7	3·4155	27-28	6·1692	17-18	8·8698	7- 8	11·5705	28-29	14·3241	18-19	17·0247
7- 8	3·4685	28-29	6·2221	18-19	8·9228	8- 9	11·6234	29-30	14·3770	19-20	17·0777
8- 9	3·5214	29-30	6·2751	19-20	8·9757	9-10	11·6764	30-31	14·4300	20-21	17·1307
9-10	3·5744	30-31	6·3280	20-21	9·0287	10-11	11·7293	OCT.		21-22	17·1836
10-11	3·6274	MAY		21-22	9·0816	11-12	11·7823	1- 2	14·4829	22-23	17·2366
11-12	3·6803	1- 2	6·3810	22-23	9·1346	12-13	11·8352	2- 3	14·5359	23-24	17·2895
12-13	3·7333	2- 3	6·4339	23-24	9·1875	13-14	11·8882	3- 4	14·5889	24-25	17·3425
13-14	3·7862	3- 4	6·4869	24-25	9·2405	14-15	11·9411	4- 5	14·6418	25-26	17·3954
14-15	3·8392	4- 5	6·5398	25-26	9·2934	15-16	11·9941	5- 6	14·6948	26-27	17·4484
15-16	3·8921	5- 6	6·5928	26-27	9·3464	16-17	12·0471	6- 7	14·7477	27-28	17·5013
16-17	3·9451	6- 7	6·6457	27-28	9·3994	17-18	12·1000	7- 8	14·8007	28-29	17·5543
17-18	3·9980	7- 8	6·6987	28-29	9·4523	18-19	12·1530	8- 9	14·8536	29-30	17·6072
18-19	4·0510	8- 9	6·7516	29-30	9·5053	19-20	12·2059	9-10	14·9066	30-31	17·6602
19-20	4·1039	9-10	6·8046	30-31	9·5582	20-21	12·2589	10-11	14·9595	DEC.	
20-21	4·1569	10-11	6·8576	JULY		21-22	12·3118	11-12	15·0125	1- 2	17·7131
21-22	4·2099	11-12	6·9105	1- 2	9·6112	22-23	12·3648	12-13	15·0654	2- 3	17·7661
22-23	4·2628	12-13	6·9635	2- 3	9·6641	23-24	12·4177	13-14	15·1184	3- 4	17·8191
23-24	4·3158	13-14	7·0164	3- 4	9·7171	24-25	12·4707	14-15	15·1713	4- 5	17·8720
24-25	4·3687	14-15	7·0694	4- 5	9·7700	25-26	12·5236	15-16	15·2243	5- 6	17·9250
25-26	4·4217	15-16	7·1223	5- 6	9·8230	26-27	12·5766	16-17	15·2773	6- 7	17·9779
26-27	4·4746	16-17	7·1753	6- 7	9·8759	27-28	12·6296	17-18	15·3302	7- 8	18·0309
27-28	4·5276	17-18	7·2282	7- 8	9·9289	28-29	12·6825	18-19	15·3832	8- 9	18·0838
28-29	4·5805	18-19	7·2812	8- 9	9·9818	29-30	12·7355	19-20	15·4361	9-10	18·1368
29-30	4·6335	19-20	7·3341	9-10	10·0348	30-31	12·7884	20-21	15·4891	10-11	18·1897
30-31	4·6864	20-21	7·3871	10-11	10·0878	31-32	12·8414	21-22	15·5420	11-12	18·2427
31-32	4·7394	21-22	7·4401	11-12	10·1407	SEPT.		22-23	15·5950	12-13	18·2956
APR.		22-23	7·4930	12-13	10·1937	1- 2	12·8943	23-24	15·6479	13-14	18·3486
1- 2	4·7923	23-24	7·5460	13-14	10·2466	2- 3	12·9473	24-25	15·7009	14-15	18·4015
2- 3	4·8453	24-25	7·5989	14-15	10·2996	3- 4	13·0002	25-26	15·7538	15-16	18·4545
3- 4	4·8983	25-26	7·6519	15-16	10·3525	4- 5	13·0532	26-27	15·8068	16-17	18·5075
4- 5	4·9512	26-27	7·7048	16-17	10·4055	5- 6	13·1061	27-28	15·8598	17-18	18·5604
5- 6	5·0042	27-28	7·7578	17-18	10·4584	6- 7	13·1591	28-29	15·9127	18-19	18·6134
6- 7	5·0571	28-29	7·8107	18-19	10·5114	7- 8	13·2120	29-30	15·9657	19-20	18·6663
7- 8	5·1101	29-30	7·8637	19-20	10·5643	8- 9	13·2650	30-31	16·0186	20-21	18·7193
8- 9	5·1630	30-31	7·9166	20-21	10·6173	9-10	13·3180	31-32	16·0716	21-22	18·7722
9-10	5·2160	31-32	7·9696	21-22	10·6703	10-11	13·3709	NOV.		22-23	18·8252
10-11	5·2689	JUNE		22-23	10·7232	11-12	13·4239	1- 2	16·1245	23-24	18·8781
11-12	5·3219	1- 2	8·0225	23-24	10·7762	12-13	13·4768	2- 3	16·1775	24-25	18·9311
12-13	5·3748	2- 3	8·0755	24-25	10·8291	13-14	13·5298	3- 4	16·2304	25-26	18·9840
13-14	5·4278	3- 4	8·1285	25-26	10·8821	14-15	13·5827	4- 5	16·2834	26-27	19·0370
14-15	5·4807	4- 5	8·1814	26-27	10·9350	15-16	13·6357	5- 6	16·3363	27-28	19·0900
15-16	5·5337	5- 6	8·2344	27-28	10·9880	16-17	13·6886	6- 7	16·3893	28-29	19·1429
16-17	5·5867	6- 7	8·2873	28-29	11·0409	17-18	13·7416	7- 8	16·4422	29-30	19·1959
17-18	5·6396	7- 8	8·3403	29-30	11·0939	18-19	13·7945	8- 9	16·4952	30-31	19·2488
18-19	5·6926	8- 9	8·3932	30-31	11·1468	19-20	13·8475	9-10	16·5482	31-32	19·3018
19-20	5·7455	9-10	8·4462	31-32	11·1998	20-21	13·9005	10-11	16·6011		
20-21	5·7985	10-11	8·4991	AUG.		21-22	13·9534	11-12	16·6541		
21-22	5·8514	11-12	8·5521	1- 2	11·2527	22-23	14·0064	12-13	16·7070		

the Table is the mean between the *preceding* and *succeeding* midnights; but if the *noon* falls after the 29th February in a leap year, the mean between the values for the *two midnights immediately following* is to be taken.

TABLE XV.—Values of p_1 (Mean Longitude of Solar Perigee) for 0 hour, January 1.

p_1 for 0 hour, January 1, 1880 = 280°·874802.
Motion per Julian year = 0°·01710693.
Motion for 365 days = 0°·01709295.

Year	p_1	Year	p_1	Year	p_1	Year	p_1
1850	280°·3614	1875	280°·7892	1900	281°·2171	1925	281°·6450
51	·3785	76	·8063	1	·2342	26	·6621
52	·3956	77	·8235	2	·2513	27	·6792
53	·4125	78	·8406	3	·2684	28	·6963
54	·4299	79	·8577	4	·2855	29	·7135
55	·4470	1880	·8748	5	·3027	1930	·7306
56	·4641	81	·8920	6	·3198	31	·7477
57	·4812	82	·9091	7	·3369	32	·7648
58	·4983	83	·9262	8	·3540	33	·7820
59	·5154	84	·9433	9	·3711	34	·7991
1860	·5325	85	·9604	1910	·3882	35	·8162
61	·5497	86	·9775	11	·4054	36	·8333
62	·5668	87	·9946	12	·4225	37	·8505
63	·5839	88	281·0117	13	·4396	38	·8676
64	·6010	89	·0289	14	·4567	39	·8847
65	·6181	1890	·0460	15	·4738	1940	·9018
66	·6352	91	·0631	16	·4909	41	·9189
67	·6523	92	·0802	17	·5081	42	·9360
68	·6694	93	·0973	18	·5252	43	·9532
69	·6866	94	·1144	19	·5423	44	·9703
1870	·7037	95	·1315	1920	·5594	45	·9874
71	·7208	96	·1486	21	·5766	46	282·0045
72	·7379	97	·1658	22	·5937	47	·0216
73	·7550	98	·1829	23	·6108	48	·0387
74	·7721	99	·2000	24	·6279	49	·0559

TABLE XVI.—Increment of p_1 since 0 hour, January 1, for certain Days of the Year.

Motion for 1 day = 0°·00004683.

Date	Increment	Date	Increment	Date	Increment	Date	Increment
Jan. 10	0°·00042	Apr. 10	0°·00464	July 9	0°·00885	Oct. 7	0°·01307
„ 20	·00089	„ 20	·00510	„ 19	·00932	„ 17	·01353
„ 30	·00136	„ 30	·00557	„ 29	·00979	„ 27	·01400
Feb. 9	·00183	May 10	·00604	Aug. 8	·01026	Nov. 6	·01447
„ 19	·00229	„ 20	·00651	„ 18	·01072	„ 16	·01494
Mar. 1	·00276	„ 30	·00698	„ 28	·01119	„ 26	·01541
„ 11	·00323	June 9	·00745	Sept. 7	·01166	Dec. 6	·01588
„ 21	·00370	„ 19	·00791	„ 17	·01213	„ 16	·01634
„ 31	·00417	„ 29	·00838	„ 27	·01260	„ 26	·01681

TABLE XVII.—Values of I, ν and ξ, corresponding to N.

N	I	ν	ξ	N	N	I	ν	ξ	N	N	I	ν	ξ	N
°	°	°	°	°	°	°	°	°	°	°	°	°	°	°
0.0	28.602	0.000	0.000	360.0	30.5	28.005	5.564	5.017	329.5	60.5	26.351	10.131	9.196	299.5
0.5	28.602	0.094	0.084	359.5	31.0	27.985	5.651	5.095	329.0	61.0	26.316	10.194	9.255	299.0
1.0	28.601	0.188	0.169	359.0	31.5	27.965	5.736	5.173	328.5	61.5	26.280	10.256	9.313	298.5
1.5	28.600	0.281	0.253	358.5	32.0	27.945	5.822	5.251	328.0	62.0	26.245	10.318	9.370	298.0
2.0	28.599	0.375	0.337	358.0	32.5	27.925	5.907	5.328	327.5	62.5	26.209	10.379	9.427	297.5
2.5	28.598	0.468	0.421	357.5	33.0	27.904	5.992	5.406	327.0	63.0	26.173	10.440	9.484	297.0
3.0	28.596	0.562	0.506	357.0	33.5	27.884	6.077	5.482	326.5	63.5	26.137	10.500	9.539	296.5
3.5	28.594	0.656	0.590	356.5	34.0	27.862	6.162	5.559	326.0	64.0	26.101	10.560	9.595	296.0
4.0	28.591	0.749	0.674	356.0	34.5	27.841	6.246	5.636	325.5	64.5	26.064	10.619	9.650	295.5
4.5	28.589	0.843	0.758	355.5	35.0	27.819	6.330	5.712	325.0	65.0	26.027	10.677	9.705	295.0
5.0	28.585	0.936	0.842	355.0	35.5	27.797	6.414	5.788	324.5	65.5	25.990	10.735	9.759	294.5
5.5	28.582	1.030	0.926	354.5	36.0	27.775	6.497	5.864	324.0	66.0	25.953	10.793	9.812	294.0
6.0	28.578	1.123	1.010	354.0	36.5	27.752	6.580	5.939	323.5	66.5	25.916	10.849	9.865	293.5
6.5	28.574	1.217	1.094	353.5	37.0	27.729	6.663	6.015	323.0	67.0	25.878	10.906	9.918	293.0
7.0	28.570	1.310	1.178	353.0	37.5	27.706	6.745	6.090	322.5	67.5	25.841	10.961	9.970	292.5
7.5	28.565	1.403	1.262	352.5	38.0	27.682	6.828	6.164	322.0	68.0	25.803	11.016	10.021	292.0
8.0	28.560	1.496	1.346	352.0	38.5	27.658	6.909	6.239	321.5	68.5	25.765	11.070	10.072	291.5
8.5	28.555	1.590	1.430	351.5	39.0	27.634	6.991	6.313	321.0	69.0	25.726	11.124	10.123	291.0
9.0	28.549	1.683	1.514	351.0	39.5	27.610	7.072	6.387	320.5	69.5	25.688	11.177	10.173	290.5
9.5	28.543	1.776	1.598	350.5	40.0	27.585	7.153	6.461	320.0	70.0	25.649	11.230	10.222	290.0
10.0	28.537	1.869	1.681	350.0	40.5	27.560	7.234	6.534	319.5	70.5	25.610	11.282	10.271	289.5
10.5	28.530	1.962	1.765	349.5	41.0	27.535	7.314	6.608	319.0	71.0	25.571	11.333	10.319	289.0
11.0	28.523	2.054	1.848	349.0	41.5	27.510	7.394	6.680	318.5	71.5	25.532	11.383	10.366	288.5
11.5	28.516	2.147	1.932	348.5	42.0	27.484	7.473	6.753	318.0	72.0	25.493	11.433	10.414	288.0
12.0	28.508	2.240	2.015	348.0	42.5	27.458	7.553	6.825	317.5	72.5	25.453	11.482	10.460	287.5
12.5	28.500	2.332	2.099	347.5	43.0	27.432	7.631	6.897	317.0	73.0	25.413	11.531	10.506	287.0
13.0	28.492	2.424	2.182	347.0	43.5	27.405	7.710	6.969	316.5	73.5	25.374	11.579	10.551	286.5
13.5	28.483	2.517	2.265	346.5	44.0	27.378	7.788	7.040	316.0	74.0	25.334	11.626	10.596	286.0
14.0	28.475	2.609	2.348	346.0	44.5	27.351	7.866	7.111	315.5	74.5	25.293	11.673	10.640	285.5
14.5	28.465	2.701	2.431	345.5	45.0	27.324	7.943	7.182	315.0	75.0	25.253	11.719	10.684	285.0
15.0	28.456	2.793	2.514	345.0	45.5	27.296	8.020	7.253	314.5	75.5	25.213	11.764	10.726	284.5
15.5	28.446	2.885	2.596	344.5	46.0	27.268	8.097	7.323	314.0	76.0	25.172	11.809	10.769	284.0
16.0	28.436	2.977	2.679	344.0	46.5	27.240	8.173	7.392	313.5	76.5	25.131	11.852	10.811	283.5
16.5	28.425	3.068	2.762	343.5	47.0	27.212	8.249	7.462	313.0	77.0	25.090	11.895	10.852	283.0
17.0	28.414	3.160	2.844	343.0	47.5	27.183	8.324	7.531	312.5	77.5	25.049	11.938	10.892	282.5
17.5	28.403	3.251	2.927	342.5	48.0	27.154	8.399	7.600	312.0	78.0	25.008	11.980	10.932	282.0
18.0	28.392	3.342	3.009	342.0	48.5	27.125	8.474	7.668	311.5	78.5	24.966	12.021	10.971	281.5
18.5	28.380	3.433	3.091	341.5	49.0	27.095	8.548	7.736	311.0	79.0	24.925	12.061	11.009	281.0
19.0	28.368	3.524	3.173	341.0	49.5	27.066	8.622	7.804	310.5	79.5	24.883	12.100	11.047	280.5
19.5	28.356	3.615	3.255	340.5	50.0	27.036	8.695	7.871	310.0	80.0	24.841	12.139	11.084	280.0
20.0	28.343	3.705	3.337	340.0	50.5	27.006	8.768	7.938	309.5	80.5	24.800	12.177	11.121	279.5
20.5	28.330	3.796	3.418	339.5	51.0	26.975	8.841	8.005	309.0	81.0	24.757	12.214	11.157	279.0
21.0	28.317	3.886	3.500	339.0	51.5	26.944	8.913	8.071	308.5	81.5	24.715	12.251	11.192	278.5
21.5	28.303	3.976	3.581	338.5	52.0	26.913	8.985	8.137	308.0	82.0	24.673	12.287	11.227	278.0
22.0	28.289	4.066	3.662	338.0	52.5	26.882	9.056	8.203	307.5	82.5	24.631	12.322	11.261	277.5
22.5	28.275	4.156	3.743	337.5	53.0	26.851	9.127	8.268	307.0	83.0	24.588	12.356	11.294	277.0
23.0	28.260	4.245	3.824	337.0	53.5	26.819	9.197	8.333	306.5	83.5	24.545	12.389	11.326	276.5
23.5	28.245	4.335	3.905	336.5	54.0	26.787	9.267	8.397	306.0	84.0	24.503	12.422	11.358	276.0
24.0	28.230	4.424	3.985	336.0	54.5	26.755	9.336	8.461	305.5	84.5	24.460	12.454	11.389	275.5
24.5	28.215	4.513	4.066	335.5	55.0	26.723	9.405	8.525	305.0	85.0	24.417	12.485	11.419	275.0
25.0	28.199	4.602	4.146	335.0	55.5	26.690	9.474	8.588	304.5	85.5	24.374	12.515	11.449	274.5
25.5	28.183	4.690	4.226	334.5	56.0	26.657	9.542	8.651	304.0	86.0	24.331	12.545	11.478	274.0
26.0	28.166	4.779	4.306	334.0	56.5	26.624	9.609	8.713	303.5	86.5	24.287	12.573	11.506	273.5
26.5	28.149	4.867	4.386	333.5	57.0	26.591	9.676	8.775	303.0	87.0	24.244	12.601	11.533	273.0
27.0	28.132	4.955	4.466	333.0	57.5	26.557	9.743	8.836	302.5	87.5	24.200	12.628	11.560	272.5
27.5	28.115	5.043	4.545	332.5	58.0	26.523	9.809	8.897	302.0	88.0	24.157	12.654	11.586	272.0
28.0	28.097	5.130	4.624	332.0	58.5	26.489	9.874	8.958	301.5	88.5	24.113	12.680	11.611	271.5
28.5	28.079	5.218	4.703	331.5	59.0	26.455	9.939	9.018	301.0	89.0	24.070	12.704	11.635	271.0
29.0	28.061	5.305	4.782	331.0	59.5	26.421	10.004	9.078	300.5	89.5	24.026	12.728	11.659	270.5
29.5	28.043	5.391	4.861	330.5	60.0	26.386	10.068	9.137	300.0	90.0	23.982	12.751	11.681	270.0
30.0	28.024	5.478	4.939	330.0										

N.B.—I is always positive. When N is between 0° and 180°, ν and ξ are positive; when N is between 180° and 360°, ν and ξ are negative.

TABLE XVII.—Values of I, ν and ξ, corresponding to N—(*Continued*).

N	I	ν	ξ	N	N	I	ν	ξ	N	N	I	ν	ξ	N
°	°	°	°	°	°	°	°	°	°	°	°	°	°	°
90.5	23.938	12.772	11.704	269.5	120.5	21.283	12.295	11.378	239.5	150.5	19.135	7.745	7.223	209.5
91.0	23.894	12.793	11.725	269.0	121.0	21.241	12.254	11.342	239.0	151.0	19.108	7.635	7.120	209.0
91.5	23.850	12.814	11.745	268.5	121.5	21.199	12.211	11.304	238.5	151.5	19.082	7.523	7.017	208.5
92.0	23.806	12.833	11.765	268.0	122.0	21.157	12.168	11.266	238.0	152.0	19.056	7.411	6.913	208.0
92.5	23.761	12.851	11.784	267.5	122.5	21.115	12.123	11.226	237.5	152.5	19.030	7.298	6.809	207.5
93.0	23.717	12.869	11.802	267.0	123.0	21.073	12.078	11.186	237.0	153.0	19.005	7.184	6.703	207.0
93.5	23.673	12.886	11.819	266.5	123.5	21.032	12.031	11.144	236.5	153.5	18.981	7.069	6.596	206.5
94.0	23.628	12.901	11.835	266.0	124.0	20.990	11.983	11.101	236.0	154.0	18.956	6.953	6.488	206.0
94.5	23.584	12.916	11.851	265.5	124.5	20.949	11.933	11.057	235.5	154.5	18.933	6.836	6.380	205.5
95.0	23.539	12.930	11.866	265.0	125.0	20.908	11.883	11.012	235.0	155.0	18.909	6.718	6.270	205.0
95.5	23.495	12.943	11.880	264.5	125.5	20.867	11.831	10.965	234.5	155.5	18.886	6.599	6.160	204.5
96.0	23.450	12.955	11.893	264.0	126.0	20.826	11.778	10.918	234.0	156.0	18.863	6.480	6.049	204.0
96.5	23.406	12.966	11.905	263.5	126.5	20.786	11.724	10.869	233.5	156.5	18.841	6.359	5.937	203.5
97.0	23.361	12.976	11.916	263.0	127.0	20.746	11.669	10.820	233.0	157.0	18.819	6.238	5.824	203.0
97.5	23.316	12.985	11.927	262.5	127.5	20.705	11.612	10.769	232.5	157.5	18.798	6.116	5.710	202.5
98.0	23.271	12.994	11.936	262.0	128.0	20.666	11.555	10.717	232.0	158.0	18.777	5.993	5.596	202.0
98.5	23.227	13.001	11.945	261.5	128.5	20.626	11.496	10.664	231.5	158.5	18.756	5.869	5.480	201.5
99.0	23.182	13.007	11.953	261.0	129.0	20.586	11.436	10.610	231.0	159.0	18.736	5.744	5.364	201.0
99.5	23.137	13.013	11.960	260.5	129.5	20.547	11.374	10.554	230.5	159.5	18.716	5.618	5.247	200.5
100.0	23.092	13.017	11.966	260.0	130.0	20.508	11.312	10.498	230.0	160.0	18.697	5.492	5.130	200.0
100.5	23.047	13.021	11.971	259.5	130.5	20.469	11.248	10.440	229.5	160.5	18.678	5.365	5.011	199.5
101.0	23.003	13.023	11.975	259.0	131.0	20.430	11.184	10.382	229.0	161.0	18.660	5.237	4.892	199.0
101.5	22.958	13.024	11.979	258.5	131.5	20.392	11.118	10.322	228.5	161.5	18.642	5.109	4.773	198.5
102.0	22.913	13.025	11.981	258.0	132.0	20.353	11.050	10.261	228.0	162.0	18.624	4.980	4.652	198.0
102.5	22.868	13.024	11.983	257.5	132.5	20.315	10.982	10.199	227.5	162.5	18.607	4.850	4.531	197.5
103.0	22.823	13.023	11.983	257.0	133.0	20.278	10.912	10.135	227.0	163.0	18.591	4.719	4.409	197.0
103.5	22.778	13.020	11.983	256.5	133.5	20.240	10.841	10.071	226.5	163.5	18.575	4.588	4.287	196.5
104.0	22.734	13.017	11.982	256.0	134.0	20.203	10.769	10.005	226.0	164.0	18.559	4.456	4.164	196.0
104.5	22.689	13.012	11.979	255.5	134.5	20.166	10.696	9.939	225.5	164.5	18.544	4.323	4.040	195.5
105.0	22.644	13.006	11.976	255.0	135.0	20.129	10.622	9.871	225.0	165.0	18.529	4.190	3.916	195.0
105.5	22.599	13.000	11.972	254.5	135.5	20.092	10.546	9.802	224.5	165.5	18.515	4.056	3.791	194.5
106.0	22.554	12.992	11.967	254.0	136.0	20.056	10.469	9.732	224.0	166.0	18.501	3.922	3.665	194.0
106.5	22.510	12.983	11.961	253.5	136.5	20.020	10.391	9.660	223.5	166.5	18.487	3.787	3.539	193.5
107.0	22.465	12.974	11.954	253.0	137.0	19.984	10.312	9.588	223.0	167.0	18.475	3.651	3.413	193.0
107.5	22.420	12.963	11.946	252.5	137.5	19.949	10.232	9.515	222.5	167.5	18.462	3.515	3.286	192.5
108.0	22.376	12.951	11.937	252.0	138.0	19.913	10.150	9.440	222.0	168.0	18.450	3.379	3.158	192.0
108.5	22.331	12.938	11.927	251.5	138.5	19.878	10.068	9.364	221.5	168.5	18.439	3.242	3.030	191.5
109.0	22.287	12.924	11.916	251.0	139.0	19.844	9.984	9.287	221.0	169.0	18.428	3.104	2.902	191.0
109.5	22.242	12.909	11.904	250.5	139.5	19.809	9.899	9.209	220.5	169.5	18.417	2.966	2.773	190.5
110.0	22.198	12.892	11.891	250.0	140.0	19.775	9.813	9.130	220.0	170.0	18.407	2.828	2.644	190.0
110.5	22.153	12.875	11.877	249.5	140.5	19.742	9.725	9.050	219.5	170.5	18.398	2.689	2.514	189.5
111.0	22.109	12.857	11.862	249.0	141.0	19.708	9.637	8.969	219.0	171.0	18.388	2.550	2.384	189.0
111.5	22.065	12.837	11.846	248.5	141.5	19.675	9.547	8.887	218.5	171.5	18.380	2.410	2.254	188.5
112.0	22.021	12.817	11.829	248.0	142.0	19.642	9.457	8.803	218.0	172.0	18.372	2.270	2.123	188.0
112.5	21.976	12.795	11.811	247.5	142.5	19.610	9.365	8.719	217.5	172.5	18.364	2.130	1.992	187.5
113.0	21.932	12.772	11.792	247.0	143.0	19.577	9.272	8.633	217.0	173.0	18.357	1.989	1.860	187.0
113.5	21.888	12.748	11.772	246.5	143.5	19.545	9.177	8.546	216.5	173.5	18.350	1.848	1.729	186.5
114.0	21.845	12.723	11.750	246.0	144.0	19.514	9.082	8.458	216.0	174.0	18.344	1.707	1.597	186.0
114.5	21.801	12.697	11.728	245.5	144.5	19.483	8.986	8.370	215.5	174.5	18.338	1.566	1.464	185.5
115.0	21.757	12.670	11.705	245.0	145.0	19.452	8.888	8.280	215.0	175.0	18.333	1.424	1.332	185.0
115.5	21.713	12.642	11.681	244.5	145.5	19.421	8.790	8.189	214.5	175.5	18.328	1.282	1.199	184.5
116.0	21.670	12.612	11.655	244.0	146.0	19.391	8.690	8.097	214.0	176.0	18.324	1.140	1.067	184.0
116.5	21.627	12.581	11.629	243.5	146.5	19.361	8.589	8.004	213.5	176.5	18.320	0.998	0.934	183.5
117.0	21.583	12.550	11.601	243.0	147.0	19.332	8.487	7.910	213.0	177.0	18.317	0.856	0.801	183.0
117.5	21.540	12.517	11.572	242.5	147.5	19.302	8.384	7.814	212.5	177.5	18.315	0.713	0.667	182.5
118.0	21.497	12.483	11.543	242.0	148.0	19.273	8.280	7.718	212.0	178.0	18.312	0.571	0.534	182.0
118.5	21.454	12.447	11.512	241.5	148.5	19.245	8.175	7.621	211.5	178.5	18.311	0.428	0.401	181.5
119.0	21.411	12.411	11.480	241.0	149.0	19.217	8.069	7.523	211.0	179.0	18.309	0.286	0.267	181.0
119.5	21.368	12.373	11.447	240.5	149.5	19.189	7.962	7.424	210.5	179.5	18.309	0.143	0.133	180.5
120.0	21.326	12.335	11.413	240.0	150.0	19.162	7.854	7.324	210.0	180.0	18.308	0.000	0.000	180.0

N.B.—I is always positive. When N is between 0° and 180°, ν and ξ are positive; when N is between 180° and 360°, ν and ξ are negative.

TABLE XVIII. (1)—Values of $1/f$ and f corresponding to various values of I, to be used in computing H and R for the Tides M_2, N, 2N, ν, MS, 2SM and Luni-Solar fortnightly.

$$\text{Argument } 1/f = \frac{\cos^4 \frac{1}{2}\,\omega \cos^4 \frac{1}{2}\,i}{\cos^4 \frac{1}{2}\,I}.$$

Values of I	$1/f$	Differences for 0°.1 of I	f	Differences for 0°.1 of I	Values of I	$1/f$	Differences for 0°.1 of I	f	Differences for 0°.1 of I
18° 18′ 30″	0·96354		1·03784		23°·5	0·99630		1·00371	
18·4	·96403		·03731		·6	·99703	73	·00298	73
·5	·96458	55	·03672	59	·7	·99775	72	·00225	73
·6	·96513	55	·03613	59	·8	·99849	74	·00152	73
·7	·96569	56	·03553	60	·9	·99922	73	·00078	74
·8	·96624	55	·03494	59	24 ·0	·99996	74	·00004	74
·9	·96680	56	·03434	60	·1	1·00070	74	0.99930	74
19·0	·96736	56	·03374	60	·2	·00145	75	·99855	75
·1	·96793	57	·03313	61	·3	·00220	75	·99781	74
·2	·96850	57	·03252	61	·4	·00296	76	·99705	76
·3	·96908	58	·03191	61	·5	·00372	76	·99629	76
·4	·96965	57	·03129	62	·6	·00448	76	·99553	76
·5	·97023	58	·03068	61	·7	·00525	77	·99478	75
·6	·97082	59	·03006	62	·8	·00601	76	·99402	76
·7	·97141	59	·02943	63	·9	·00679	78	·99326	76
·8	·97199	58	·02882	61	25 ·0	·00757	78	·99249	77
·9	·97259	60	·02819	63	·1	·00835	78	·99172	77
20·0	·97318	59	·02756	63	·2	·00913	78	·99095	77
·1	·97379	61	·02692	64	·3	·00993	80	·99017	78
·2	·97439	60	·02628	64	·4	·01072	79	·98939	78
·3	·97500	61	·02564	64	·5	·01151	79	·98862	77
·4	·97561	61	·02500	64	·6	·01231	80	·98783	79
·5	·97622	61	·02436	64	·7	·01312	81	·98705	78
·6	·97684	62	·02371	65	·8	·01393	81	·98626	79
·7	·97746	62	·02306	65	·9	·01474	81	·98547	79
·8	·97808	62	·02241	65	26 ·0	·01556	82	·98468	79
·9	·97871	63	·02175	66	·1	·01638	82	·98388	80
21·0	·97935	64	·02109	66	·2	·01721	83	·98308	80
·1	·97999	64	·02042	67	·3	·01803	82	·98229	79
·2	·98062	63	·01976	66	·4	·01887	84	·98148	81
·3	·98127	65	·01909	67	·5	·01970	83	·98068	80
·4	·98191	64	·01842	67	·6	·02054	84	·97988	80
·5	·98256	65	·01775	67	·7	·02138	84	·97907	81
·6	·98321	65	·01707	68	·8	·02223	85	·97826	81
·7	·98387	66	·01639	68	·9	·02308	85	·97744	82
·8	·98453	66	·01572	67	27 ·0	·02394	86	·97662	82
·9	·98519	66	·01503	69	·1	·02480	86	·97580	82
22·0	·98586	67	·01434	69	·2	·02566	86	·97498	82
·1	·98653	67	·01365	69	·3	·02653	87	·97416	82
·2	·98721	68	·01296	69	·4	·02740	87	·97333	83
·3	·98788	67	·01226	70	·5	·02828	88	·97250	83
·4	·98857	69	·01157	69	·6	·02916	88	·97167	83
·5	·98925	68	·01087	70	·7	·03005	89	·97083	84
·6	·98994	69	·01016	71	·8	·03093	88	·97000	83
·7	·99063	69	·00945	71	·9	·03183	90	·96916	84
·8	·99133	70	·00875	70	28 ·0	·03272	89	·96832	84
·9	·99203	70	·00803	72	·1	·03363	91	·96747	85
23·0	·99273	70	·00732	71	·2	·03453	90	·96662	85
·1	·99344	71	·00661	71	·3	·03544	91	·96577	85
·2	·99415	71	·00589	72	·4	·03635	91	·96492	85
·3	·99486	71	·00516	73	·5	·03727	92	·96407	85
·4	·99558	72	·00444	72	·6	·03819	92	·96321	86
·5	·99630	72	·00371	73	28° 36′ 6″	·03821		·96320	

TABLE XVIII. (2)—Values of $1/f$ and f corresponding to various values of I, to be used in computing H and R for the Tides O and Q. Also used in determining $1/f$ for the Tide M_1.

$$\text{Argument } 1/f = \frac{\text{Sin } \omega \text{ Cos}^2 \tfrac{1}{2} \omega \text{ Cos}^4 \tfrac{1}{2} i}{\text{Sin } I \text{ Cos}^2 \tfrac{1}{2} I}.$$

Values of I	$1/f$	Differences for 0°·1 of I	f	Differences for 0°·1 of I	Values of I	$1/f$	Differences for 0°·1 of I	f	Differences for 0°·1 of I
18° 18′ 30″	1·24126		0·80563		23°·5	0·99434		1·00569	
18·4	·23563		·80932		·6	·99073	361	·00936	367
·5	·22951	612	·81333	401	·7	·98715	358	·01303	367
·6	·22350	601	·81734	401	·8	·98360	355	·01667	364
·7	·21753	597	·82135	401	·9	·98009	351	·02032	365
·8	·21163	590	·82534	399	24·0	·97660	349	·02396	364
·9	·20581	582	·82933	399	·1	·97316	344	·02758	362
19·0	·20003	578	·83331	398	·2	·96974	342	·03121	363
·1	·19435	568	·83729	398	·3	·96635	339	·03482	361
·2	·18870	565	·84126	397	·4	·96300	335	·03843	361
·3	·18313	557	·84523	397	·5	·95966	334	·04204	361
·4	·17762	551	·84919	396	·6	·95637	329	·04563	359
·5	·17214	548	·85314	395	·7	·95310	327	·04921	358
·6	·16676	538	·85708	394	·8	·94986	324	·05280	359
·7	·16142	534	·86102	394	·9	·94666	320	·05636	356
·8	·15614	528	·86496	394	25·0	·94347	319	·05992	356
·9	·15092	522	·86889	393	·1	·94033	314	·06347	355
20·0	·14573	519	·87281	392	·2	·93720	313	·06702	355
·1	·14064	509	·87672	391	·3	·93410	310	·07055	353
·2	·13557	507	·88062	390	·4	·93103	307	·07408	353
·3	·13056	501	·88453	391	·5	·92798	305	·07761	353
·4	·12561	495	·88842	389	·6	·92497	301	·08112	351
·5	·12069	492	·89231	389	·7	·92198	299	·08463	351
·6	·11585	484	·89619	388	·8	·91902	296	·08812	349
·7	·11105	480	·90006	387	·9	·91608	294	·09161	349
·8	·10629	476	·90393	387	26·0	·91316	292	·09510	349
·9	·10159	470	·90779	386	·1	·91028	288	·09857	347
21·0	·09692	467	·91164	385	·2	·90742	286	·10203	346
·1	·09233	459	·91549	385	·3	·90458	284	·10549	346
·2	·08776	457	·91933	384	·4	·90177	281	·10894	345
·3	·08325	451	·92316	383	·5	·89897	280	·11238	344
·4	·07878	447	·92698	382	·6	·89622	275	·11581	343
·5	·07434	444	·93080	382	·7	·89347	275	·11924	343
·6	·06997	437	·93461	381	·8	·89076	271	·12265	341
·7	·06563	434	·93842	381	·9	·88806	270	·12606	341
·8	·06133	430	·94222	380	27·0	·88538	268	·12946	340
·9	·05708	425	·94601	379	·1	·88273	265	·13285	339
22·0	·05286	422	·94979	378	·2	·88010	263	·13623	338
·1	·04870	416	·95357	378	·3	·87750	260	·13961	338
·2	·04457	413	·95734	377	·4	·87491	259	·14297	336
·3	·04047	410	·96111	377	·5	·87234	257	·14634	337
·4	·03643	404	·96486	375	·6	·86981	253	·14968	334
·5	·03241	402	·96861	375	·7	·86729	252	·15302	334
·6	·02845	396	·97235	374	·8	·86478	251	·15636	334
·7	·02451	394	·97608	373	·9	·86231	247	·15968	332
·8	·02061	390	·97981	373	28·0	·85985	246	·16299	331
·9	·01675	386	·98353	372	·1	·85741	244	·16630	331
23·0	·01292	383	·98725	372	·2	·85499	242	·16961	331
·1	·00915	377	·99095	370	·3	·85260	239	·17289	328
·2	·00539	376	·99464	369	·4	·85022	238	·17617	328
·3	·00167	372	·99834	370	·5	·84785	237	·17945	328
·4	0·99799	368	1·00202	368	·6	·84551	234	·18272	327
·5	·99434	365	·00569	367	28° 36′ 6″	·84547		·18277	

TABLE XVIII. (3)—Values of $1/f$ and f corresponding to various values of I, to be used in computing H and R for the Tide J; and for determining k_1, used in the preparation of Table XVIII. (6).

$$\text{Argument } 1/f = \frac{\text{Sin } \omega \text{ Cos } \omega \ (1 - \tfrac{3}{2} \text{ Sin}^2 i)}{\text{Sin } I \text{ Cos } I}.$$

Values of I	$1/f$	Differences for 0°·1 of I	f	Differences for 0°·1 of I	Values of I	$1/f$	Differences for 0°·1 of I	f	Differences for 0°·1 of I
18° 18′ 30″	1·20958		0·82673		23°·5	0·98648	319	1·01371	329
18·4	·20442	560	·83028	388	·6	·98329	316	·01700	328
·5	·19882	550	·83416	385	·7	·98013	313	·02028	327
·6	·19332	547	·83801	385	·8	·97700	309	·02355	325
·7	·18785	539	·84186	384	·9	·97391	308	·02680	325
·8	·18246	532	·84570	383	24 ·0	·97083	303	·03005	323
·9	·17714	529	·84953	382	·1	·96780	301	·03328	322
19·0	·17185	518	·85335	380	·2	·96479	297	·03650	320
·1	·16667	515	·85715	380	·3	·96182	294	·03970	319
·2	·16152	509	·86095	379	·4	·95888	293	·04289	319
·3	·15643	502	·86474	377	·5	·95595	287	·04608	316
·4	·15141	499	·86851	377	·6	·95308	286	·04924	316
·5	·14642	490	·87228	375	·7	·95022	284	·05240	314
·6	·14152	486	·87603	375	·8	·94738	279	·05554	313
·7	·13666	480	·87978	373	·9	·94459	278	·05867	312
·8	·13186	474	·88351	373	25 ·0	·94181	274	·06179	310
·9	·12712	472	·88724	371	·1	·93907	272	·06489	309
20·0	·12240	463	·89095	370	·2	·93635	269	·06798	308
·1	·11777	459	·89465	369	·3	·93366	266	·07106	306
·2	·11318	455	·89834	367	·4	·93100	265	·07412	306
·3	·10863	448	·90201	367	·5	·92835	260	·07718	303
·4	·10415	445	·90568	366	·6	·92575	259	·08021	302
·5	·09970	438	·90934	364	·7	·92316	256	·08323	302
·6	·09532	435	·91298	364	·8	·92060	253	·08625	300
·7	·09097	430	·91662	362	·9	·91807	252	·08925	299
·8	·08667	424	·92024	361	26 ·0	·91555	248	·09224	296
·9	·08243	422	·92385	361	·1	·91307	246	·09520	296
21·0	·07821	414	·92746	359	·2	·91061	243	·09816	295
·1	·07407	412	·93105	358	·3	·90818	241	·10111	293
·2	·06995	406	·93463	356	·4	·90577	240	·10404	292
·3	·06589	402	·93819	356	·5	·90337	236	·10696	290
·4	·06187	400	·94175	355	·6	·90101	234	·10986	290
·5	·05787	392	·94530	353	·7	·89867	232	·11276	287
·6	·05395	391	·94883	352	·8	·89635	229	·11563	286
·7	·05004	385	·95235	351	·9	·89406	228	·11849	286
·8	·04619	381	·95586	349	27 ·0	·89178	224	·12135	284
·9	·04238	379	·95935	349	·1	·88954	224	·12419	283
22·0	·03859	372	·96284	347	·2	·88730	220	·12702	280
·1	·03487	370	·96631	347	·3	·88510	218	·12982	280
·2	·03117	366	·96978	345	·4	·88292	217	·13262	278
·3	·02751	361	·97323	344	·5	·88075	214	·13540	277
·4	·02390	359	·97667	343	·6	·87861	212	·13817	275
·5	·02031	353	·98010	341	·7	·87649	210	·14092	274
·6	·01678	352	·98351	340	·8	·87439	208	·14366	272
·7	·01326	346	·98691	340	·9	·87231	207	·14638	272
·8	·00980	344	·99031	337	28 ·0	·87024	203	·14910	269
·9	·00636	341	·99368	337	·1	·86821	202	·15179	269
23·0	·00295	335	·99705	335	·2	·86619	199	·15448	267
·1	0·99960	333	1·00040	335	·3	·86420	198	·15715	265
·2	·99627	329	·00375	333	·4	·86222	197	·15980	265
·3	·99298	326	·00708	332	·5	·86025	194	·16245	264
·4	·98972	324	·01040	331	·6	·85831		·16509	
·5	·98648		·01371		28° 36′ 6″	·85828		·16513	

TABLE XVIII. (4)—Values of $1/f$ and f corresponding to various values of I, to be used in computing H and R for the Tide Mf.

$$\text{Argument } 1/f = \frac{\sin^2 \omega \cos^4 \frac{1}{2} i}{\sin^2 I}.$$

Values of I	$1/f$	Differences for 0°.1 of I	f	Differences for 0°.1 of I	Values of I	$1/f$	Differences for 0°.1 of I	f	Differences for 0°.1 of I
18° 18′ 30″	1·59903		0·62538		23°·5	0·99238		1·00768	
18·4	·58377		·63145		·6	·98450	788	·01579	811
·5	·56720	1657	·63808	663	·7	·97667	783	·02392	813
·6	·55107	1613	·64476	668	·8	·96895	772	·03207	815
·7	·53508	1599	·65146	670	·9	·96134	761	·04026	819
·8	·51937	1571	·65820	674	24 ·0	·95378	756	·04846	820
·9	·50394	1543	·66496	676	·1	·94638	740	·05670	824
19·0	·48865	1529	·67175	679	·2	·93903	735	·06495	825
·1	·47375	1490	·67858	683	·3	·93179	724	·07324	829
·2	·45898	1477	·68544	686	·4	·92463	716	·08155	831
·3	·44448	1450	·69232	688	·5	·91753	710	·08988	833
·4	·43021	1427	·69924	692	·6	·91058	695	·09825	837
·5	·41607	1414	·70618	694	·7	·90367	691	·10663	838
·6	·40229	1378	·71316	698	·8	·89685	682	·11504	841
·7	·38862	1367	·72017	701	·9	·89012	673	·12348	844
·8	·37519	1343	·72720	703	25 ·0	·88344	668	·13193	845
·9	·36198	1321	·73427	707	·1	·87689	655	·14043	850
20·0	·34888	1310	·74136	709	·2	·87039	650	·14893	850
·1	·33610	1278	·74849	713	·3	·86398	641	·15747	854
·2	·32343	1267	·75564	715	·4	·85764	634	·16603	856
·3	·31096	1247	·76283	719	·5	·85135	629	·17461	858
·4	·29871	1225	·77004	721	·6	·84518	617	·18323	862
·5	·28655	1216	·77727	723	·7	·83904	614	·19186	863
·6	·27468	1187	·78455	728	·8	·83300	604	·20051	865
·7	·26291	1177	·79185	730	·9	·82702	598	·20920	869
·8	·25133	1158	·79919	734	26 ·0	·82109	593	·21789	869
·9	·23993	1140	·80655	736	·1	·81527	582	·22663	874
21·0	·22862	1131	·81392	737	·2	·80949	578	·23538	875
·1	·21758	1104	·82135	743	·3	·80378	571	·24415	877
·2	·20663	1095	·82879	744	·4	·79814	564	·25296	881
·3	·19584	1079	·83626	747	·5	·79254	560	·26177	881
·4	·18523	1061	·84376	750	·6	·78704	550	·27062	885
·5	·17470	1053	·85128	752	·7	·78158	546	·27948	886
·6	·16441	1029	·85885	757	·8	·77619	539	·28838	890
·7	·15420	1021	·86644	759	·9	·77086	533	·29729	891
·8	·14414	1006	·87405	761	27 ·0	·76557	529	·30622	893
·9	·13424	990	·88169	764	·1	·76037	520	·31519	897
22·0	·12441	983	·88935	766	·2	·75521	516	·32417	898
·1	·11480	961	·89706	771	·3	·75010	511	·33318	901
·2	·10527	953	·90478	772	·4	·74506	504	·34221	903
·3	·09588	939	·91254	776	·5	·74006	500	·35125	904
·4	·08663	925	·92032	778	·6	·73514	492	·36033	908
·5	·07745	918	·92812	780	·7	·73025	489	·36942	909
·6	·06847	898	·93596	784	·8	·72542	483	·37853	911
·7	·05955	892	·94382	786	·9	·72065	477	·38767	914
·8	·05077	878	·95171	789	28 ·0	·71591	474	·39683	916
·9	·04211	866	·95963	792	·1	·71125	466	·40602	919
23·0	·03352	859	·96756	793	·2	·70662	463	·41521	919
·1	·02511	841	·97554	798	·3	·70205	457	·42444	923
·2	·01677	834	·98354	800	·4	·69752	453	·43369	925
·3	·00854	823	·99156	802	·5	·69303	449	·44295	926
·4	·00043	811	·99962	806	·6	·68859	444	·45224	929
·5	0·99238	805	1·00768	806	28° 36′ 6″	·68852		·45239	

TABLE XVIII. (5)—Values of $1/f$ and f corresponding to various values of I, to be used in computing H and R for the Tide Mm.

$$\text{Argument } 1/f = \frac{(1 - \frac{3}{2}\,\text{Sin}^2\,\omega)\;(1 - \frac{3}{2}\,\text{Sin}^2\,i)}{1 - \frac{3}{2}\,\text{Sin}^2\,I}.$$

Values of I	$1/f$	Differences for 0°.1 of I	f	Differences for 0°.1 of I	Values of I	$1/f$	Differences for 0°.1 of I	f	Differences for 0°.1 of I
18° 18′ 30″	0·88401		1·13121		23°·5	0·98904		1·01108	
18·4	·88550		·12931		·6	·99155	251	·00853	255
·5	·88713	163	·12723	208	·7	·99407	252	·00597	256
·6	·88879	166	·12512	211	·8	·99660	253	·00341	256
·7	·89046	167	·12301	211	·9	·99917	257	·00083	258
·8	·89215	169	·12090	211	24·0	1·00174	257	0·99826	257
·9	·89384	169	·11877	213	·1	·00435	261	·99567	259
19·0	·89554	170	·11665	212	·2	·00697	262	·99308	259
·1	·89727	173	·11450	215	·3	·00962	265	·99048	260
·2	·89900	173	·11234	216	·4	·01229	267	·98786	262
·3	·90076	176	·11018	216	·5	·01498	269	·98524	262
·4	·90252	176	·10800	218	·6	·01770	272	·98261	263
·5	·90430	178	·10582	218	·7	·02044	274	·97998	263
·6	·90611	181	·10362	220	·8	·02319	275	·97734	264
·7	·90792	181	·10142	220	·9	·02598	279	·97469	265
·8	·90974	182	·09922	220	25·0	·02878	280	·97203	266
·9	·91159	185	·09699	223	·1	·03162	284	·96936	267
20·0	·91344	185	·09476	223	·2	·03447	285	·96669	267
·1	·91531	187	·09252	224	·3	·03735	288	·96400	269
·2	·91719	188	·09028	224	·4	·04025	290	·96131	269
·3	·91910	191	·08803	225	·5	·04317	292	·95862	269
·4	·92102	192	·08576	227	·6	·04613	296	·95591	271
·5	·92295	193	·08348	228	·7	·04910	297	·95320	271
·6	·92491	196	·08119	229	·8	·05210	300	·95048	272
·7	·92687	196	·07890	229	·9	·05513	303	·94776	272
·8	·92885	198	·07660	230	26·0	·05818	305	·94502	274
·9	·93085	200	·07429	231	·1	·06127	309	·94227	275
21·0	·93286	201	·07197	232	·2	·06437	310	·93952	275
·1	·93490	204	·06964	233	·3	·06751	314	·93677	275
·2	·93695	205	·06730	234	·4	·07067	316	·93400	277
·3	·93902	207	·06494	236	·5	·07385	318	·93123	277
·4	·94110	208	·06259	235	·6	·07707	322	·92845	278
·5	·94319	209	·06023	236	·7	·08031	324	·92566	279
·6	·94532	213	·05785	238	·8	·08359	328	·92286	280
·7	·94746	214	·05546	239	·9	·08689	330	·92006	280
·8	·94960	214	·05308	238	27·0	·09021	332	·91726	280
·9	·95178	218	·05067	241	·1	·09358	337	·91444	282
22·0	·95397	219	·04825	242	·2	·09696	338	·91162	282
·1	·95618	221	·04583	242	·3	·10037	341	·90878	284
·2	·95840	222	·04341	242	·4	·10382	345	·90594	284
·3	·96064	224	·04098	243	·5	·10729	347	·90310	284
·4	·96290	226	·03853	245	·6	·11081	352	·90025	285
·5	·96518	228	·03608	245	·7	·11434	353	·89739	286
·6	·96748	230	·03362	246	·8	·11791	357	·89453	286
·7	·96980	232	·03115	247	·9	·12152	361	·89165	288
·8	·97214	234	·02866	249	28·0	·12514	362	·88878	287
·9	·97450	236	·02617	249	·1	·12881	367	·88589	289
23·0	·97687	237	·02368	249	·2	·13250	369	·88300	289
·1	·97926	239	·02118	250	·3	·13624	374	·88011	289
·2	·98167	241	·01867	251	·4	·14000	376	·87720	291
·3	·98412	245	·01614	253	·5	·14378	378	·87429	291
·4	·98657	245	·01361	253	·6	·14763	385	·87137	292
·5	·98904	247	·01108	253	28° 36′ 6″	·14769		·87132	

TABLE XVIII. (6)—Values of $1/f$ and f corresponding to various values of I, to be used in computing H and R for the Tide K_1.

$$\text{Argument } 1/f = \frac{1{\cdot}46407 \times k_1}{\left\{1 + (0{\cdot}46407 \times k_1)^2 + 0{\cdot}92814\, k_1 \operatorname{Cos} \nu\right\}^{\frac{1}{2}}} \text{ where } k_1 = \frac{\operatorname{Sin} \omega \operatorname{Cos} \omega \left(1 - \frac{3}{2} \operatorname{Sin}^2 i\right)}{\operatorname{Sin} I \operatorname{Cos} I}.$$

Values of I	$1/f$	Differences for 0°·1 of I	f	Differences for 0°·1 of I	Values of I	$1/f$	Differences for 0°·1 of I	f	Differences for 0°·1 of I
18° 18′ 30″	1·13424		0·88165		23°·5	0·99619	224	1·00383	226
18·4	·13142	307	·88385	240	·6	·99395	223	·00609	226
·5	·12835	303	·88625	238	·7	·99172	222	·00835	227
·6	·12532	303	·88863	240	·8	·98950	220	·01062	224
·7	·12229	302	·89103	241	·9	·98730	220	·01286	226
·8	·11927	299	·89344	239	24 ·0	·98510	217	·01512	225
·9	·11628	299	·89583	241	·1	·98293	217	·01737	225
19·0	·11329	294	·89824	237	·2	·98076	214	·01962	222
·1	·11035	295	·90061	240	·3	·97862	213	·02184	224
·2	·10740	292	·90301	239	·4	·97649	214	·02408	224
·3	·10448	290	·90540	239	·5	·97435	208	·02632	221
·4	·10158	291	·90779	240	·6	·97227	210	·02853	222
·5	·09867	287	·91019	238	·7	·97017	209	·03075	222
·6	·09580	286	·91257	239	·8	·96808	206	·03297	220
·7	·09294	283	·91496	238	·9	·96602	206	·03517	221
·8	·09011	282	·91734	238	25 ·0	·96396	202	·03738	219
·9	·08729	283	·91972	240	·1	·96194	203	·03957	220
20·0	·08446	278	·92212	237	·2	·95991	201	·04177	218
·1	·08168	279	·92449	239	·3	·95790	200	·04395	218
·2	·07889	275	·92688	237	·4	·95590	199	·04613	219
·3	·07614	274	·92925	237	·5	·95391	196	·04832	216
·4	·07340	273	·93162	238	·6	·95195	196	·05048	217
·5	·07067	271	·93400	237	·7	·94999	195	·05265	215
·6	·06796	269	·93637	236	·8	·94804	192	·05480	215
·7	·06527	269	·93873	238	·9	·94612	193	·05695	216
·8	·06258	265	·94111	235	26 ·0	·94419	191	·05911	215
·9	·05993	266	·94346	237	·1	·94228	189	·06126	213
21·0	·05727	262	·94583	235	·2	·94039	187	·06339	211
·1	·05465	262	·94818	236	·3	·93852	186	·06550	213
·2	·05203	259	·95054	235	·4	·93666	187	·06763	213
·3	·04944	257	·95289	234	·5	·93479	184	·06976	211
·4	·04687	258	·95523	236	·6	·93295	183	·07187	210
·5	·04429	254	·95759	234	·7	·93112	182	·07397	211
·6	·04175	256	·95993	235	·8	·92930	180	·07608	209
·7	·03919	250	·96228	233	·9	·92750	180	·07817	209
·8	·03669	250	·96461	233	27 ·0	·92570	177	·08026	207
·9	·03419	251	·96694	235	·1	·92393	178	·08233	209
22·0	·03168	246	·96929	232	·2	·92215	173	·08442	205
·1	·02922	245	·97161	232	·3	·92042	175	·08647	206
·2	·02677	246	·97393	233	·4	·91867	174	·08853	206
·3	·02431	242	·97626	232	·5	·91693	172	·09059	205
·4	·02189	241	·97858	231	·6	·91521	170	·09264	204
·5	·01948	238	·98089	230	·7	·91351	170	·09468	204
·6	·01710	239	·98319	231	·8	·91181	168	·09672	202
·7	·01471	237	·98550	231	·9	·91013	167	·09874	203
·8	·01234	235	·98781	230	28 ·0	·90846	166	·10077	201
·9	·00999	235	·99011	230	·1	·90680	165	·10278	201
23·0	·00764	230	·99241	228	·2	·90515	162	·10479	198
·1	·00534	231	·99469	229	·3	·90353	163	·10677	200
·2	·00303	228	·99698	228	·4	·90190	162	·10877	200
·3	·00075	229	·99926	228	·5	·90028	160	·11077	198
·4	0·99846	227	1·00154	229	·6	·89868		·11275	
·5	·99619		·00383		28° 36′ 6″	·89865		·11279	

TABLE XVIII. (7)—Values of $1/f$ and f corresponding to various values of I, to be used in computing H and R for the Tide K_2.

$$\text{Argument } 1/f = \frac{1{\cdot}46407 \times k_2}{\left\{1 + (0{\cdot}46407 \times k_2)^2 + 0{\cdot}92814 k_2 \operatorname{Cos} 2\nu\right\}^{\frac{1}{2}}} \text{ where } k_2 = \frac{\operatorname{Sin}^2\omega\,(1 - \frac{3}{2}\operatorname{Sin}^2 i)}{\operatorname{Sin}^2 I}.$$

Values of I	$1/f$	Differences for 0°·1 of I	f	Differences for 0°·1 of I	Values of I	$1/f$	Differences for 0°·1 of I	f	Differences for 0°·1 of I
18° 18′ 30″	1·33764		0·74759		23°·5	1·01137		0·98876	
18·4	·33173		·75090		·6	·00566	571	·99437	561
·5	·32518	655	·75462	372	·7	0·99996	570	1·00004	567
·6	·31871	647	·75832	370	·8	·99429	567	·00574	570
·7	·31218	653	·76209	377	·9	·98865	564	·01148	574
·8	·30564	654	·76591	382	24 ·0	·98304	561	·01725	577
·9	·29915	649	·76974	383	·1	·97748	556	·02303	578
19·0	·29257	658	·77365	391	·2	·97194	554	·02887	584
·1	·28608	649	·77756	391	·3	·96643	551	·03474	587
·2	·27953	655	·78154	398	·4	·96095	548	·04064	590
·3	·27301	652	·78554	400	·5	·95548	547	·04659	595
·4	·26650	651	·78958	404	·6	·95008	540	·05254	595
·5	·25993	657	·79369	411	·7	·94469	539	·05855	601
·6	·25345	648	·79780	411	·8	·93932	537	·06459	604
·7	·24693	652	·80197	417	·9	·93401	531	·07065	606
·8	·24042	651	·80618	421	25 ·0	·92871	530	·07677	612
·9	·23394	648	·81041	423	·1	·92347	524	·08287	610
20·0	·22740	654	·81473	432	·2	·91823	524	·08905	618
·1	·22096	644	·81903	430	·3	·91303	520	·09526	621
·2	·21448	648	·82340	437	·4	·90788	515	·10147	621
·3	·20800	648	·82782	442	·5	·90273	515	·10775	628
·4	·20157	643	·83224	442	·6	·89765	508	·11403	628
·5	·19510	647	·83675	451	·7	·89257	508	·12036	633
·6	·18872	638	·84124	449	·8	·88754	503	·12671	635
·7	·18228	644	·84582	458	·9	·88254	500	·13309	638
·8	·17590	638	·85042	460	26 ·0	·87755	499	·13954	645
·9	·16952	638	·85505	463	·1	·87263	492	·14596	642
21·0	·16313	639	·85975	470	·2	·86772	491	·15245	649
·1	·15682	631	·86444	469	·3	·86284	488	·15896	651
·2	·15049	633	·86919	475	·4	·85801	483	·16549	653
·3	·14418	631	·87399	480	·5	·85317	484	·17210	661
·4	·13790	628	·87881	482	·6	·84842	475	·17867	657
·5	·13161	629	·88370	489	·7	·84366	476	·18531	664
·6	·12539	622	·88858	488	·8	·83893	473	·19200	669
·7	·11915	624	·89354	496	·9	·83425	468	·19868	668
·8	·11296	619	·89851	497	27 ·0	·82959	466	·20542	674
·9	·10678	618	·90352	501	·1	·82499	460	·21214	672
22·0	·10060	618	·90860	508	·2	·82039	460	·21894	680
·1	·09449	611	·91366	506	·3	·81583	456	·22575	681
·2	·08838	611	·91880	514	·4	·81131	452	·23258	683
·3	·08230	608	·92396	516	·5	·80679	452	·23948	690
·4	·07624	606	·92916	520	·6	·80234	445	·24636	688
·5	·07019	605	·93442	526	·7	·79789	445	·25330	694
·6	·06421	598	·93966	524	·8	·79351	438	·26023	693
·7	·05823	598	·94498	532	·9	·78912	439	·26723	700
·8	·05226	597	·95033	535	28 ·0	·78477	435	·27425	702
·9	·04635	591	·95570	537	·1	·78048	429	·28127	702
23·0	·04043	592	·96114	544	·2	·77619	429	·28834	707
·1	·03457	586	·96658	544	·3	·77193	426	·29546	712
·2	·02873	584	·97207	549	·4	·76772	421	·30257	711
·3	·02292	581	·97759	552	·5	·76351	421	·30974	717
·4	·01714	578	·98315	556	·6	·75934	417	·31693	719
·5	·01137	577	·98876	561	28° 36′ 6″	·75928		·31703	

TABLE XIX.—Values of ν' corresponding to I, to determine initial argument of Tide K_1.

$$\operatorname{Tan} \nu' = \frac{\operatorname{Sin} \nu}{\operatorname{Cos} \nu + 0{\cdot}46407 \times k_1} \quad \text{where } k_1 = \frac{\operatorname{Sin} \omega \operatorname{Cos} \omega \left(1 - \frac{3}{2} \operatorname{Sin}^2 i\right)}{\operatorname{Sin} I \operatorname{Cos} I}.$$

I	ν'	Differences for 0°·1 of I	I	ν'	Differences for 0°·1 of I	I	ν'	Differences for 0°·1 of I	I	ν'	Differences for 0°·1 of I	I	ν'	Differences for 0°·1 of I
° ′ ″	°	°	°	°	°	°	°	°	°	°	°	°	°	°
18 18 30	0·000		20·4	7·363	0·124	22·5	8·819	0·021	24·6	8·567	0·049	26·7	6·672	0·145
18·4	1·557	0·958	·5	7·487	·112	·6	8·840	·019	·7	8·518	·053	·8	6·527	·155
·5	2·515	·534	·6	7·599	·109	·7	8·859	·016	·8	8·465	·057	·9	6·372	·158
·6	3·049	·491	·7	7·708	·103	·8	8·875	·011	·9	8·408	·059	27·0	6·214	·172
·7	3·540	·406	·8	7·811	·096	·9	8·886	·010	25·0	8·349	·065	·1	6·042	·177
·8	3·946	·358	·9	7·907	·093	23·0	8·896	·004	·1	8·284	·068	·2	5·865	·187
·9	4·304	·334	21·0	8·000	·085	·1	8·900	·001	·2	8·216	·071	·3	5·678	·200
19·0	4·638	·286	·1	8·085	·081	·2	8·901	·001	·3	8·145	·076	·4	5·478	·206
·1	4·924	·274	·2	8·166	·077	·3	8·900	·006	·4	8·069	·078	·5	5·272	·228
·2	5·198	·251	·3	8·243	·070	·4	8·894	·008	·5	7·991	·085	·6	5·044	·237
·3	5·449	·231	·4	8·313	·068	·5	8·886	·013	·6	7·906	·088	·7	4·807	·256
·4	5·680	·223	·5	8·381	·061	·6	8·873	·015	·7	7·818	·092	·8	4·551	·280
·5	5·903	·199	·6	8·442	·059	·7	8·858	·019	·8	7·726	·097	·9	4·271	·292
·6	6·102	·193	·7	8·501	·053	·8	8·839	·022	·9	7·629	·100	28·0	3·979	·344
·7	6·295	·180	·8	8·554	·049	·9	8·817	·025	26·0	7·529	·107	·1	3·635	·372
·8	6·475	·169	·9	8·603	·047	24·0	8·792	·029	·1	7·422	·111	·2	3·263	·427
·9	6·644	·163	22·0	8·650	·040	·1	8·763	·032	·2	7·311	·116	·3	2·836	·553
20·0	6·807	·149	·1	8·690	·038	·2	8·731	·036	·3	7·195	·123	·4	2·283	·614
·1	6·956	·144	·2	8·728	·034	·3	8·695	·040	·4	7·072	·125	·5	1·669	1·642
·2	7·100	·136	·3	8·762	·029	·4	8·655	·041	·5	6·947	·136	·6	0·027	
·3	7·236	·127	·4	8·791	·028	·5	8·614	·047	·6	6·811	·139	28° 36′ 6″	·000	
·4	7·363		·5	8·819		·6	8·567		·7	6·672				

N.B.—In the above table ν' is positive when N is between 0° and 180°, and negative when N is between 180° and 360°; thus it is necessary to observe what is the value of N, because I is always positive.

TABLE XX.—Values of $2\nu''$ corresponding to I, to determine initial argument of Tide K_2.

$$\text{Tan } 2\nu'' = \frac{\text{Sin } 2\nu}{\text{Cos } 2\nu + 0{\cdot}46407 \times k_2} \quad \text{where } k_2 = \frac{\text{Sin}^2 \omega \, (1 - \frac{3}{2} \text{Sin}^2 i)}{\text{Sin}^2 I}.$$

I	$2\nu''$	Differences for 0°·1 of I	I	$2\nu''$	Differences for 0°·1 of I	I	$2\nu''$	Differences for 0°·1 of I	I	$2\nu''$	Differences for 0°·1 of I	I	$2\nu''$	Differences for 0°·1 of I
° ′ ″	°	°	°	°	°	°	°	°	°	°	°	°	°	°
18 18 30	0·000		20·4	13·963	0·265	22·5	17·422	0·072	24·6	17·463	0·078	26·7	13·931	0·291
18·4	2·810	1·738	·5	14·228	·246	·6	17·494	·068	·7	17·385	·086	·8	13·640	·309
·5	4·548	0·983	·6	14·474	·240	·7	17·562	·059	·8	17·299	·095	·9	13·331	·319
·6	5·531	·907	·7	14·714	·227	·8	17·621	·052	·9	17·204	·099	27·0	13·012	·348
·7	6·438	·758	·8	14·941	·216	·9	17·673	·047	25 0	17·105	·113	·1	12·664	·359
·8	7·196	·674	·9	15·157	·210	23·0	17·720	·035	·1	16·992	·117	·2	12·305	·383
·9	7·870	·632	21·0	15·367	·194	·1	17·755	·032	·2	16·875	·128	·3	11·922	·409
19·0	8·502	·549	·1	15·561	·189	·2	17·787	·023	·3	16·747	·136	·4	11·513	·423
·1	9·051	·528	·2	15·750	·179	·3	17·810	·016	·4	16·611	·142	·5	11·090	·470
·2	9·579	·488	·3	15·929	·168	·4	17·826	·012	·5	16·469	·157	·6	10·620	·491
·3	10·067	·454	·4	16·097	·164	·5	17·838	·000	·6	16·312	·162	·7	10·129	·529
·4	10·521	·438	·5	16·261	·149	·6	17·838	·004	·7	16·150	·172	·8	9·600	·583
·5	10·959	·399	·6	16·410	·145	·7	17·834	·012	·8	15·978	·184	·9	9·017	·609
·6	11·358	·388	·7	16·555	·136	·8	17·822	·020	·9	15·794	·189	28·0	8·408	·720
·7	11·746	·364	·8	16·691	·126	·9	17·802	·024	26·0	15·605	·207	·1	7·688	·780
·8	12·110	·346	·9	16·817	·122	24 0	17·778	·036	·1	15·398	·213	·2	6·908	·901
·9	12·456	·336	22 0	16·939	·109	·1	17·742	·040	·2	15·185	·226	·3	6·007	1·166
20·0	12·792	·310	·1	17·048	·105	·2	17·702	·049	·3	14·959	·239	·4	4·841	1·300
·1	13·102	·303	·2	17·153	·097	·3	17·653	·056	·4	14·720	·245	·5	3·541	3·483
·2	13·405	·286	·3	17·250	·088	·4	17·597	·061	·5	14·475	·268	·6	0·058	
·3	13·691	·272	·4	17·338	·084	·5	17·536	·073	·6	14·207	·276	28° 36′ 6″	·000	
·4	13·963		·5	17·422		·6	17·463		·7	13·931				

N.B.—In the above table $2\nu''$ is positive when N is between 0° and 180°, and negative when N is between 180° and 360°; thus it is necessary to observe what is the value of N, because I is always positive.

PART VII.

Levelling Operations.

CHAPTER I. INSTRUMENTS AND THEIR ADJUSTMENTS.

Introduction.

1. The lines of levels executed by the Trigonometrical Branch of the Survey of India Department are generally of very great length, and the only independent means of verifying their accuracy is by comparison with the sea-level at their extremities: consequently, it is of the utmost importance that errors of every description be eliminated, continually, during the progress of the levelling, a matter which cannot be too strongly impressed on any one undertaking the work.

Levels.

2. The levels which have hitherto been used on the main lines are of three distinct patterns, *viz*:—

A Rectangular level by Troughton and Simms.

Three Cylindrical levels by Troughton and Simms.

A Cushing's Reversible level by Cooke and Sons.

Rectangular level.

3. The telescope in the rectangular level has a total length of 24 inches. The object-glass has an effective diameter of $2\frac{1}{4}$ inches and a focal length of 21 inches. There are two eye-pieces of powers 39 and 50 respectively.

In order to secure rigidity the telescope is enclosed in a prism, the transverse section of which is a square of 3-inch side.

There are two levels mounted on the prism, each having an effective length of 9 inches: the value of one division of the level-scale of A is $0''\cdot960663$, and of B $1''\cdot04347$: from the latter a subtense table has been constructed on the basis that 1 division = $\cdot00033388$ feet at a distance of one chain. Level A has never been used, as it is considered sufficient to depend on the readings of B. To enable the observer to read A, if required, without shifting his position, a couple of upright sockets are fixed to the top of the prism between the two levels; on each of these a swinging mirror can be placed, capable of adjustment to any angle required for giving, by reflection, the readings of the ends of the bubble. This instrument has a much higher magnifying power than any of the others and, under exceptional circumstances, it may be used to read the ordinary staff at a distance of 20 chains.

Cylindrical levels.*

4. The cylindrical levels are numbered 1, 3, and 4, and are all alike in general construction, each having a clamp and tangent-screw fixed to the boss of the tribrach. The telescope has a total length of 24 inches. The object-glass has an effective diameter of 2¼ inches and a focal length of 21 inches. There are two eye-pieces with each instrument, giving a power of about 35. The effective length of the level tube which in Nos. 1 and 4 is protected by an outside glass case, is 8¾ inches.

The value of one division of the level-scale is for

No. 1.	No. 3.	No. 4.
3″·0534:	1″·70915:	1″·01728,

and from these values, tables of subtenses have been prepared on the basis that 1 division at 1 chain distance = 0·0009770 feet for No. 1; 0·000546889 for No. 3; and 0·0003255 for No. 4.

Cushing's Reversible level.†

5. The telescope in Cushing's reversible level has a total length of 23 inches. The object-glass has an effective diameter of 2 inches and a focal length of 21 inches. There are two eye-pieces with an arrangement for pushing forward a dark glass when required: the magnifying power of these is however low, being only about 25. The effective length of the level tube is 6¾ inches, and the value of 1 division of the level-scale is 5″·0525, from which a subtense table has been prepared on the basis that 1 division = 0″·001617 feet at 1 chain distance. The object and eye-ends of the telescope are readily interchangeable by means of the following arrangement:—To the internal tube of the telescope is fixed a gun-metal socket, turned and ground with a short conical fitting and wide flange to receive the eye-end with its eye-piece and diaphragm. On the opposite end of the outer tube a precisely similar fitting receives the cell containing the object-glass, both of the ends being identical as regards the fitting. The eye-piece is attached to the telescope by two screws placed 180° apart in the flange of the socket: these screws are

* *See* Mathematical Instrument Office Descriptive Catalogue, fig. 131.
† The description is taken from a paper "On Cushing's Reversible Level" by Major E. H. Courtney, R.E.

not intended to be taken out, but corresponding holes in the flange of the eye-end allow the latter to be inserted in the socket, when a short rotatory motion from left to right will bring it into its proper position against a stop. The object-glass has precisely the same kind of attachment, and will, like the eye-end, fit the socket at either end of the telescope.

Instead of threads, lines are finely engraved on a glass disc which fits into a sliding diaphragm. The horizontal plate and tribrach are both cast hollow underneath, securing maximum strength and rigidity with minimum weight. A new feature is introduced in the attachment of the supports of the telescope: the support nearest the object-end is in contact with the plate, but is capable of a slight rocking motion in the direction of the axis of the telescope, so as to admit of the second adjustment (hereafter described), whilst the other support is provided with two large nuts for clamping and permanently securing the telescope to the plate when this adjustment has been performed.

Adjustment of the cylindrical and rectangular levels.—Bubble.

6. The adjustments of these levels before commencing work are performed as follows :—The instrument is levelled very carefully by means of the foot-screws, so that the one end of the level reads the same while the instrument makes a complete rotation round the vertical axis ; then by means of the adjusting screws of the level itself, the bubble is brought as nearly as possible into the centre of its scale, care being taken that the edge of the scale corresponds with the longitudinal axis of the bubble.

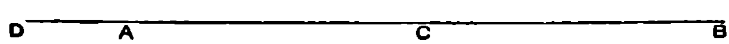
Telescope.

7. If the staves are set up at exactly equal distances from the instrument, all errors arising from imperfect adjustment of the level to the optical axis of the telescope, are cancelled. But although this is the case, it is still very important that the telescope should be, as nearly as possible, truly adjusted. These adjustments are two-fold :—

Firstly. To bring the horizontal wire of the diaphragm into the visual axis of the telescope.

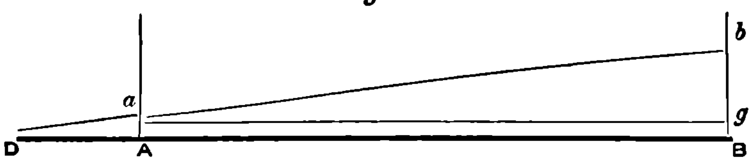
The instrument is set up at C,

Fig. 1.



exactly mid-way and in a line with two staves at A and B, distant two or three chains from each other, carefully levelled and the difference of readings, x, of the staves at A and B found: this is the true value, notwithstanding the errors of adjustment, since C is the middle point of AB. The instrument is then shifted to a point D near A, say $\frac{1}{4}$ chain off, and so placed that both staves are simultaneously in the field of the telescope. It is carefully levelled and the difference of readings, y, of the staves at A and B found. If x is equal to y, the horizontal wire is in the visual axis of the telescope, but if not the difference $y-x$ will be equal to bg in Fig. 2,

Fig. 2.



where Db is the apparent line of sight and DB the true line. The point B on the staff is determined from the formula

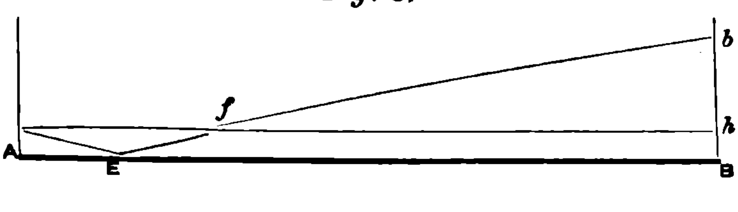
$$b\,\mathrm{B} = (y - x)\,\frac{\mathrm{DB}}{\mathrm{AB}}$$

and the horizontal wire brought opposite the point B by means of the diaphragm screws, the instrument being kept level. The horizontal wire will then be in the visual axis of the telescope.

Secondly. To make the visual axis of the telescope perpendicular to the axis of rotation :—

The instrument is set up at E, Fig. 3, between the staves, but much nearer to A than to B. It is carefully levelled and the difference of readings, z, of the staves at A and B found. If z is equal to x, the visual axis of the telescope is perpendicular to the axis of rotation, but if not $z-x$ is equal to bh in Fig 3.

Fig. 3.



The point B is determined from the equation

$$bB = (z - x)\frac{EB}{EB - EA}$$

and the horizontal wire is made to intersect B by means of the adjusting screws under and near each extremity of the horizontal plate. This, of course, disturbs the bubble of the level: it must now be corrected by its own screws, and the whole operation repeated.

The rectangular level has no horizontal plate with adjusting screws: its visual axis must be set at right angles to the axis of rotation by moving the screws of the diaphragm; as the first adjustment cannot be made, the greatest care must be taken to have the instrument at equal distances from the staves during field work.

Adjustment of Cushing's level.

8. The adjustment of the bubble is the same as described in para. 6, and there are three other adjustments for this instrument:—*firstly*, to eliminate vertical collimation; *secondly*, to make the line of collimation perpendicular to the vertical axis; and *thirdly*, to make the bubble tube parallel to the line of collimation. These are effected as follows:—

Firstly. The instrument is set up on its stand, with one foot-screw under the telescope. The small fixing-screw at the top of the object-glass cell is taken out and the cross lines focused; the telescope is then directed on any convenient object, *e.g.* a small circular dot on a sheet of paper placed about 20 feet from the instrument, the object focused and bisected with the horizontal wire. The eye-end is then carefully turned in its socket, from right to left, until the holes in the flange of the eye-end are opposite the heads of the screws in the socket and removed. It is replaced again but in an inverted position, care being taken to turn it from left to right until it comes to a stop, when the diaphragm will be in its proper position. If the point is still bisected the collimation is perfect, but if not half the deviation from horizontal line is corrected by the foot-screw under the telescope, and half by the two screws that give vertical motion to the diaphragm. This process is repeated till the adjustment is perfect.

Secondly. The object being now bisected and all parallax eliminated, the eye-end and the object-glass cells are removed from their respective sockets and placed in the opposite ends of the telescope. If the object is still bisected on turning the telescope half round, the line of collimation is perpendicular to the vertical axis; but if not, half the error is corrected by the large clamping nuts at one end of the horizontal limb, and the other half by the foot-screw under the telescope. As soon as it is found that the object and eye-ends can be reversed without any apparent change in the position of the object intersected, the small fixing screw should be returned and the object cell made secure. It is important in changing the object-glass from end to end to keep that part of the cell which has the small screw hole in it, always uppermost.

Thirdly. The instrument stand is levelled approximately by the legs, the telescope turned so that its axis is parallel to a line joining two foot-screws, and the bubble brought by motion of the latter to the centre of its run. If it remains so on turning the telescope half round, the level is correct; but if not, the bubble is brought half way back by the foot-screws over which it stands, and the other half by the two opposing nuts at the eye-end of the bubble-tube. The levelling is completed by turning the telescope a quarter round, so that one end of the level is over the third foot-screw by which the bubble must be brought to the centre of its run. The bubble should now remain in the centre during a complete revolution, and the small cross-level can then be adjusted.

Staves.

9. The staves are graduated on both sides, one face being painted white with black divisions and graduated to hundredths of a foot from 0 to 10·00 feet, the other face being black, with white divisions, similarly divided from 5·55 to 15·55 feet. They are both shod and capped with brass, the extreme graduations being laid off on the brass. They are supplied with plummets let into their sides and visible through glass doors, so that they may be adjusted to within an inch or two of the true perpendicular, and they are held in position by ropes attached to a swivel on their summits.

The units of the levelling staves must be determined in terms of the 10-foot standard of the G. T. Survey. For this purpose a portable metal bar, on which the length of the standard bar has been laid off, is taken with each party of levellers. The staves should be compared with it at least twice during the course of the season, *viz.*, at the beginning and end of the season's work. It is also advisable that additional comparisons be made every six weeks or two months, especially if the rise or fall is great, and the value of the work likely to be affected by the slight variations in length to which the staves are liable. As the wood of the staves has a tendency to shrink, a partial separation from the brass at the ends may be caused. In this case an exact foot should be taken off, with a beam compass, from any of the intermediate feet (1 to 9) all of which are defined by dots on brass pins let into the staff and the spot where one end of the compass falls on the brass should be marked, the other end being on the nearest foot. The mark will indicate the true position of the zero before separation, and if it is referred to when the staff is compared with the portable standard, no corrections for the separation will be required.

In choosing staves, at the commencement of the field operations, the observer must be careful to examine the differences between the zeros of the black and white faces of each staff. In most staves it is 5·55 but in some it is 5·60 feet; both of a pair should have the same difference in common, otherwise the observer will be perplexed and delayed by troublesome discrepancies between the results from black and white faces.

Comparison of staves.

10. The comparison of staves is generally done by two observers who make independent measures of the difference in length between the standard bar and 10 feet of the staff by means of a finely graduated scale, the temperature being noted at the time of observation.

The following table shows how the comparison is conducted and needs no remark except that it is the mean of the two observations which is entered in the fifth column :—

COMPARISON OF STAVES.

Value of the 10-foot Portable Standard Bar No. 3 (Edge A) = 9·999424 feet at 62° Fahrenheit
Factor of expansion = 0·0000064 per degree per foot.

Date of Comparison	Temperature	Staves: No. of Staff	Staves: Face of Staff	Value of Staves in terms of Portable Standard Bar when observed	Correction for Temperature to Portable Standard Bar	Corrected length of Portable Standard Bar when observations were made	Corrected Value of Staves	Corrected Mean of both Faces	Difference of Length of Staves from 10 feet	Mean Difference of Length pairs of Staves from 10 feet	Mean Correction for 10 feet for pairs of Staves used in different Sections	Remarks
	° ′											
15th January	79·25	B_1	W	10·000700	+ ·001104	10·000528	10·001228					
	80·0		B	10·002100	·001152	576	2676	10·001952	+ ·001952			For Section from Navánúr to Lakhpat G. T. S. Station
	80·0	B_2	W	10·001600	·001152	576	2176			+ ·002064		
	80·0		B	10·001600	·001152	576	2176	·002176	·002176			
14th March	81·25	B_1	W	9·999600	·001232	10·000656	10·000256				+ ·001174	
	81·65		B	9·999650	·001258	682	0332	·000294	·000294			
	81·8	B_2	W	10·000350	·001267	691	0041			+ ·000283		
	81·95		B	9·999800	·001277	701	0501	·000271	·000271			
22nd May	87·0	B_1	W	9·999450	·001600	10·001024	10·000474				+ ·000291	For Section from Lakhpat G.T.S. Station to Tatta.
	87·0		B	9·999100	·001600	1024	0124	·000299	·000299			
	87·0	B_2	W	9·999450	·001600	1024	0474			+ ·000299		
	87·0		B	9·999100	·001600	1024	0124	·000299	·000299			

Run of a level.

11. The best way of determining the run of a level is by means of an instrument, called a 'bubble-tester,' specially constructed for the purpose. One of these is kept in the Mathematical Instrument Office, Calcutta, and another in the Trigonometrical Branch Office, Dehra Dún.

Another simple way is by attaching the level to the vertical circle of an alt-azimuth instrument or a theodolite; the readings of the ends of the bubble and of the micrometers or verniers are then taken in different positions of the bubble along its scale, and the number of divisions of the level-scale compared with angle given by the circle, in each of the different positions, are used to obtain a series of values of one division of the level-scale in terms of seconds of arc. The mean of these will give the run of the level.

Subtense table.

12. When the run of a level is known, it is easy to ascertain the correction to a staff reading for any amount of dislevelment within the range of the bubble of the level. Suppose, for example, that the value of 1 division of the scale is equal to $1''\cdot709$, then the amount subtended by an angular deflection measured by the movement of the bubble through 1 division will be equal to $1\cdot709$ times the amount subtended by an angle of $1''$ at the given distance. At a distance of 10 chains or 660 feet from the instrument, the maximum distance at which the staff is usually set up, the subtense for 1 division of the level will be

$$= 1\cdot709 \times \sin 1'' \times 660 \text{ feet}$$
$$= 1\cdot709 \times \cdot0000048481 \times 660 \text{ feet}$$
$$= \cdot0054684 \text{ feet.}$$

With such a datum, it is necessary to construct a table of subtenses for varying amounts of dislevelment and ranges of distances. As the staves are divided into tenths and hundredths and the readings are estimated to the thousandths of a foot, the subtenses must also be expressed in thousandths of a foot as follows :—

TABLE OF SUBTENSES.

Divisions	Distance in chains										Divisions
	1	2	3	4	5	6	7	8	9	10	
0·1	·000	·000	·000	·000	·000	·000	·000	·000	·000	·001	0·1
·2	·000	·000	·000	·000	·001	·001	·001	·001	·001	·001	·2
·3	·000	·000	·000	·001	·001	·001	·001	·001	·001	·002	·3
·4	·000	·000	·001	·001	·001	·001	·002	·002	·002	·002	·4
·5	·000	·001	·001	·001	·001	·002	·002	·002	·002	·003	·5
·6	·000	·001	·001	·001	·002	·002	·002	·003	·003	·003	·6
·7	·000	·001	·001	·002	·002	·002	·003	·003	·003	·004	·7
·8	·000	·001	·001	·002	·002	·003	·003	·003	·004	·004	·8
·9	·000	·001	·001	·002	·002	·003	·003	·004	·004	·005	·9
1·0	·001	·001	·002	·002	·003	·003	·004	·004	·005	·005	1·0
·1	·001	·001	·002	·002	·003	·003	·004	·004	·005	·006	·1
·2	·001	·001	·002	·002	·004	·004	·005	·005	·006	·006	·2
·3	·001	·001	·002	·003	·004	·004	·005	·005	·006	·007	·3
·4	·001	·001	·003	·003	·004	·004	·006	·006	·007	·007	·4
·5	·001	·002	·003	·003	·004	·005	·006	·006	·007	·008	·5
·6	·001	·002	·003	·003	·005	·005	·006	·007	·007	·008	·6
·7	·001	·002	·003	·004	·005	·005	·007	·007	·008	·009	·7
·8	·001	·002	·003	·004	·005	·006	·007	·007	·009	·009	·8
·9	·001	·002	·003	·004	·005	·006	·007	·008	·009	·010	·9
2·0	·001	·002	·003	·004	·005	·007	·008	·009	·010	·011	2·0
·1	·001	·002	·003	·004	·005	·007	·008	·009	·010	·012	·1
·2	·001	·002	·003	·004	·006	·008	·009	·010	·011	·012	·2
·3	·001	·002	·003	·005	·006	·008	·009	·010	·011	·013	·3
·4	·001	·002	·004	·005	·006	·008	·010	·011	·012	·013	·4
·5	·001	·003	·004	·005	·006	·009	·010	·011	·012	·014	·5
·6	·001	·003	·004	·005	·007	·009	·010	·012	·013	·014	·6
·7	·001	·003	·004	·006	·007	·009	·011	·012	·013	·015	·7
·8	·001	·003	·004	·006	·007	·010	·011	·012	·014	·015	·8
·9	·001	·003	·004	·006	·007	·010	·011	·013	·014	·016	·9
3·0	·002	·003	·005	·006	·008	·010	·011	·013	·014	·016	3·0
·1	·002	·003	·005	·006	·008	·010	·011	·013	·014	·017	·1
·2	·002	·003	·005	·006	·009	·011	·012	·014	·015	·017	·2
·3	·002	·003	·005	·007	·009	·011	·012	·014	·015	·018	·3
·4	·002	·003	·006	·007	·009	·011	·013	·015	·016	·018	·4
·5	·002	·004	·006	·007	·209	·012	·013	·015	·016	·019	·5
·6	·002	·004	·006	·007	·010	·012	·013	·016	·017	·019	·6
·7	·002	·004	·006	·008	·010	·012	·014	·016	·017	·020	·7
·8	·002	·004	·006	·008	·010	·013	·014	·016	·018	·020	·8
·9	·002	·004	·006	·008	·010	·013	·014	·017	·018	·021	·9
4·0	·002	·004	·007	·009	·011	·013	·015	·018	·020	·022	4·0
·1	·002	·004	·007	·009	·011	·013	·015	·018	·020	·023	·1
·2	·002	·004	·007	·009	·012	·014	·016	·019	·021	·023	·2
·3	·002	·004	·007	·010	·012	·014	·016	·019	·021	·024	·3
·4	·002	·004	·008	·010	·012	·014	·017	·020	·022	·024	·4
·5	·002	·005	·008	·010	·012	·015	·017	·020	·022	·025	·5
·6	·002	·005	·008	·010	·013	·015	·017	·021	·023	·025	·6
·7	·002	·005	·008	·011	·013	·015	·018	·021	·023	·026	·7
·8	·002	·005	·008	·011	·013	·016	·018	·021	·024	·026	·8
·9	·002	·005	·008	·011	·013	·016	·018	·022	·024	·027	·9
5·0	·003	·005	·008	·011	·014	·016	·019	·022	·024	·027	5·0

If, however, neither bubble-tester, alt-azimuth instrument nor a well graduated theodolite is available, a subtense table may be made by directing the levelling instrument on a staff set up at any convenient distance at which it can be read with accuracy, taking a series of staff readings and comparing their difference with the differences of the corresponding readings of the bubble. The observations should be taken in the morning before the atmosphere is agitated by the sun's rays, or in the afternoon a couple of hours before sunset, and a cloudy day is preferable to one of bright sunshine for the purpose. Suppose that the staff is set up at 10, 8 and 5 chains from the instrument, and that the means of 20 observations at each distance are for 1 division of the level-scale equal respectively to ·0057 feet, ·0042 feet and ·0028 feet. Then the value to be adopted for the subtense of 1 division at a distance of 10 chains

$$= \tfrac{1}{3}\left\{\cdot 0057 + \frac{10}{8} \times \cdot 0042 + \frac{10}{5} \times \cdot 0028\right\} = \cdot 0055 \text{ feet,}$$

and the subtense table will be computed in the same form as before by simple proportion.

Dislevelment.

13. The dislevelment must be deduced from the readings of the bubble and may be expressed in terms of the divisions of the scale. As the numbering of the divisions is carried from the centre outwards, the readings of the ends are identical when the bubble is exactly in the middle of its tube; hence it is evident, that if the level were truly adjusted to the visual axis of the telescope, the instrument would be in a truly horizontal position when the readings of the two ends of the bubble were identical, and that when the readings were not identical, half their differences would indicate the amount of deflection of the instrument from horizontality. As however there is no certainty that the level is in exact adjustment to the visual axis of the telescope, it is necessary after having taken a pair of readings of the bubble to turn the instrument through an exact semi-revolution and take a fresh pair of readings: the four readings must then be combined together in such a manner as to eliminate the effect of the error of adjustment, in deducing the amount of dislevelment. In practice it is not necessary to reverse the instrument and take two pairs of readings for the mere elimination of the errors of the adjustment of the level to the visual axis of the telescope,

as this purpose is answered by the readings which are taken when the telescope is pointed in succession to the back and forward staves.

If the telescope has been pointed to the back staff in the first instance, and d_b and d_f be the readings of the ends of the level towards the back and forward ends respectively, and if when the telescope is pointed to the forward staff d'_b and d'_f be the readings towards the back and forward ends respectively, then the amount of dislevelment expressed in divisions of the scale will be

$$= \frac{1}{4}\left\{(d_b + d'_b) - (d_f + d'_f)\right\},$$
$$= \frac{1}{4}\left\{(d_b - d_f) + (d'_b - d'_f)\right\};$$

if this is positive, the end of the instrument towards the back staff will be above the horizontal plane passing through the centre of the instrument.

The effect of this instrumental dislevelment on the rise or fall deduced from the staff readings will evidently be equal to the sum of the subtense on the back and forward staves, or twice the value given by the subtense table. Hence in practice half the expression within the brackets is taken, instead of one quarter, as the quantity with which to enter the subtense table.

The above argument is only strictly true when the instrument is in a line with the staves. If the instrument is on either side of this line, one end of the level is towards the staff on which the telescope is pointed, and the other is towards the opposite direction and not towards the other staff. In this case twice the difference between the respective amounts of dislevelment in two vertical planes passing through the instrument and the staves is determined, and this is all that is required for the correction of the observations, provided that the essential condition of setting up the instrument at equal distances from the staves has been fulfilled.

CHAPTER II. PRECAUTIONS AGAINST ERRORS.

Errors in staff readings.

1. The chief cause of the inaccuracies so frequently found at the close of a line of levels is due to errors of staff readings. Even when an observer is careful to repeat the readings at each station, before proceeding to the next, he

may occasionally make a mistake without finding it out. It is to guard against this class of error that the staves are graduated differently on the two faces, so that if an observer make a mistake in one reading he is not likely to make a similar mistake in the reading on the opposite face, and the error will be shown up immediately by the different results from the black and white faces, if the computation has been correctly performed.

Errors in writing down results.

2. Errors of reading are, however, of such an uncommon occurrence, and the results from black and white faces so constantly coincide, that the recorder in taking out the second result immediately under the first is liable to be biassed by it and to fancy that they coincide, when in reality there is a difference. If only one observer was working, such a mistake would probably not be found out until the preparation of the duplicate records some months afterwards, and it could not then be rectified. To remove this source of error, a second observer follows the first over the line, station by station, with an independent set of instruments.

Instrumental errors.

3. Errors of permanent instrumental adjustments, as well as those which might be caused by normal atmospheric refraction and the earth's curvature, are wholly eliminated when the staves are at equal distances from the instrument. Errors caused by instrumental deflection from horizontality are obviated by reading the ends of the level, and applying a correction for dislevelment to the difference of the staff readings.

Errors in reading the level.

4. The instrument generally stands at such a height that it is impossible for the observer to look down on it. From above, the bubble is seen to be sharply defined and the scale can be read with great accuracy, but the observer usually stands with his eye nearly on a level with the instrument, in which position the rim round the bubble, caused by the adhesion of the liquid to the sides of the glass tube, becomes so prominent that, in certain lights, its extremities may be taken for the true ends of the bubble. When light falls obliquely on the instrument, the outer edge of the rim, being towards the light, is more clearly defined than the inner edge, while at the opposite end of the bubble, the inner edge or true end is most clearly defined. Hence there is a liability to read

one end erroneously, and thus introduce one or two divisions of level error. This must be most carefully guarded against, and the observer should take as many opportunities as he can of reading the bubble with his eye above the instrument, and again with his eye on the same level, until he is satisfied that his readings are correct.

Cumulative errors.

5. Cumulative errors are caused by the constant recurrence of small errors, either personal, instrumental or atmospheric; and, though too small to attract attention at any one station, become manifest when the results of different observers on the same line of levels are compared, or those of a single observer returning station by station over a line from terminus to origin.

Small errors recurring in a constant order—such as might be caused by a uniformly rising or sinking refraction, or by a tendency in the instrument to settle on its axis more one way than another on being set up for observation—may be cancelled by alternating the order of observation to the back and forward staves, at the successive stations, *i.e.*, by observing the back staff first at one station, and the forward staff first at the next. This method might be still further amplified by the following circuit system, by which an observer working only in one direction has a means of finding out whether there are such errors in his work, by comparing the differences between the respective reductions to origin from black and white faces, one pair of which may be treated so as to give the results of an 'up line' and the other of a 'down line':—

Odd stations	1	Forward Staff Back ,,	White Face	Down Line.
	2	Back ,, Forward ,,	Black ,,	Up ,,
Even stations	1	Back ,, Forward ,,	Black ,,	Up ,,
	2	Forward ,, Back ,,	White ,,	Down ,,

This amplification is however seldom carried out in practice.

Errors recurring in a constant order might be caused by an irregularity in the socket of the axis, if the instrument be invariably set up in the same direction with regard to the line followed. This can be obviated by marking one of the legs of the tripod stand, or of the instrument, and directing it back and forward at alternate stations.

Circuit system.

6. The circuit system which requires an observer to close on his origin is theoretically the best; but a circuit, when levelled by a single observer, is exposed to a greater chance of accidental error than a single line executed by two levellers consecutively and independently. For both observers to work twice over the same line would more than double the time and cost of the undertaking, as they would never be able to increase the distance between the staves, when favoured by clear weather, except at the cost of losing all intermediate comparisons, but would be compelled to adopt short unvarying distances. If, however, an amount of extra trouble, equivalent to both observers marching over the whole line once, be accepted, all the advantages of closing on an origin may be obtained by the simple expedient of dividing the line into as nearly as possible equal sections and both observers working alternate sections in opposite directions. The sections should be so arranged that the sum of the forward ones is equal to that of the back ones. If, as is usual, the sections are each a day's work—in good ground 4 or 5 miles—and circumstances will permit of encamping near their extremities, it is easy to walk the 4 or 5 miles to the commencement of a section, without fatigue, in the early morning before sunrise and then level back to camp; in this way the delay of extra marching is avoided and steady progress made at a rapid rate. This section system has apparently contributed, more than anything else, to reduce cumulative errors in the operations, and should invariably be adopted.

Cumulative level errors.

7. A tendency is sometimes observed, in a long line of levels, for the negative level corrections to exceed the positive ones or *vice versâ*. It is of importance that they should cancel each other, as any error in the determination of the runs of the levels is thus eliminated. The observer must therefore be prepared to prevent a preponderance in any direction by setting up his instrument with a slight tilt in the opposite direction.

Cumulative errors caused by the sun.

8. It has also been found that the sun exercises a constant dis-levelling effect on instruments, tending to raise the end towards itself and lower the opposite end. This is shown by the excess of negative corrections for level errors in lines worked from south to north and of positive corrections in the opposite lines. Thus an error in the adopted value of the run of the level would affect the results by the same fraction of the accumulated correction that it is of the run. In practice this may be cancelled by watching the signs of the correction and occasionally tilting the instrument to counteract the dispositon to rise towards the sun.

There is however another order of error which no *modus operandi* will wholly cancel; for, however carefully the instrument may be shaded, the action of the sun tends to cause a constantly recurring displacement in the level in the interim between the reading of the staff and that of the bubble ends: these readings should be strictly simultaneous in order to compare with each other, but they are necessarily consecutive; and short as the interval between them may be, it is enough to cause slight error, of a cumulative nature, which may amount to a large quantity at the end of a long line of levels. It would be maximum on a meridional line, but would not affect a line carried from east to west or *vice versâ*. This error is of a class which cannot be eliminated by working in a circuit: so long as the cause remains constant, the effect is the same in both up and down operations; thus the opposite points of a circuit, which closes without apparent error, may yet be considerably erroneous.

Leaving this class of error out of consideration, atmospheric influences must tend to cancel each other, on a long line of operations, except under the following circumstances:—

Error due to refraction.

9. *Firstly*—when more stations are observed before than afternoon: during the forenoon the refraction will generally be sinking and rising in the afternoon, consequently the first of a pair of staff readings will have a tendency to be more refracted than the second, thus introducing cumulative error unless the precaution is taken to alternate the order of observation by taking the back staff first at one station and the forward staff first at the next.

10. *Secondly*—when operations are carried over a line of country which slopes uniformly in one direction, as in proceeding from the sea to the foot of a range of mountains, the rays of light from the up-staff to the observer are generally nearer to the ground than those from the down-staff, and they must therefore be more subject to extremes of refraction. The influence of this source of error will vary with the seasons, and it is evidently beyond the control of the observer.

Error due to slope of ground.

11. The sinking of a peg that supports a staff, in the interval between its being the forward end of one section and the back end of the next, has often been suggested as a cause of error. But there appears to be as much reason for a firmly driven peg to be raised by the reaction of the ground as to be lowered by the weight of the staff, and there are nearly as many instances in which the signs of the errors indicate the possibility of the one event having happened as the other.

Error due to sinking of peg.

12. On the whole, then, when the precautions which are taken to guard against accidental gross errors, such as misreadings the feet and tenths, mistaking the identity of the pegs on the line, or confounding one bench-mark with another, are such as to preclude the possibility of any error arising from these causes, certain other precautions must still be taken against the accumulation of minute errors which may arise (1) from instrumental defects, (2) from the rising of the pegs on hard ground or their sinking on soft ground, (3) from rising or sinking refraction, (4) from personal bias, (5) from the effect of the sun in disturbing the horizontality of the instrument in the interval between the staff and level readings, (6) from unequal illumination of the ends of the bubble, (7) from working continuously over a long slope, on which the average height above the surface of the ground will be sensibly different for up and down rays.

Conclusions.

The alternation of the order of observation of the back and forward staves should eliminate all error arising from the first three sources. Those which cannot be eliminated by this method may be met to some extent by multiplying the number of observers, in order to diminish the effect of individual bias, and by working under every possible variation of climate.

CHAPTER III. PROCEDURE IN THE FIELD.

Instructions to observers.

1. Every main line of levels is executed by two observers, each working with his own instrument quite independently of the other, one being a station in advance of the other.

Before beginning work, each observer must be careful to see that the instruments are in perfect adjustment.

Adjustments. Stand.

2. These adjustments are as follows:—The stand must be rigid, and there must be no play between it and the telescope. In soft or sandy ground, the feet of the stand should rest on wooden pegs firmly driven.

Telescope.

3. The telescope must be adjusted so that, *firstly*, the horizontal wire of the diaphragm is in the visual axis of the telescope, and, *secondly*, the visual axis of the telescope is perpendicular to the axis of rotation. For the methods of effecting these adjustments, see page 283 for the cylindrical levels and the rectangular level, and page 285 for Cushing's reversible level.

Bubble.

4. When the instrument is level, the bubble should be adjusted so as to be in the centre of its run and should be bisected crosswise by the edge of the scale; it should remain thus in any position of the instrument.

Staves.

5. The staves must be compared with a standard 10-foot bar at the beginning and end of the season's work and also during the field season as often as is convenient, see page 286.

In choosing a pair of staves the observer must carefully examine that the difference between the zeros of the black and white faces of each staff is the same, see page 287.

Chains.

6. The chains should also be compared occasionally with the 10-foot bar.

Level-scale.

7. Before beginning work, the observer should determine the value of the level-scale and prepare a subtense table, giving the value in feet of one division of the level-scale at various distances, see page 289.

Support for staves.

8. The staves must invariably rest on wooden pins driven very firmly into the ground. A hemispherical brass brad should also be let into the head of each pin after it is driven, to offer a point instead of an uncertain surface, for the staves to stand on, and so that they may be rotated freely to present each face in succession to the observer. The brad also affords a common point of reference for the successive observers, whose results may thus be compared rigorously station by station.

Distance between staves.

9. The staves must be set up at equal distances from the instrument; if the ground will not permit of the latter being set up midway on the line between the staves, then by shifting the positions of the instrument and of the forward staff some point can always be found where the instrument will occupy the vertex of an isosceles triangle of which the line between the staves is the base. Unequal distances should always be for them excepting on unusually rough

299	last sentence but one of para. 9	„ On a good day . . . . to 10 o'clock	„	On a good day the maximum distance between the staves is about 16 chains in the early morning and about 12 chains up to 10 o'clock.

distances should be carefully measured with a chain.

Re-adjustment of staves.

10. When the difference of level is small and a staff has once been set up and observed, it should not be re-adjusted if it has to be observed at the next station, because then any error caused at the first station by deflection of the staff will in a great measure be cancelled at the next. But the staff should be carefully re-adjusted whenever, having been read near the bottom at one station, it is liable to be read near the top at the next.

Readings.

11. The instrument being set up at the same distance from the two staves, must be carefully levelled. The white face of the back staff should then be intersected and the reading taken to three decimal places: when the staves are very

7. Before beginning work, the observer should determine the value of the level-scale and prepare a subtense table, giving the value in feet of one division of the level-scale at various distances, see page 289.

Level-scale.

8. The staves must invariably rest on wooden pins driven very firmly into the ground. A hemispherical brass brad should also be let into the head of each pin after it is driven, to offer a point instead of an uncertain surface, for the staves to stand on, and so that they may be rotated freely to present each face in succession to the observer. The brad also affords a common point of reference for the successive observers, whose results may thus be compared rigorously station by station.

Support for staves.

9. The staves must be set up at equal distances from the instrument; if the ground will not permit of the latter being set up midway on the line between the staves, then by shifting the positions of the instrument and of the forward staff some point can always be found where the instrument will occupy the vertex of an isosceles triangle of which the line between the staves is the base. Unequal distances should always be avoided: there is no excuse for them excepting on unusually rough ground when the observer is pressed for time and then the distances should be made as short as possible. On a good day the maximum distance between the instrument and the staves is from 16 to 20 chains in the early morning and about 8 chains up to 10 o'clock. The distances should be carefully measured with a chain.

Distance between staves.

10. When the difference of level is small and a staff has once been set up and observed, it should not be re-adjusted if it has to be observed at the next station, because then any error caused at the first station by deflection of the staff will in a great measure be cancelled at the next. But the staff should be carefully re-adjusted whenever, having been read near the bottom at one station, it is liable to be read near the top at the next.

Re-adjustment of staves.

11. The instrument being set up at the same distance from the two staves, must be carefully levelled. The white face of the back staff should then be intersected and the reading taken to three decimal places: when the staves are very

Readings.

close a fourth place of decimals might be estimated, but this is so seldom the case that it may be treated as exceptional. The ends of the bubble must then be read with great care, as quickly as possible after the staff reading, see page 293. The white face of the forward staff must next be intersected and the level readings taken: after this the process must be repeated on the black faces of the staves. The observations of the back and forward staves should follow each other as quickly as possible, consistently with accuracy, in order that they may be strictly differential. If the differences of height deduced from the white and black faces agree within ·005, and the observer is otherwise satisfied with them, he should proceed to the next station. Should they differ by a greater amount, they must be repeated once or twice according to discretion. If the differences are very large and obviously due to grazing rays, the station should be rejected and a better one selected if possible.

After finishing the first station, the observer must leave his forward staff as it was, so that it may become the back staff at the second station and its zero error be eliminated. In observing at the second station the forward staff should first be read and then the back one, and so on alternately, see page 294: this is not apparent from the field books which are arranged to facilitate computation, but it must be attended to.

Position of instrument.

12. The observer in setting up the instrument ought to mark one of the legs of the tripod, or of the instrument, and always point the marked leg to the same staff-holder throughout a section, see page 295.

First day's work.

13. Proceeding in this way, the observer should finish the first day's work of say 4 miles, if in good ground, between day light and 10 o'clock, and his camp ought to be waiting for him at the end of his work.

Relevelling closing station.

14. As the day's work will necessarily close more frequently on temporary pins, over which the line of levels is carried, than on permanent bench-marks, the closing station of the previous day must be relevelled next morning to test the permanence of the pins, before carrying on operations beyond them.

15. The next morning before day break, the observer should march out to the end of his second section and level back into camp, see page 295, afterward relevelling his closing station of the previous day.

Second day's work.

16. In consequence of alternating the direction of operations, the verification is made in the opposite direction to the first measurement, consequently the two results have opposite signs: they cannot be conveniently combined together and the mean employed, as when the work is carried only in one direction, and there is a liability to confusion at the time, and subsequent misapprehension if the results are brought up by any one but the leveller himself. This should be carefully guarded against by the method of recording the repetition in the Field Book. A clear space of two inches should be left between the record of the extreme station of the section in progress and that of the repeated station of the adjacent section, and the reduction to the origin of the section in progress should be entered above the record of the repetition. Opposite the repetition a remark should be made as follows:—

Precautions to be taken at the end of a section.

By up line fall	2·327
By down line rise	2·331

The terminal pins should be large and strong, firmly driven and covered over with thorns for protection. A trench should be cut across the line at the extremity of each section to guard against the possibility of introducing into one section the stations of another.

17. The observer should proceed thus levelling in opposite directions on alternate days till the line is finished, and during the whole work he must be careful to prevent a preponderance of level error as described on page 295.

Level error to be guarded against.

18. The results as deduced by each observer from the white and black faces of the staves must agree within ·005, and the mean of these as determined by each observer must also agree within ·005, otherwise the observations at the particular station must be repeated.

Agreement of results.

19. At intervals of every 10 or 12 miles an 'embedded' bench-mark should be laid down. A stone 18 × 12 × 12 inches is generally built into a block of masonry

Bench-marks.

from about 3 to 4 feet cube, with the upper surface of the stone flush with the upper surface of the masonry which is about 6 inches below the ground level, and the whole block covered with earth. A small hollow square, a quarter of an inch deep and 5 inches square, is cut in the upper surface of the stone for the foot of the levelling staff to rest in. The letters G.T.S. B.M. and the date A.D. 18 . are also cut on the bench-mark. These bench-marks are as a rule laid down in the vicinity of masonry buildings, and to mark the spot the letters B.M. are cut somewhere on the building close by so that identification is unmistakeable. Occasionally, bench-marks are placed in the verandah floors of travellers' bungalows and in other such places; in these cases the stone is placed flush with the floor level.

'Inscribed' bench-marks should be placed on such points as the copings of railway platforms, parapets of bridges, &c. The letters G.T.S. B.M. and a small circle should be cut deep into the masonry, the circle showing the position which the bottom of the staff occupied. In the neighbourhood of tidal observatories, pairs of test bench-marks should be laid down for the detection of secular changes of relative level between the land and the sea. They should be placed some 8 or 10 miles inland, and connected very carefully with the tide-gauge: suitable embedded bench-marks on the line of levels will of course answer this purpose.

Protection of bench-mark.

20. Each embedded bench-mark must be transferred for safe custody to the care of responsible railway or civil authorities. The transfer papers are in triplicate: one copy is kept for record in the office of the Tidal and Levelling Party, the second by the responsible authority and the third by the individnal entrusted by the reponsible authority with the actual care of the bench-mark. The papers are signed by the last mentioned individual and by the officer in charge of the Levelling Operations.

Connection with triangulation.

21. If possible, all principal stations of the Great Trigonometrical Survey, in the neighbourhood of a line of levels, should be connected by branch lines.

Exceptional cases. Levelling to a tower station.

22. There are a few cases which require exceptional treatment. Thus supposing the height of a tower station has to be determined, *firstly* when the foot can be

levelled up to, and *secondly* when it cannot be reached. In the first case the levelling should be carried up to a peg or mark at the foot of the tower and a tape suspended from a staff laid horizontally on the summit of the tower and levelled by means of a spirit-level placed upon it. By this means a direct measurement of the difference of height between the summit of the tower and the mark levelled to, is obtained. In the second case a levelling staff should be erected on the summit of the tower and the angular elevation of two divisions, near the top and bottom of the staff, measured with a small theodolite centered over the last point levelled. The height of the summit of the tower above the eye of the observer and consequently above the levelled point can then be readily deduced.

Levelling up steep ground.

23. Again supposing a great length of steep ground intervenes between the summit of a hill station and the last point levelled to. In this case a staff should be erected over the last point and a second staff laid transversely with one end resting on a convenient spot on the side of the slope while the other is supported by a man and raised or lowered till the indications of a small bubble placed on it show that it is horizontal: the reading on the vertical staff corresponding to the lower edge of the transverse staff should then be taken several times and the difference of level deduced. In this way by a succession of measurements, the top of the hill will be reached and the difference of level found.

Levelling across a river or creek.

24. Again supposing the levels have to be carried across a river or creek whose width is too great to admit of the ordinary staves being read. In this case staves of Strange's pattern graduated to tenths and subdivided to two-hundredths of a foot are used: they can be read pretty fairly with the rectangular level at 50 or 60 chains, and several measurements should be taken on two or more days at the most favourable time so as to reduce the chances of error to a minimum.

If however the river or creek is too wide to admit of even this alternative, two staves should be embedded on opposite sides of the water and as nearly as possible at right angles to the direction of the current and a large number, say 2 or 300, of simultaneous readings of the height of the water on the staves taken at intervals of 5 minutes or so, and the difference of height on the two sides deduced from these.

Field Book.

25. The following is a specimen of the Field Book used in the levelling operations of the Trigonometrical Branch of the Survey of India Department. The 1st column gives the number of the station and indicates the horizontal lines on which the observations to the respective staves are entered; six of these lines are required to each station, three for observations to the black faces and their reduction, and three for the white faces. The 2nd column gives the distances and bearings of the staves from the instrument, the latter being necessary to enable the line of levels to be protracted. The 3rd and 4th columns give the back and forward end readings of the bubble, the 5th and 6th the calculations of dislevelment and the 7th the corresponding correction obtained from the subtense table, expressed in thousandths of a foot, omitting decimal points and cyphers. The 8th column gives the staff readings; the 9th and 10th the approximate results deduced by subtracting the reading of the forward from that of the back staff, one column being allotted to rises and the other to falls. The 11th and 12th columns give the true differences of level, *i.e.*, the approximate values of the 9th and 10th columns corrected for dislevelment; in these columns the mean of the results from black and white faces is also entered. The 13th column gives the reduction of the levels to the origin of the line, which should be deduced station by station and checked by summing up the rises and falls at the foot of the page as indicated. The 14th column is reserved for remarks and descriptions of the bench-marks: the latter should be sufficiently lucid to enable any person who may have occasion to visit a bench-mark, either for the purpose of commencing or closing a new line of levels, to ascertain the exact position of the mark without the slightest doubt; drawings of the bench-marks should also be given with their distances and bearings from any prominent buildings in the neighbourhood. The remaining columns on the right hand side are required for the entry of details regarding the small cumulative errors described in pages 295 and 296. The level and staff readings are recorded by a native assistant who calculates the results before handing the Field Book over to the observer, by whom they are checked before proceeding to the next station. The headings, remarks and descriptions of stations should invariably be written by the observer.

SPECIMEN OF FIELD BOOK.

SURVEY OF INDIA DEPARTMENT.

FORWARD SECTION KALIANPUR TO DEORI VILLAGE.

With No. 3 Standard—value of 1 Division of Scale = 1″·709. Length of Chain = 66 feet. Magnetic North by Compass East of true North by 2½°.

RULE FOR CORRECTING DISLEVELMENT.—*Consider Back End level readings to be −, and Forward End to be +. Find their Difference and enter it with sign of whichever is greater. Half the Algebraical sum of the Differences is the quantity for which a correction is to be taken from Subtense Tables. The correction to have the same sign as the half sum.*

Station No. and Staff Positions	Distances and Bearings of Staves from Instrument	Level Readings—Dislevelment and Corrections					Staff Readings	Approximate Differences of Level		Corrected Differences of Level		Level reduced to Origin	Remarks	Level Correction	Cumulative level Correction
		Back End −	Forward End +	Diffce.	½ Sum	Corrections		Rise +	Fall −	Rise +	Fall −				
Back ...	10·00	73·9	73·1	− 0·8	...	...	11·234						Commenced forward section on yesterday's initial picket, having first tested the stability of it by re-observing station 1 of that day, 11th February, 1862.		
Forward	10·00	74·6	72·3	− 2·3	...	...	11·755	...	0·521	...	0·529				
1			Sum	− 3·1	1·55	− 8									
Back ...	340°	73·9	73·1	− 0·8	...	...	5·686	...	0·539	...	0·528				
Forward	158	71·0	76·0	+ 5·0	...	...	6·225								
			Sum	+ 4·2	2·10	+ 11	...	...	...	...	0·529	−0·529		+ 2	+ 2

Back ...	11·24	76·1	70·4	− 5·7	...	...	9·334						Forward staff on stone Bench-mark imbedded at Pipalwála Chauki in lands of Jorah or Jori village. The stone is about 100 feet west of the high road, and 50 feet west of a Pipal tree, under which a Road Chauki has been placed.		
Forward	11·24	70·3	76·0	+ 5·7	...	...	8·196	1·138	...	1·138					
2			Sum	0·0		0									
Back ...	335°	76·4	69·4	− 7·0	...	...	3·785	1·141	...	1·130					
Forward	154	71·4	74·9	+ 3·5	...	...	2·644								
			Sum	− 3·5	1·75	−11									
Back ...	...	76·7	69·0	− 7·7	...	...	9·336						Repeated on account of discrepancy between first and second results. The general mean of all the observations is the value finally adopted.		
Forward	...	70·4	75·1	+ 4·7	...	...	8·195	1·141	...	1·132					
Repetition			Sum	− 3·0	1·50	− 9									
Back ...	...	73·0	72·5	− 0·5	...	...	3·766	1·121	...	1·133					
Forward	...	70·5	75·0	+ 4·5	...	...	2·645								
			Sum	+ 4·0	2·00	+12	...	...	...	1·133	...	+0·604		− 2	0
Back ...	10·50	72·0	71·5	− 0·5	...	...	9·075								
Forward	10·50	69·7	73·9	+ 4·2	...	...	10·414	...	1·339	...	1·328				
3			Sum	+ 3·7	1·85	+11									
Back ..	348°	71·4	72·1	+ 0·7	...	...	3·524	...	1·341	...	1·326				
Forward	155	69·9	74·1	+ 4·2	...	...	4·865								
			Sum	+ 4·9	2·45	+15	...	...	...	...	1·327	−0·723		+13	+13
Recorded by			Examined by					Totals ...		1·133	1·856	−0·723	Levelled by		

CHAPTER IV. DUTIES IN RECESS.

Duplicate field books.

1. It is a practice in the Levelling Party to prepare duplicates of the field books. This is done by transcribing from the original, the observations only, that is the entries in columns 1 to 4, 8 and 14, see pages 306 and 307. The remaining columns of the form are next filled in from computation made quite independently, and compared with the original. The advantage of this method is that errors of calculation are often brought to light and corrected in the original.

Preparation of abstract.

2. The next step is the preparation of the abstract of which a sample is here given. As all the necessary information is contained in the headings of the form, it needs no further explanation. The abstract should first be made out for the sectional points of the main line, next for the intermediate points, and lastly for points on the several branch lines. The abstract is also prepared in duplicate by two computers working independently.

Press copy of spirit-levelled heights.

3. The press copy of pamphlets of heights is also prepared in the office of the Levelling Party, a copy of which is given in the original field book. In addition to the descriptions and heights above sea level, &c., of the bench-marks, exhibited in a tabular form, section by section, there is added an introduction giving a brief account of the operations, mentioning the names of the observers and the instruments employed, also the error generated in the course of the work.

Chart.

4. A skeleton chart is also prepared by the field party showing the season's operations.

SECTIONAL POINTS.

MAIN LINE FROM NAVANAR TO TATTA.

SEASON 1889-90.

The Heights have been reduced to Mean Level of Sea at Navánár.

Points determined		By Rectangular Level			By Cylindrical Level No. 3			Mean Height above Mean Level of Sea	Remarks
		Sectional Values		Height above Mean Sea Level	Sectional Values		Height above Mean Sea Level		
		Observed	Corrected to Standard Unit		Observed	Corrected to Standard Unit			
G.T.S. 1 □ B.M.	Navánár Tidal Station	...	...	10·409*	...	...	10·409*	...	* From Spirit-levelled Heights No. 1 Bombay Presidency, 1879.
C. G.T.S. 2 □ B.M.	Tapaska Dhoi	+ 1·274	+ 1·274	11·683	+ 1·312	+ 1·312	11·721	11·702	
G.T.S. 6 ○ B.M.	Kapáia Mota village at entrance to ...	+72·995	+73·004	84·687	+73·003	+72·987	84·708	84·698	Devji Khoja's House.
G.T.S. 9 ○ B.M.	Kálághogha, Stone W. of Village ...	+60·297	+60·304	190·684	+60·330	+60·317	190·681	190·683	
INTERMEDIATE POINTS.									
G.T.S. 6 ○ B.M.	Kapáia Mota village entrance to ...	...	...	84·687	...	...	84·708	...	Devji Khoja's House.
G.T.S. 3 □ B.M.	Mundra embedded B.M.	−49·324	−49·330	35·357	−49·349	−49·338	35·370	35·364	
G.T.S. 4 ○ B.M.	Mundra. Jairám Shivji's ...	−43·365	−43·370	41·317	−43·387	−43·378	41·330	41·324	Tank.
BRANCH LINE POINTS.									
G.T.S. 9 ○ B.M.	Kálághogha, Stone W. of Village ...			190·684			190·681		
G.T.S. 1/9 ○ B.M.	Khára Nála Well	−55·261	−55·267	135·420	−55·292	−55·280	135·395	135·408	

PART VIII.

Pendulums.

It is altogether beyond the province of this Handbook to enter upon the details of such an extensive and difficult subject as the swinging of pendulums to determine variations of the force of gravity; more especially as there is no immediate probability of any such work being again undertaken in India. It is, however, such an important factor in a geodetic survey that it cannot be entirely passed over without reference; but it will be sufficient for the purposes of this Handbook if a list of the standard works on the subject is brought within the reach of the reader to enable him to master the details, should he ever be called upon to take part in such operations.

It fortunately happens that Colonel Herschel, R.E., the officer to whom the writing of the Volume of the G. T. Survey of India (Vol. V) on Pendulums, was entrusted, has made a particular study of the bibliography of Pendulums, and has added to that volume an appendix containing a list of nearly three hundred books and papers on this subject. In addition to the list the volume gives a detailed account of the instruments used, and the whole procedure adopted in India, and the reader is referred thereto as a trustworthy guide, if at any time observations of the same kind should be again required.

PART IX.

Miscellaneous Subjects.

CHAPTER I. ANNUAL REPORTS, &c.

General remarks on annual reports.

1. The operations of the Trigonometrical Branch are so varied in their character, that it is almost impossible to lay down exact rules by which all Executive Officers may be guided in the preparation of their annual reports.

Executive Officers must always keep in mind that the object of these reports is to enable the authorities to know exactly what is done from year to year; they must therefore so draw them up, that all who are interested in such matters will have sufficient details available to enable them to see and judge for themselves of the progress and quality of the work turned out.

General remarks on annual reports.

2. In all cases the dates of going to, and returning from, the field must be shown, the strength of the establishment, and the duties allotted to each member of it. The amount of work done and the programme for the ensuing field season, as well as a report on the conduct and efficiency of the subordinates attached to the party, must be given. To this extent the circumstances of all the parties are nearly similar. For details executive officers may with advantage consult the published annual reports of the Survey of India Department which will give them a clear idea of what it is advisable to insert.

Annual report of a triangulation party.

3. For instance, an officer engaged upon Triangulation may with propriety describe the country over which his work extends; and if it is a more or less unknown district, he should make the best use he can of his opportunities for collecting notes about the inhabitants, their manners, customs, language, &c., for insertion in his report. He should also discuss any peculiarities of his instrument, noting any weak points in it, and describing any precautions taken to guard against any evil effects therefrom; he should state his average triangular error, the number of azimuths observed, giving also a general description of the stations, and the precautions taken for their protection. Any political difficulties met with in the past, or anticipated in the future, should also be touched upon.

Annual report of a latitude party.

4. An officer employed upon Latitude observations should describe in detail the programme of star observations at each station, the instrument he used, and any peculiarities noticed in it; also any remedies he devised to cancel them. He should also draw up a tabular statement giving the values of the latitudes observed, with their probable errors, the number of stars from which they were deduced and the differences between the astronomic and geodetic latitudes of the stations occupied. Any hints for future observers deduced from his own experience may properly find a place in his report.

5. A report on Electro-Telegraphic Longitude Operations should contain a list of the arcs measured with the name of the observer at each station, in the order of measurement. A discussion of any peculiarities or abnormal results met with should follow, with suggestions for future avoidance. The method adopted for ascertaining the observer's personal equations is important, and must be described, and a statement of the results at each determination must be prepared.

Annual report of a longitude party.

The values of the several arcs, as determined astronomically, and geodetically, must be compared, and the circuit errors presented in a convenient form. The form of battery used, and the number of cells in circuit, may be added with advantage.

6. The reports of the Tidal and Levelling party are somewhat voluminous. They should contain a brief account of the working of all the instruments at each tidal observatory, specifying the date of inspection, and the steps taken to remedy any defects found. Statements must also be furnished showing the tidal constants both for short-period, and long-period tides at all the observatories, also tabular statements showing :—

Annual report of a Tidal and Levelling party.

(1) Percentage and amount of errors in predicted times of high water
(2) " " " low "
(3) " " heights of high "
(4) " " " low "
(5) Table of average errors in predicted times and heights of high and low water. The progress and present state of the tide tables, as worked out by the Tide Predictor, must also be noticed.

The lines of levels completed should be specified, and the errors in closing on bench-marks noted, as well as any discrepancies found between the heights of principal stations as determined by triangulating, and by levelling, respectively. New bench-marks must also be noticed, when they are in connection with tidal stations.

7. In addition to the annual reports explained above it is necessary for every Executive Officer in charge of a party to submit a precis of his report. This precis must be a brief account of the operations of the party, so written as to be as far as possible intelligible to a non-professional reader, and

Precis of annual report.

free from technicalities. Its object is to enable a Secretary to Government, or other persons not conversant with scientific details, to gain a fair idea of the progress and state of the work: it should be written in the third person, and the services of the individual members of the party should not be alluded to, unless they are particularly praiseworthy, when they may be mentioned in a footnote.

As this precis is intended to be inserted *verbatim* in Part II of the Annual Reports of the Survey of India, it saves a great deal of trouble in compiling these reports if attention is paid to making the form and wording of the precis suitable for this purpose, so as to avoid as far as possible the necessity of subsequent alteration in the Deputy Surveyor General's Office. Specimens may be found in the Annual Reports of each year, which may be taken as guides to the Executive Officer. When Indian terms are used a translation of them should be given.

Confidential reports.

8. All reports on survey operations or on any other kindred subjects, beyond the limits of British India, and of the feudatory Native states, are to be considered in the first instance as strictly confidential.

Visits to other survey offices in the same station.

9. Officers of the senior division must consider it part of their duty during recess to visit the offices of any other survey parties that may be recessing in the same station. It frequently happens that the procedure of different parties varies in some particulars, and mutual visits between the officers concerned tends to utilize to the utmost their varied experiences.

Visits to Head-Quarters offices.

10. Officers of both divisions visiting the presidency should, unless incapacitated by ill-health, be officially conducted through the offices of the Survey of India; and be made acquainted as far as possible with the procedure of the Drawing, Engraving, Mathematical Instrument, Photographic and Lithographic offices. Advantage should be taken by officers passing through Dehra of the opportunity to pay an official visit to the offices of the Trigonometrical Branch, after having obtained permission from the Deputy Surveyor General.

Arrivals to be reported to senior survey officer.

11. Officers and surveyors of all grades will in future not only report in person their arrival at the Head-Quarters of the Survey of India at Calcutta and

Dehra, but at all stations where an office of the Survey of India is established, they will visit that office, and intimate their addresses to the officer in charge. In the case of a station where there are several parties quartered together, the visit will be made to the senior officer present.

CHAPTER II. MANAGEMENT OF THE HEALTH OF A PARTY.*

General remarks.

1. Although in general the occupation of a surveyor may be reckoned more healthy than any other mode of life in India, on account of the fresh air, constant exercise, and interesting nature of the employment, still there are some tracts of country in which trigonometrical survey parties are exposed to considerable risk from jungle fever and other diseases which prevail with greater or less virulence for several months in the year.

2. As these parts of the country must, however, be surveyed it becomes an important matter to consider the best sanitary arrangements for the protection of survey parties, from jungle fever in the first instance; and secondly, the proper treatment and line of conduct to be pursued in the event of the disease breaking out as an epidemic in camps.

3. That it is practicable, by judicious sanitary arrangements, to carry on operations with comparative safety for several months in such tracts, has been proved by the experience of the Survey Department in all parts of India; while it has also been demonstrated that by proper medical treatment, disaster may be prevented, and the establishment restored to efficiency after being attacked by jungle fever.

4. Although it may, at first, appear to be beyond the province of a non-medical person to enter on a subject of this nature, still surveyors are of necessity thrown upon their own resources, and compelled to qualify themselves, as well as they can, to perform this work of humanity, in addition to their own all-absorbing duties. Fortunately, the treatment of jungle fever, if taken in hand on the first appearance of premonitory symptoms, and the patient be not unnecessarily exposed to aggravating circumstances, is sufficiently simple to be learned by any

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* This section is reprinted from an Extract from Remarks on the Sanitary arrangements of Survey parties in the jungly tracts of India, by Colonel Sir Andrew Scott Waugh, Surveyor General of India; Dehra Dún, October 1861.

person of common sagacity, and will prove successful in nineteen cases out of twenty; though, if once the disease be allowed to gain head, it will require the utmost skill and refinement of the physician's science to reduce it.

5. Whatever rules are here given have been derived, in the first instance, from the advice of medical friends; they have been tested by experience, and are believed to be consistent with modern practice, as far as they go. When once a moderate degree of skill is acquired by experience, combined with the advice which medical officers are always willing to afford, it will be found that a degree of confidence will be inspired among all parties in camp,—a feeling indispensable to success in all arduous undertakings.

6. Although several cases of failure have occurred in the progress of the Indian surveys, owing to the disastrous effects of jungle fever, these have generally happened from the want of experience; and instances are numerous where the work has been carried out with success, notwithstanding the prevalence of this disease. There are even instances where one portion of a party has escaped on the same tract in which another portion has suffered, merely owing to the difference of habit and treatment.

Long fasts to be avoided.

7. The benefit of breakfasting the whole party before starting has been proved by the practice of a number of officers; and to this, as well as to the superior stamina derived from good living, may be attributed the circumstance, that Mussalmans are less subject to the influence of miasma than Hindus, who cannot conveniently cook oftener than once a day. Nevertheless, the latter class of people should be induced to eat parched grain or something similar to break their fast, or be compelled to cook a regular meal in the early morning, if employed in very insalubrious places.

8. Early in the season, when marching to the field of survey, and afterwards on returning to quarters, when the weather is very hot, it is usual to move at night, to save the men from exposure to sun. It is impossible so far to break up established habits as to breakfast in the middle of the night; but it is usual for officers to take a cup of coffee and a biscuit; and as a further precaution, if likely to be detained

out long, it is a good plan to fill the pockets with biscuits, ginger-bread-nuts, or something of that sort, and carry a bottle of cold tea or cold weak coffee without milk.*

9. In the survey season, surveyors are liable to be out all day, from morning till night. In such circumstances it is desirable to be accompanied with a basket containing sandwiches, &c.; for the great object is to avoid having to labour under a feeling of exhaustion arising from mere want of food—a state in which the human frame is peculiarly susceptible to malarious influences. For this reason the men should have a lunch of parched grain and sugar, when out at work through the whole day.

Predisposing causes in case of native establishment.

10. Now, it may be remarked that most officers take these precautions for themselves: they also sleep under canvas, protected from the dew, and upon beds raised above the exhalations of the ground. But the experience of the Department shows that a large proportion of the native establishment and servants may be laid up with fever and brought to the verge of death, without the officer in charge or assistants being affected in the smallest degree. This will always be the case, unless the Europeans are of weakly constitution, peculiarly susceptible to fever, or neglectful of their comforts.

Proper protection at night.

11. It is difficult in the case of Hindus to get them to cook their meals at proper times, but as far as possible it should be done; and all surveyors who are careful of their men are sedulous to protect them from dew at night, and to furnish them with straw to sleep on. The use of camel carriage in Upper India renders it easy to carry charpoys for some of the upper servants; and when men are detached to show signals from hill stations, they are always directed to hut themselves, which can easily be effected in the jungles at the expense of two or three rupees. In the case of a party recently employed in malarious tracts in the Central Provinces, the executive officer has furnished his men with tea, blankets and filters. The effects have been so to increase their efficiency that the cost of these comforts has been abundantly repaid by increased outturn of work.

* Make coffee as usual and pour into a bottle till half full, fill up with hot water and add sugar according to taste: a man can work all day on this beverage.

Quinine as a preventive.

12. In addition to the ordinary precautions above directed, it is usual in the case of persons very susceptible of fever to fortify themselves by taking quinine while they are employed in unhealthy localities; and this certainly has a good preventive effect.

Importance of open camping grounds and good water.

13. In order to avoid the particular spots which are unhealthy, all due enquiry should be made of the indigenous inhabitants, with whom relations of amity should be maintained. The cleared and inhabited parts should be preferred for camping on, or high open spots, and the vicinity of rivers avoided, as well as closely wooded places, where there is no circulation of air or sunshine on the ground. Every enquiry should also be made for good drinking water; when it is likely to be scarce, it should always be carried on from the last stage, in order that time may be given to find out the best water at the new encampment: many persons use boiled water—a precaution which even the native inhabitants practise in some parts. In some parts of India well water is considered the most wholesome, while that from streams is reckoned deleterious. In other places well water is considered to occasion fluxes, but this is only where the soil is impregnated with saline substances. As a general rule, in all tracts much covered with vegetation, river water is dangerous, and resort should be had to wells. But it must be recollected that the water in old wells not in daily use, and filled with rubbish, is always stagnant, putrid, and more dangerous than any other water.

Hospital Assistants.

14. Survey parties are usually attended by hospital assistants, who compound the medicines and act according to the orders of the officer in charge. These men are very useful in attendance on the sick, in general arrangement and hospital duties, and some are pretty skilful in the use of medicines; but, as a general rule, they are wanting in a strong mind to fall back on for advice in times of difficulty and danger, being either disposed to act with timidity or rashness. Be this as it may, the attendance of the officer on the sick prevents neglect, and cheers the poor men in their misfortunes. It sometimes happens, however, that the hospital assistant requires to be left at some distance, while observations are being taken on hill tops, moreover it usually happens that some of the subordinates being detached are unable to avail themselves of the

hospital assistant's services. Under such conditions in order to avoid the weighing and mixing of medicines which is not a very pleasant business at any time, and particularly irksome to a man fagged by a hard day's survey work, it is a good plan to furnish each person with a set of medicines prepared and weighed out beforehand, whereby all trouble will be saved.

Folly of neglecting reasonable precautions.

15. It may be said that the sedulous attention here recommended to ensure the comfort of every individual in camp would substitute, for the ancient hardy habits of the Indian surveyors, a sybaritic regard for luxury, inconsistent with the nature of the duties. It may look much more manly to be able to brave malaria on an empty stomach: but what is the use of hardy habits and contempt for comfort if no work is done? It is a very good thing to be able to brave an Indian sun with impunity; but it will be found that an umbrella will enable any man to accomplish more survey work than he could without that protection. The quantity of work is the only criterion of good habit and arrangement, and all other experience shows that, unless a party take the field well equipped with tents aud other comforts, health is endangered, and the interests of Government are sacrificed.

Healthiness of camp life.

16. The number of diseases to which a party under canvas are exposed are comparatively few, provided the health of the persons composing the party was good at starting, that the coolies &c., are well provided with warm blankets during the cold season, aud that every care is taken to keep the camp clean, dry, and within easy reach of a supply of good drinking water.

17. Most of the diseases to which survey parties are liable are treated of at considerable length in Moore's Family Medicine for India, a book which all European members of a Survey party should carry with them. The price of this work is Rs. 3 to all government officials receiving less than Rs. 500 per mensem, and Rs. 4 to all others, and to the public at large. A certificate declaring that the book is to be purchased only for the personal use of the officer is necessary for the lower rate. It may be convenient however to give here a brief description of the symptoms and treatment of the commoner forms of sickness prevalent in survey camps.

Fever.

18. Malarious fevers are beyond question the diseases which most tend to interfere with the progress of Survey parties in India; and a clear conception as to how they should be treated in a general way in cases of emergency is of very great value to every person liable to exposure to malarial influences. The fevers are referred to under various designations in this country, such as *Fever and Ague, Terai fever, Marsh fever, Bengal fever, Jungle fever, &c.* The affections commonly referred to by these names correspond to what medical men describe as *Intermittent* and *Remittent* forms of malarious fevers. By intermittent fever is meant that form of fever which, whilst recurring daily, on alternate days, or every third day, leaves the patient fever-free in the intervals; there is an *intermission* of feverish symptoms—in the first case of one day, in the second of two, and in the last of three, more or less, clear days. In the case of remittent fever, however, the feverishness does not completely disappear between paroxysms, there is simply a diminution—a *remission* of the attack. It is the severer attacks of this latter form of malarious disease that is usually meant when the terms *jungle* and *terai* fever are used, whilst the former is commonly referred to as *ague*.

It is, however, frequently difficult, even for the experienced physician, to decide definitely whether a person is suffering from the intermittent or the remittent form of the malady, as the symptoms often 'shade off' one into another—the fever being intermittent for a few days, then remittent, and again, perhaps, intermittent.

Fortunately the general line of treatment to be adopted is the same in both forms, and it will probably be found that when working in some malarious localities, the two forms of the disease may prevail simultaneously.

Prophylactic measures.

19. When it has been decided to visit a locality known to be productive of fever, it should be laid down as a general rule that every member of the party should receive small doses of some cinchona preparation during a few days previously, and twice daily during the stay in the locality. Two to three grains of quinine should be taken morning and evening, or, what is equally efficacious and very much cheaper, the same quantity of the *Cinchona febrifuge*, prepared at the Government cinchona

plantation. The expense of a prophylactic measure of this kind would be very trifling, especially if the *febrifuge* be resorted to.

Intermittent fever.

20. An attack of ague consists of three more or less distinctly marked stages: a *cold* stage, often not well marked, but it may be severe and last from one to four hours; a *hot* stage, which may last from three to six hours—seldom more than twelve; and a *sweating* stage, which terminates in a couple of hours, and is followed by complete relief. The average duration of the attack is from five to six hours.

Treatment Intermittent fever.

21. When the slightest symptoms exist of an impending attack of fever, the condition of the bowels should be attended to, and if any tendency to constipation exist, a mild purgative may be administered—a small dose of castor oil being the safest. Should it be considered advisable to administer a purgative, wait for an hour after its administration, and then give 10 to 15 grains of quinine [or an equivalent dose of the cinchona febrifuge], and repeat the dose in about six hours. Early and vigorous treatment of this kind, if not always successful in warding off the attack altogether, tends greatly to diminish its severity.

Should the symptoms continue, and shivering set in, marking the advent of the *cold* stage, the patient should be well wrapped in blankets; hot bottles, or hot bricks wrapped in flannel, should be applied to the feet, and warm tea, toast and water, or some such simple fluid given him to drink. Should constipation have existed which the castor oil had not relieved, a mild purgative may again be administered during this stage. Say a drachm of compound jalap powder in a wine-glassful of water, or a podophyllin pill.

Should, however, there be only slight constipation, await the advent of the *hot* stage, and administer two teaspoonfuls of granular citrate of magnesia or a laxative dose of pyretic saline in half a tumbler of water. Half a teaspoonful of this granular preparation, added, as required, to a little sweetened water, and administered every hour or so, will form a pleasant effervescing febrifuge. If, however, the bowels are relaxed, it will be better to keep to cold tea or toast and water, according to the patient's inclination. The bed clothes may be gradually lessened as the hot stage progresses, and attempts should be made to relieve headache by applying wet cloths to the head.

As soon as the *sweating* stage sets in, 10 grains of quinine [or an equivalent quantity of the Cinchona febrifuge] should be given every eight hours or so during the *intermission*. The quinine (or the cinchona febrifuge) may either be taken in powder, or dissolved by means of half a teaspoonful of the juice of a lime, and taken in a wine-glassful of water. Care should be taken to avoid a chill during the hot and sweating stages. Should diarrhœa be present, a dose of chlorodyne may be administered during the intermission, and each dose of quinine combined with 3 grains of Dover's powder.

Treatment of Remittent, or jungle fever.

22. The general rules thus laid down for the treatment of intermittent fever are equally applicable to the treatment of the generally more serious form of malarious disease called jungle fever. The premonitory symptoms should be treated on precisely similar principles, with the addition that it is more advisable to abstain from partaking solid food shortly before an attack is anticipated, as vomiting, which is a prominent symptom here, is apt to be more aggravated than when the stomach is empty.

Should the mild purgative and preliminary doses of quinine not have sufficed to ward off an attack, the cold stage will be ushered in by more or less distinctly marked shivers; but it will be found that, as a rule, this stage is less marked here than in an ordinary attack of ague; the succeeding or hot stage, however, is more prolonged, and generally more severe. The temperature runs high, the pulse is very quick, often bilious vomiting of a very obstinate character occurs; the tongue is furred, and sometimes jaundice sets in. The patient is very restless, and his remarks often incoherent.

This stage may last from six to eight hours or longer, and, instead of terminating in severe sweating, followed by complete relief from all painful sensations, as in intermittent fever, there is only a subsidence in the severity of the symptoms—a *remission*, not a cessation, of the fever. The duration of the paroxysm is uncertain—may last from 6 to 24 hours, and an attack may recur daily for a week, 10 days, or even longer.

When the cold stage has passed away, and the pulse becomes full and the face flushed, administer frequent small doses of the granular citrate of magnesia as above directed, and apply cold to the head: avoid stimulants of all kinds, but attempts should be constantly made to get the patient to partake of beef tea, Liebig's extractum carnis, and such like nutritious food in a liquid form.

If the patient becomes delirious, the lower part of the back of the head should be shaved, and a blister (about 2 inches × 3 inches) raised, by means of blistering fluid, across the nape of the neck. If this be of no avail, and the pulse continues full and bounding, and the patient be a strong man, four to six leeches may be applied to the temples—the bleeding being arrested when the leeches have dropped off. It is advisable to defer giving quinine until the remission sets in. This will be known by the diminution in the temperature, and the appearance of a gentle perspiration, together with, probably, the disappearance of the head symptoms. Five grains of quinine may mow be given every three hours in the form of a mixture, or dissolved in the juice of a lime with water. Nourishing food should be given, with a very little stimulant, if desired. Should the patient suffer from diarrhœa also, the 5 grain quinine doses given during the remission may be combined with 3 grains of Dover's powder. If the head symptoms have been severe, it will perhaps be advisable to keep to quinine, but otherwise an equivalent dose of the cinchona febrifuge may be safely substituted for it.

If the attack has been severe, have the patient removed, if practicable, as quietly and as expeditiously as possible, to the nearest station where medical aid can be procured. This is especially desirable, if the disease has recurred two or three times without any appreciable diminution in the aggravation of the successive attacks.

Treatment of diarrhœa and dysentery.

23. Diarrhœa and dysentery sometimes appear suddenly in a camp, and in aggravated forms, and require prompt attention, especially as the former is very apt to pass into the latter in malarious districts.

The first precaution to be taken on the advent of diarrhœa is abstinence from solid food; nutritious food in a liquid, or semi-solid form should be substituted, and cold drinks should be avoided. Should the diarrhœa have been immediately preceded by constipation, a small dose of castor oil should be taken, and possibly after this has acted, the bowels may return to their ordinary condition.

If, however, the diarrhœa has not been preceded by constipation, 30 drops of chlorodyne may be taken in half a wine-glassful of lukewarm water. Should there be pain in the bowels, warm water fomentations should be applied to the stomach, and a flannel belt tightly drawn round. Three or four hours after taking the chlorodyne, 5 grains of quinine, or 5 grains of the cinchona febrifuge should be taken in combination with 5 grains of Dover's powder, and the patient sent to bed. It is probable that after prompt action of this kind, the patient may find himself perfectly well by the next morning: if not, let the chlorodyne be repeated, followed by the quinine and Dover's powder, twice daily, for a day or two, or until perfect recovery.

Should the diarrhœa pass on to dysentery, which may be inferred if irregular attacks of griping pain in the abdomen persist with increasing tendency to strain at stool, the condition of the stools themselves should be carefully noted, and if traces of blood or fleshy shreds be observed, it is absolutely necessary that complete rest should be enjoined. The patient should be given a grain of opium, or 30 drops of chlorodyne in a little water, to be followed in about quarter of an hour by 20 grains of ipecacuanha powder in half a wine-glassful of water, or the powder may be given as a bolus. This medicine will probably be succeeded by nausea, but every attempt should be made to keep it down. This is best done by the patient remaining on his back; indeed throughout the attack the patient should be made to lie down. Repeat the ipecacuanha powder in about three hours, and continue the dose twice a day for two or three days, or until the disease stops. Hot bran or linseed meal poultices should be applied constantly to the stomach; and should there be severe pain, apply a turpentine stupe (made by pouring about a tablespoonful of turpentine on a piece of flannel wrung out of hot water) over the painful part.

If the patient does not recover in about a week after a fair trial of this treatment, he should be carried to the nearest station where medical aid may be procured.

Cholera.

24. When cholera breaks out in a camp, the first step to be taken is, to shift the tents to some considerable distance; and if this does not suffice, shift again, across a river if practicable: of course the greatest possible attention should be paid to the sanitary state of the camp, and especial care taken during periods of this kind in particular to avoid unwholesome food and impure water.

Treatment of cholera.

25. Unfortunately no very certain remedy is known for this disease, but much may be done by early treatment. The premonitory diarrhœa, if present, should be treated with 30 drops of chlorodyne every three or four hours, and complete rest enjoined. Should, however, the stools become watery in appearance and mixed with bran-like flocculi, the pulse become small and thready, the skin blue and pinched, and the voice husky, 10 drops, of the '*Cholera drops*,' supplied by the Medical Stores Department, should be given every half an hour in a tablespoonful of water. Or, if this medicine be not at hand, 15 drops of sal volatile should be given instead, in half a wine-glassful of water.

When severe cramps of the limbs or of the muscles of the chest come on, the painful parts should be sedulously rubbed with a mixture of ginger (or mustard) powder and linseed meal, easily obtainable in any bazaar, and cold drinks (soda water and the like) given when desired to quench the great thirst, from which the patient will in all probability suffer.

As soon as reaction sets in, the pulse regain its strength, and the unexposed parts of the body become warmer to the applied hand, the stimulating medicine (whether 'cholera drops' or ammonia) should be discontinued. Attempts should be made to get the patient to swallow a little beef tea, Liebig's extractum carnis, or such other nutritious food as may be available of an allied character. Occasionally, small effervescing draughts, made by adding half a teaspoonful of citrate of magnesia to a little sweetened water may be given. Should the urinary functions not have become restored towards the end of the second day, the loins should be well fomented with warm water, followed by a turpentine stupe applied over the kidneys, with a linseed or bran poultice over the bladder. Should indications of stupor be observed, and the tongue become dry, the back of the head should be shaved, and the nape of the neck painted two or three times with blistering fluid. As a rule, when the urinary secretion becomes re-established, convalescence is rapid.

Sun-stroke.

26. The Europeans of a party are more liable to suffer from sun-stroke than the natives, and it not uncommonly happens that when one attack has occurred, it is

followed during the next few days by others. It is therefore especially necessary that the tents should be pitched in shaded positions, and that great care should be taken to avoid unnecessary exposure to the sun's influence for some days after a case of this kind has occurred.

No very clear description can be given of the symptoms which precede an attack, and very frequently none whatever are observed: a person may become suddenly faint, and fall without having given any previous intimation of his state.

Treatment of sun-stroke.

27. The *treatment* for such a case is, to lay the patient on his back in the shade, have his limbs well rubbed, and a dose of sal volatile (40 drops) in water administered, or, if ammonia be not at hand, a little brandy and water may be substituted.

Should, however, the face get flushed, the pulse full, and the heat of the body to the applied hand very great, with stertorous respiration, the *bhisties* should be called and directed to pour a continuous stream of cold water from their *mussucks* upon the patient's head for several minutes, the head being, meanwhile, kept well elevated. Should this procedure not prove successful in improving the patient's condition, the back of his head should be shaved, and blistering fluid painted over the part, so as to raise a blister about the size of a man's hand.

Two drachms of compound jalap powder should be administered in a wine-glassful of water, followed in the course of half an hour by 20 grains of quinine, in solution if possible. Attempts should also be made to clear the bowels by means of warm water enemata, if practicable. During recovery absolute rest must be insisted on, nourishing food should be given, with only a minimum of stimulants, and the patient not allowed to resume work in the sun for some weeks.

Supply of medicines for detached parties.

28. A short list is appended of the medicines which would be found useful in cases of emergency of the kind referred to above. Only the simpler kind of remedies have been mentioned, as it would be useless to do more than suggest the general line of treatment which meets with the approval of the majority of the profession. Were more than this done, it would tend to confuse non-professional persons, and an elaborate list of remedies would imply a great addition to the weight to be carried.

A LIST OF MEDICINES, &c., WHICH, IT IS SUGGESTED, SHOULD BE TAKEN BY SMALL DETACHED SURVEY PARTIES OF ABOUT 25 MEN.

½ oz. *Sulphate of Quinine.*

2 oz. '*Cinchona Febrifuge*' or 'Mixed Cinchona Alkaloids.'

1 oz. *Chlorodyne.*

2 oz. '*Cholera Drops,*' as issued by Medical Stores Department. Ol. Anisi, Ol. Cajeput, Ol. Juniper, each ½ ounce, Æther ½ ounce; Liquor Acid. Halleri ½ ounce; Tinct. Cinnam. 2 ounces; mix. Useful as a stimulant in the cold stage of cholera, and in other cases where a stimulant is desirable: 10 to 15 drops in water for a dose.

2 oz. *Sal Volatile* (Sp. Ammon. Aromat).

8 oz. Granular *Citrate of Magnesia.*

8 oz. *Epsom Salts.*

8 oz. *Castor Oil.*

2 oz. *Compound Jalap Powder:* a brisk purgative: dose 1 or 2 drachms.

1 oz. *Dover's Powder.*

25 One-grain opium pills [in a well-corked phial].

50 *Podophyllin pills* [Recipe, Podophylli Resinæ gr. ¼; Extr. Hyoscyam. gr. 1; Pil. Colocynth Co. gr. 3. Make a pill]. Preserve in a well-corked phial.

50 Two-and-a-half grain *Quinine pills.* In a well-corked phial.

25 *Gallic acid* (three grains) and *opium* (half a grain) *pills.* In a well-corked phial. Useful in mild cases of diarrhœa: one or two for a dose.

1 oz. *Ipecacuanha Powder.* For treatment of dysentery.

1 oz. *Blistering Fluid.* [In order to produce a blister, paint the part three or four times with a feather].

2 oz. *Solution of Acetate of Lead* [Liq. Plumbi Subacetas]. For making Goulard's Lotion, add about a tablespoonful to a pint of water. A useful application to sprains and to indolent sores.

1 oz. *Tincture of Steel.* Useful for stopping hæmorrhage. Dip a cotton rag into the solution, press it into the wound, and apply a bandage.

1 oz. *Tincture of Iodine.* Useful in reducing swollen glands (buboes) in the armpit, groin, &c. Paint the part twice daily with the tincture.

4 oz. *Zinc Ointment*, in a wide-mouthed stoppered bottle. Useful for sores, burns, scalds, &c.
3 Rolls of 1½-inch calico *bandages*.
1 Stick of *Caustic* (Nitrate of Silver,) mounted in a quill and put into a phial. Useful for sores, bites, &c.
1 Small roll of *Isinglass adhesive plaster* (*i. e.*, Court plaster). This is preferable to ordinary sticking-plaster, as the latter, owing to the heat, often becomes useless. To be moistened before application.

Care of medicines.

29. As several of the powder-medicines deteriorate greatly when kept in paper packages, and frequently, owing to damp, &c., become useless when thus previously weighed out, it is recommended that phials should be adopted in all cases and the doses weighed as required; a pair of small scales and a 2-oz. graduated glass measure being packed with the medicines. A small tin box, divided into compartments, could readily be devised; and the whole need not weigh more than 8 or 10 pounds.

Apothecaries' Weights and Measures.

Solids.

20 grains	=	1 scruple.
3 scruples	=	1 drachm.
8 drachms	=	1 ounce.
12 ounces	=	1 pound.

Fluids.

60 minims	=	1 fluid drachm.
8 drachms	=	1 fluid ounce.
20 ounces	=	1 pint.

Approximate Fluid Measures.

1 minim	=	about 1 drop.
1 fluid drachm	=	1 teaspoonful.
2 ,, drachms	=	1 dessertspoonful.
4 ,, drachms	=	1 tablespoonful.
1 ,, ounce	=	2 tablespoonfuls.
2 ,, ounces	=	1 wine-glassful.

CHAPTER III. CARE AND TREATMENT OF ELEPHANTS.

Necessity of supervision.

1. In many parts of India and Burma elephants are almost indispensable for Trigonometrical survey parties, when employed on Triangulation, and as both the purchase and keep of these animals form a somewhat heavy charge, it is very desirable that executive officers should themselves exercise a careful supervision over their treatment in order to ensure their efficiency.

Two principal causes of disablement.

2. It would be impossible to enter here into all the diseases to which elephants are subject, but there are two which are the most frequent cause of temporary disablement, and which are generally preventible by reasonable care. These two are, (1) injuries to the feet, and (2) sore backs.

Injuries of the feet are very liable to occur in marching over ground from which bushes and small jungle have been cut down, with short sharp stumps left standing in the ground. An elephant treading on these, or even on small sharp pieces of stone, may cripple itself more or less: when such a road has to be traversed it is a good plan to harden the soles of the feet by the application of a paste, of which the ingredients are known to mahouts, and generally used by them for this purpose.

Injuries to the feet, and their treatment.

3. There are two kinds of disease peculiar to the feet of elephants, named 'Tawakh' and 'Sarjan'. They are both contracted through neglect, and from allowing the animal to stand for many consecutive days on the same 'tahan' without cleansing it. The best treatment is by nitric acid, applied with thin strips of copper, which should be dipped into the bottle containing the acid, and rubbed over the wound. The feet must be well pared and washed first; the acid should be applied morning and evening.

Countinued.

4. One of the best native remedies for 'Sarjan' is the following:— 'Tua' or 'Chowdara', one fruit, cut up, powdered, and mixed with one seer of mustard oil; this must be kept on the fire until it burns, and becomes thick and black. The mixture is spread over the wounded surface whilst it is quite hot. An ordinary paint-brush is the best means of applying it.

Carbolic lotion is useful for dressing wounds on the feet of the elephant, and should be applied on plugs of tow inserted in the wounds after they have been carefully cleaned out, and all proud flesh cut away.

Sore backs, their cause and treatment.

5. Sore backs are generally caused by the padding being out of order, in consequence of which the loads press directly on the back, and thus occasion inflammation, and very frequently suppuration to an extent that sometimes require months to cure. If on the first symptom of a swelling being observed, the place is well fomented and rubbed, a sore back may probably be avoided. But mahouts, as a class, are proverbially unobservant and heedless in the performance of their duties; and the chances are that the swelling will not be noticed by them, until it becomes so large that it is impossible to avoid seeing it. The mischief is then done, and suppuration, which must be allowed to take its course, has most probably set in. These suppurating abscesses usually take place a little behind the tips of the shoulder blades. When they are quite ripe they should be opened with a long lancet-shaped knife, four incisions in the lower surface being made—two on either side. It is better to open them thoroughly at first, otherwise the pus is liable to find its way into the surrounding cellular tissue, and an enormous diffused abscess is the result. After the matter has been well pressed out, the cavity should be thoroughly syringed with lukewarm water, twice a day, morning and evening—the cavity being filled between the syringings with tow steeped in carbolic acid lotion of the following strength, *viz.*, 1 oz. of carbolic acid to 10 oz. of water. The wound should be constantly protected by a broad pad of cloth, otherwise its cure will be considerably retarded by the habit elephants have of blowing dust or dirt over their bodies—more particularly when they have sores on any part of them.

Elephant gear discribed.

6. In the case of sore backs prevention is easier than cure, and as the "gadhela" or pads are generally the cause, too much attention cannot be given to ensure their being kept in a proper condition. They are very liable to become rotten when exposed to rain, and should therefore be always kept under cover when not in actual use; a single month of monsoon weather is sufficient to destroy them, if recklessly exposed to it. The pads and other furniture at present in use for the elephant are as follows:—

1st. The "namda" which consists of hair well felted together, is about an inch thick, and two yards square, and has a covering of gunny on the upper side, and one of coarse cloth on the under. The latter comes in immediate contact with the animal's back.

2nd. The "gadhela" placed on the top of the namda, consists of two bags of gunny filled with bulrushes, a foot thick, and two feet broad, the length being four to five feet, according to the size of the animal. These bags are joined edge to edge at either extremity, the open space between them being intended to receive the spinal ridge of the animal's backbone.

3rd. The "Nímgadhi" which is of similar construction to the gadhela, but smaller in its dimensions.

4th. The "jhúl" a coarse cloth consisting of gunny which is thrown over the whole.

5th. A rope about one inch in diameter by which the above are fixed on the back—one end of this rope is converted into a loop, sufficiently long nearly to surround the body of the animal and the pads. It is tied on the top, then passed singly round the neck, then along the upper sides of the pads, then below the tail, and finally tied again at the place of the first knot on the top. The parts which pass under the belly and tail are sheathed with leather, to prevent abrasion of the elephant's skin.

Stoppage of attendants' pay.

7. It may be assumed as a principle that sore backs and injuries to the feet of elephants are the result of negligence on the part of the attendants, and it is therefore a good rule to enforce, that so long as the animals are under treatment for such injuries, half of the attendants' pay should be stopped; unless good cause can be shown, in individual cases, why the rule should not be put in operation.

Head pads.

8. Elephants are occasionally very useful in pushing down trees; when so occupied pads should be provided to defend their heads. If an elephant bruise or injure itself when so employed, and this it is liable to do, it will probably refuse to give assistance again in this way.

Elephants to be picketed in the shade.

9. Elephants should not be picketed out in a burning sun; inflammation of the brain, and other similar diseases may be caused by such exposure. If shade be not available a white padded covering should be fastened over the head.

Precautions against chills.

10. On coming off a march, an elephant should be allowed to cool previously to being washed; negligence in this particular is apt to occasion serious illness. The animal may be allowed to drink water when in a heated state without any untoward effects resulting, but it should not be allowed to throw water over itself.

Watering elephants.

11. Elephants should be watered twice daily; at each time they take about 15 or 16 gallons of water. They prefer river water to all others, and willingly drink that procured by digging holes in the sandy beds of rivers, after it has stood for a few minutes to allow the sediment to sink. Elephants cannot be kept with impunity for more than 24 hours without water.

CHAPTER IV. EQUIPMENT, NATIVE ESTABLISHMENT, &c.

Constitution of native establishment.

1. The entertainment of low caste men for the greater portion of the native establishment such as "kahárs", "korís", "chamárs" &c., is advisable, though it is useful to have a small proportion of Brahmins and Muhammadans. Young hands should be entertained for survey work, and executive officers should be careful not to employ old men, unless of superior attainments and merit. Every khalási should be provided with a service book on joining. All inefficient men should be discharged at the close of the field season. Others not absolutely required in recess-quarters, or at the field depôt, may be allowed leave of absence during the recess on such portion of their salary, not exceeding half, as may be deemed advisable by the executive officer, to induce them to return to the party at the commencement of the next field season; a certificate to this effect being furnished to each man, showing the date and place ordered for his return. This leave does not constitute a break in service.

Guarding stores.

2. Stores and public property should not be kept in a closed tent, with a single sentry outside; for such an arrangement is unsafe, and unfair to the man on guard. The most secure method of guarding property is to collect it in an open spot, from which the sentry can have a clear view on all sides; the articles should be raised above the influence of damp ground and of white ants by means of stones and bricks. Delicate instruments liable to injury from exposure can be guarded securely if placed in a "shuldári" or open "pál." When the strength of the guard is insufficient for furnishing a double night sentry, the khalásis of the establishment must take their turn of such duty.

Office work when marching.

3. Assistants are not exempted from all office work for the days on which they may have marched. The daily office hours will be regulated with due consideration to the length of the day's march, and to the exigency of the work in hand. Every detail concerning the arrangement of camps and order of marching is under the control of the officer in charge of the party.

Duration of field season.

4. The field season generally commences about the 15th of October, and lasts about six months, but there are few districts in India or Burma, where this whole period can be utilized. In malarious tracts little benefit is gained by commencing field work before the middle of December, and in others, such as Rajputana, out-of-door work becomes almost impossible for Europeans after April. In Burma little in the way of triangulation or reconnaissance can be done after the middle of March, owing to the dense haze which obscures all distant points. Executive officers must apply for orders from the Surveyor General, or the Deputy Surveyor General of their branch, as to the times of taking and leaving the field. They are on no account, unless by special sanction, to return to recess-quarters until all the instruments and stores have been properly stowed away in the field depôt, and all accounts with the field establishment properly adjusted.